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Multiscale probability distribution of pressure fluctuations in fluidized beds

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Abstract. Analysis of flow in fluidized beds, a common chemical reactor, is of much current interest due to its fundamental as well as industrial importance. Experimental data for the successive increments of the pressure fluctuations time series in a fluidized bed are analyzed by computing a multiscale probability density function (PDF) of the increments. The results demonstrate the evolution of the shape of the PDF from the short to long time scales. The deformation of the PDF across time scales may be modeled by the log-normal cascade model. The results are also in contrast to the previously proposed PDFs for the pressure fluctuations that include a Gaussian distribution and a PDF with a power-law tail. To understand better the properties of the pressure fluctuations, we also construct the shuffled and surrogate time series for the data and analyze them with the same method. It turns out that long-range correlations play an important role in the structure of the time series that represent the pressure fluctuation.

Keywords: turbulence, stochastic processes (theory), computational fluid dynamics

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1. Introduction

Fluidized bed (FBs) are packed beds of particles through which a fluid or fluids flow at such high velocities that the particles are loosened and suspended in the fluid. A stream of gas blows upward through a layer of fine solid particles and sets them in motion. At a certain gas velocity, usually referred to as the *minimum fluidization velocity* $v_{\rm m}$, the gravity and drag force are balanced, leading to the suspension of the particles without being transported. If the gas velocity exceeds $v_{\rm m}$, then the excess gas flows through the bed of the particles as bubbles, the bed is said to be fluidized, and the mixture of the fluids and particles acts as though it is a fluid. Fluidized beds represent a common chemical reactor that have been studied for a long time [1]–[4].

Despite being seemingly simple, the fluid mechanics of the FBs, which represents a two-phase system, is extremely complex. Given that the FBs are used in combustion of solid fossil fuels and biomass, many exothermic reactions in the chemical industry, oil refinery operations, and biochemical and environmental cleanup processes, as well as in heating, cooling, drying, and coating particles in the pharmaceutical industry, gaining better understanding of the fluid mechanics of the FBs is highly important.

Similar to many other fluid mechanical systems, the dynamic characteristics of the FBs manifest themselves in terms of the fluctuations of their properties, and in particular density and pressure of the system. Moreover, the FBs exhibit highly stochastic behavior, which is due to a variety of factors, including the jetting and bubbling of the FB, and the motion of the fluidized particles. The stochastic nature of the properties of the FBs and the fluctuations in the pressure and density give rise to complex time series, the analysis of which is fraught with difficulties and pitfalls. As is well known, the analysis of stochastic time series remains an active area of research [5]. One purpose of the present paper is to present a method of analyzing such data that arise in FBs.

Due to the stochastic nature of the pressure in a FB, it is natural to develop statistical methods for investigating the times series that represent it. Indeed, research

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efforts in this direction have a relatively long history [6]–[9]. Various approaches for analyzing the pressure fluctuations have been proposed, including the classical continuum equations [6]–[9], the idea [10, 11] that the pressure fluctuation series follow a fractional Brownian motion [12, 13], or that the probability density function (PDF) of the fluctuations [14] may be described by a Tsallis distribution [15].

In the first half of the 1990s, Daw and co-workers [16, 17] and Schouten and van den Bleek [18] proposed the application of techniques from chaos analysis to pressure fluctuation data from a FB, which were subsequently applied extensively in the second half of the 1990s. An attractive feature of the approach was that it could handle FB dynamics in a way that was different from many conventional methods: rather than a reductionist approach to capture the movement of all the individual particles, it considered the spatio-temporal patterns encountered in a FB as a whole. But, similar to many other real-life systems, such as climatological, economical and physiological ones, it is extremely difficult, if not impossible, to prove that a FB is a chaotic system, or that it exhibits certain chaotic features [19]. For example, van der Schaaf *et al* [20, 21] showed that the interpretation of the entropy of pressure series of a FB should be done with great care.

On the other hand, Gheorghiu *et al* [22] proposed the study of the PDF of the successive pressure increments, $\Delta p = p(t + \Delta t) - p(t)$, over a variable time delay Δt, instead of the pressure series itself. The study of such increments was inspired by signal processing methods in turbulence research, where the relevant variable is typically the velocity fluctuations. The advantage of using the pressure increments is that the resulting series is robust to long-term trends and that, implicitly, the time scale and dynamics of the pressure are included. Gheorghiu *et al* [22] argued that the PDF is not Gaussian, but has a power-law tail. Bai *et al* [23] argued that the PDF may be well described by the Student's t-distribution, and claimed that the fundamental origin of the power-law distribution of pressure fluctuations is the broad, power-law nature of the bubble or void size distribution, and not any spatial or temporal long-range correlations that are often proposed as the cause of power-law statistics. The bubble polydispersity was attributed to bubble growth that involves coalescence and breakup.

In a recent paper [24] we analyzed extensive new data for pressure fluctuations in a FB, and demonstrated that the data possess multifractal properties. In the present paper we shed further light on the properties of the pressure fluctuations time series by computing the evolution with the time of a multiscale PDF for the data.

The rest of this paper is organized as follows. In section 2 the data and their measurement are described. Section 3 describes the new method of analyzing the data. Section 4 analyzes the PDF of the successive increments of the pressure data by constructing its shuffled and surrogate series. Section 5 describes briefly the similarities between pressure fluctuations in the FBs and local velocity fluctuations in turbulent flows. The paper is summarized in section 6.

2. Measurement of the data

The data [24] were measured in a FB of sand particles, with an inside diameter of 80 cm, a height of 93 cm, and a bed mass of 700 kg. Sand particles of diameters 356, 532 and 760 μm were used. The superficial gas velocity was 40 cm s^{-1}. The bed was in the bubbling fluidization regime (also called 'freely' bubbling bed). The data were measured at the wall

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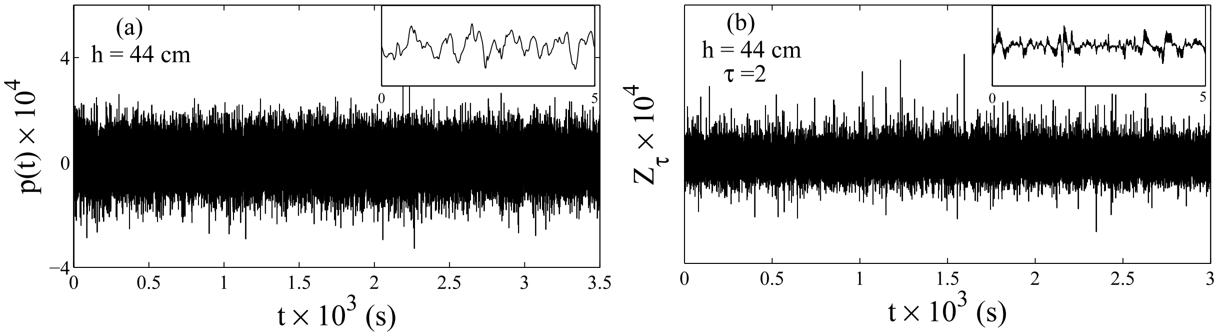

Figure 1. (a) The data at the depth of 44 cm. (b) The detrended increments of the data shown in (a).

at 34, 44, and 54 cm above the bottom of the FB with a measurement frequency of 200 Hz. The pressure fluctuations were recorded and the average was set to zero. In each experiment the local pressure fluctuation measurements were performed, following the procedure described by van Ommen and Mudde [25]. The pressure probes were 10 cm long with an internal diameter of 4 mm, which guaranteed an undisturbed transfer of the signal in the frequency range of interest. The end of each probe was covered by a wire gauze in order to prevent the fluidized particles from intruding and blocking the probe, and thus affecting the pressure measurements. The gauze has no effect on the pressure signal. Piezoelectric pressure sensors of the Kistler-type 7261 connected to the probes were used to measure the pressure fluctuations. The time series were then low-pass filtered with a cutoff frequency of 60 Hz. 16 bits analog-to-digital conversion was subsequently applied at a sample frequency of 200 Hz. The pressure fluctuation time series with a duration of 60 min were measured. Each of the three data sets contained a little over 720 000 data points. Figure 1 presents the data at height $h = 44$ cm. For clarity we also show in the inset the fluctuations over a much shorter time scale. The data at the other two heights are similar to those shown in figure 1 [24].

While the pressure fluctuations are straightforward to measure, their interpretation is not so. It has been shown that the pressure series is a combination of the local bubble passages and nonlocal compression waves, with the latter resulting from a number of hydrodynamic phenomena. When a gas bubble rises upward through the FB and passes the measurement point, the pressure fluctuates with a characteristic shape that is described by the so-called Davidson model [1, 2]. The model assumes that the FB is infinitely wide. Since small diameter columns are typically used in such experiments, the effect of moving bed mass must be taken into account [20, 21]. In the lower part of a bubbling bed the bubble passage makes a relatively small contribution to the pressure fluctuations. But, the contribution becomes more significant when moving upward [20, 21], because the bubble diameter increases with increasing height, whereas the amplitude of the compression waves decreases with increasing height.

On the other hand, compression waves can propagate both upwards and downwards through the bed. The amplitude of a downward-traveling compression wave remains constant, if the diameter of the bed is small, whereas that of an upwards-moving wave decreases linearly and vanishes at the bed [19]. The fast-traveling compression waves may originate from bubble generation, coalescence, and eruption, as well as being due to fluctuations in the gas flow.

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3. Analysis of the data

Before embarking on the analysis of the data, we should point out that an important point must be taken into account when analyzing any time series. Recall that if we have an ensemble of data sets, the assumption of homogeneity of the variance implies that within each of the data sets the variance is the same. But, for a single data set, homogeneity implies that if we divide the set into many segments, the variances of all the segments are equal and, thus, it is essentially equivalent to a second-order stationary process. On the other hand, if neither of the two conditions is fulfilled, slow convergence to a Gaussian or even a non-Gaussian PDF with fat tail—one with a long tail such that the stochastic variable has a nonvanishing probability of occurring—can arise. To quantitatively characterize the non-Gaussian property at any scale, the standardized PDF (variance normalized to one) is constructed.

The data are first detrended in order to remove any possible trends in the time series, and then are processed by the procedure described below. To do so, we fit $p(t)$ in each interval $[1+\tau(k-1), \tau k]$ of length 2τ (where k is the index of the box) to a polynomial function of order three. Higher-order polynomials may also be used, but as pointed out by Kiyono *et al* [26, 27], there is no significant difference between the detrended data with respect to the order of detrending polynomials if the order is greater than two. This is also in agreement with our past experience with detrending many distinct types of time series [5]. Thus, in the following analysis we use third-order polynomials that are accurate and computationally efficient. The fitted polynomials represent the trends in the corresponding segment. The detrended increments on the scale τ are defined by $Z_\tau(t) = p^*(t+\tau) - p^*(\tau)$, where $\tau \in [1+\tau(k-1), \tau k]$, with $p^*(t)$ being the detrended series, i.e. the deviation of $p(t)$ from its fitted values. Figure 1 also presents the data for the detrended $Z_\tau(t)$ for $\tau = 2$ with its values over a short time scale in the inset.

We now argue that for a fixed time t, the fluctuations at scales τ and λ_τ are related through the cascading rule

$$Z_{\lambda_\tau} = W_\lambda Z_\tau(t), \qquad \forall\tau, \quad \lambda > 0, \tag{1}$$

where $\ln(W_\lambda)$ is a random variable that relates the two scales. Iterating equation (1) forces (implicitly) the random variable W_λ to have a log infinitely-divisible distribution [28]. It has been demonstrated [26, 27, 29] that one of the simplest candidates for the cascading process (1) is a non-Gaussian PDF with fat tails that is modeled by a random multiplicative process

$$Z_\tau(t) = \chi_\tau(t) \exp[\omega_\tau(t)], \tag{2}$$

where χ_τ and ω_τ are both Gaussian random variables with zero mean and variances σ_χ^2 and σ_ω^2, respectively, which are independent of each other. The PDF of $Z_\tau(t)$ has fat tails, depending on the variance of ω_τ, and is expressed by [30]

$$P_\tau(Z_\tau) = \int \frac{1}{\sigma} F_\tau(Z_\tau/\sigma) G_\tau(\ln\sigma)\, \mathrm{d}\ln\sigma, \tag{3}$$

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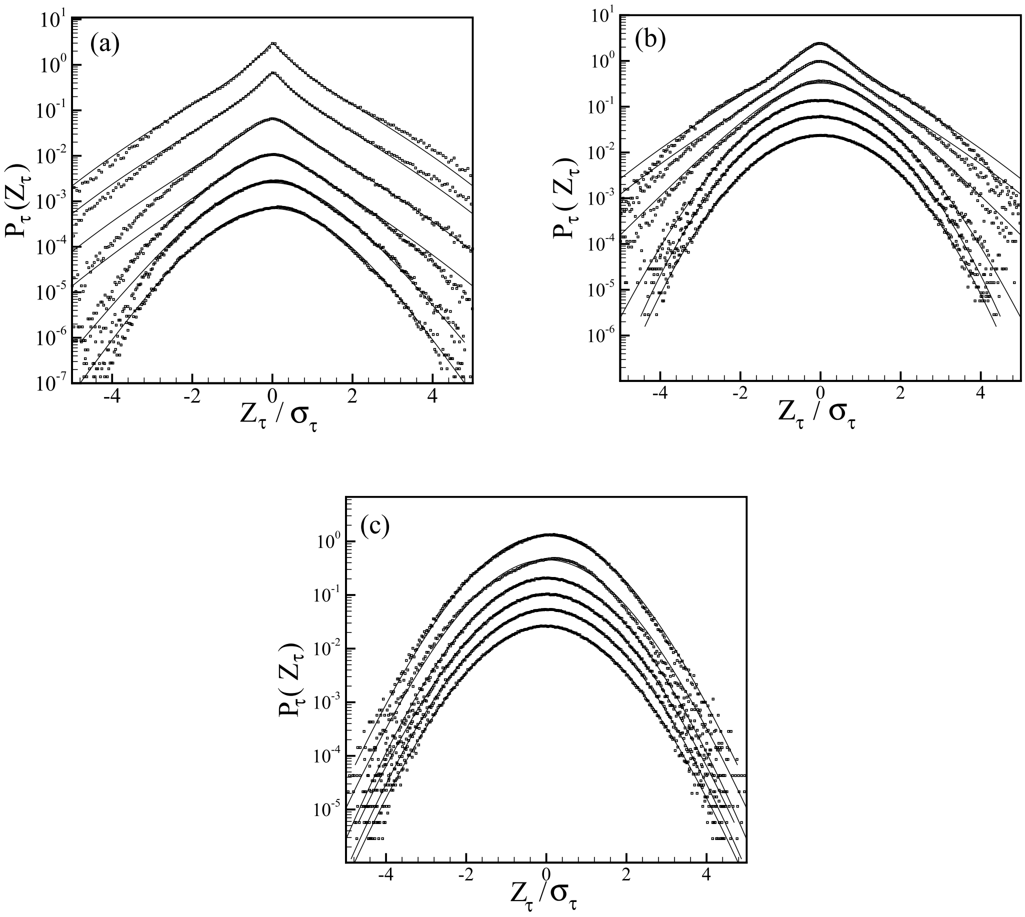

Figure 2. Deformation of standardized PDFs of the increments Z_τ across the scales, where σ_τ is the standard deviation of Z_τ. (a) The standardized PDFs (in semi-logarithmic scale) for time scales (from top to bottom) $\tau = 2, 5, 10, 20, 30$ and 50, and height $h = 44$ cm. Solid curves show the functions obtained from the cascade equation. (b) and (c) show the PDFs for the surrogate and shuffled series, respectively.

where F_τ and G_τ are both Gaussian with zero mean and variances σ_τ^2 and λ_τ^2, respectively. It is usually assumed [5] that G_τ is given by

$$G_\tau(\ln \sigma) = \frac{1}{\sqrt{2\pi}\lambda_\tau} \exp(-\ln^2 \sigma / 2\lambda_\tau^2), \tag{4}$$

and similarly for F_τ [30]. As λ_τ increases, fat tails and a peak around the mean value become evident. The PDF converges to a Gaussian when $\lambda_\tau \to 0$. Equation (3) is equivalent to that for a log-normal cascade model that has been developed for velocity fluctuations in fully-developed turbulence [31, 32]. For reasons that are not understood yet, equation (3) also provides accurate approximations for the non-Gaussian PDFs that describe such diverse phenomena and systems as foreign currency exchange markets [28, 29], fluctuations in heartbeat intervals [33], and seismic time series [34, 35], which are essential to understanding earthquakes.

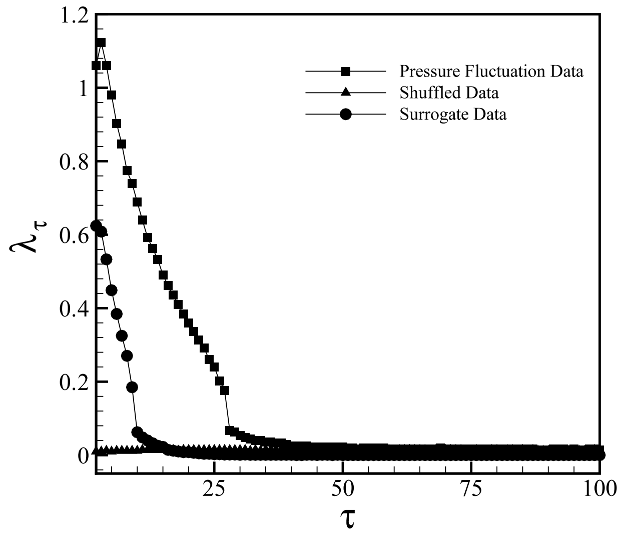

Figure 3. Dependence of the fitting parameter λ_τ of the cascade equation for detrended data on the time scale τ at height $h = 44$ cm.

We computed the PDF of the increments Z_τ, fitted it to equation (3), and estimated the variance λ_τ^2 of $G_\tau(\omega_\tau)$. Figure 2 presents the resulting standardized PDFs of the detrended increments Z_τ across scales τ. It is clear that the cascade modeling provides an accurate fit of the data for the increments across the time scale τ (except, in some cases, in the tail of the PDF). Figure 2 also indicates that the PDFs exhibit continuous deformation as the time scale τ varies. The Gaussian PDF emerges only at large τ. This finding is in contrast to the previous suggestions for the PDF that we compute, which included a Gaussian distribution [36] at all times, as well as one with a power-law tail [22].

In addition to the accurate fit of the PDF by the equation (3), we also obtained a rapidly-decaying λ_τ as a function of τ. Figure 3 depicts the resulting fitting parameter λ_τ of the equation (3) for the detrended data. As pointed out earlier, when $\lambda_\tau \to 0$, one has a Gaussian PDF. The logarithmic behavior of λ_τ is of special interest, because it is related to the power-law scaling of all the moments of the increments Z_τ, as can be seen easily by integrating equation (3), provided that the variance of Z_τ follows a power-law scaling in τ. Such a power-law scaling is similar to the structure of local velocity fluctuations in turbulent flow [30].

It is important to understand the effect of detrending on the results. Thus, we define the pressure increments on the scale τ by, $Z_\tau(t) = p(t + \tau) - p(\tau)$, where $\tau > 0$ and $p(t)$ represents the data *without* detrending. The resulting successive increments $Z_\tau(t)$ for the height $h = 44$ cm are presented in figure 4. We then constructed the PDF of $Z_\tau(t)$ for various τ by the method described earlier, and estimated the fitting parameter λ_τ. The results for the PDF of the data increments without detrending and measured at height $h = 44$ cm are shown in figure 5 for various time scales τ. Figure 6 presents the resulting fitting parameter λ_τ. Though the numerical values of $\lambda_\tau(\tau)$ for the data without detrending is smaller than those with detrending, a comparison of the results for the detrended and undetrending data indicates that the PDFs and the fitting parameters are qualitatively similar with and without detrending, hence indicating that the original data

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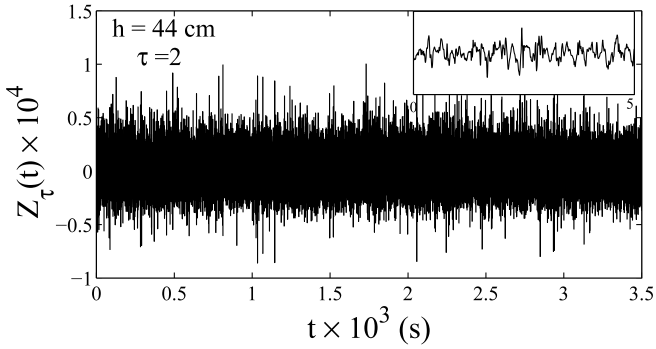

Figure 4. The increments Z_τ of the data shown in figure 1, but without detrending.

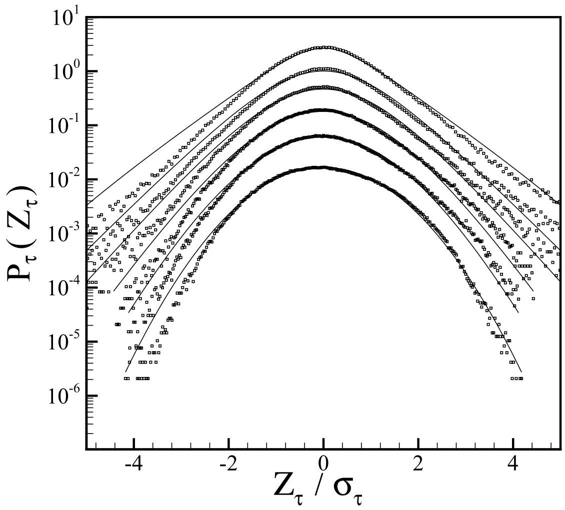

Figure 5. Deformation of standardized PDFs of the increments Z_τ of the data *without* detrending across time scales, with σ_τ being the standard deviation of Z_τ, for time scales (from top to bottom) $\tau = 2, 5, 10, 20, 30$ and 50, and height $h = 44$ cm. Solid curves show identical functions obtained from cascade equation.

did not contain any trends. Similar results were obtained for the data measured at 34 and 54 cm above the bottom of the FB.

4. The surrogate and shuffled series

We further studied the PDF of the successive increments of the pressure fluctuations, in order to better understand the origin of the properties of the time series [37]. To do so, we computed the surrogate [38, 39] and shuffled time series of the original data. To form the shuffled series, we randomized the order of the data points in the underlying series, but preserved their values. To generate a surrogate series, we proceeded as follows. Suppose that the time series $p(t) = \{p_1, p_2, \ldots, p_N\}$ contains N data points. The discrete Fourier

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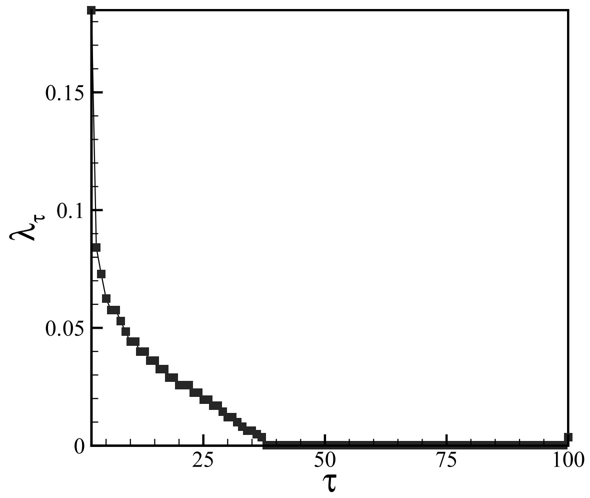

Figure 6. Dependence of the fitting parameter λ_τ of the cascade equation on the time scale τ at height $h = 44$ cm. The results are for undetrended data.

transform of $p(t)$, corresponding to times $t = 0, \Delta t, 2\Delta t, \ldots, (N-1)\Delta t$, is then computed,

$$\mathcal{F}(\omega) = \sum_{n=0}^{N-1} p(t_n) \exp(2\mathrm{i}\pi\omega N\Delta t). \tag{5}$$

The computed Fourier transform was then written as $\mathcal{F}(\omega) = A(\omega) \exp[\mathrm{i}\phi(\omega)]$, with $A(\omega)$ being the amplitude and $\phi(\omega)$ the phase. $\mathcal{F}(\omega)$ was evaluated at discrete frequencies, $\omega = -N\Delta\omega/2, \ldots, -\Delta\omega, 0, \Delta\omega, \ldots, N\Delta\omega/2$, where $\Delta\omega = 1/(N\Delta t)$. The phase at each frequency was then randomized and uniformly distributed in $[0, 2\pi]$:

$$\tilde{\mathcal{A}}(\omega) = A(\omega) \exp\{\mathrm{i}[\phi(\omega) + \psi(\omega)]\}, \tag{6}$$

where $\psi(\omega)$ is a random variable, selected uniformly in the range $[0, 2\pi)$. By numerically computing the inverse Fourier transform we obtained the surrogate time series, which has the same spectral density and autocorrelation function as that of the original data set [38, 39]. Note that the shuffling of the time series destroys the correlations in the series. Therefore, if the time scale-dependence of the PDF $P(Z_\tau)$ is due to only the correlations, the shuffled series must have scale-invariance properties.

After constructing the shuffled and surrogate series, we analyzed them by the method described in section 3. The resulting PDFs for both series are also shown in figure 2 for the detrended data measured at $h = 44$ cm. The scale-dependence of the fitting parameter λ_τ for the same detrended data is also shown in figure 3. Note that the surrogate series has the same type of PDF as the original series, with the same type of evolution to a Gaussian at large times, and the same type of the fitting parameter λ_τ. On the other hand, the shuffled series takes on the well-defined Gaussian PDF shape for all the time scales, hence indicating that the cascade equation (3) arises as a result of long-range correlations in the pressure fluctuation time series.

5. Similarity with turbulence

As mentioned earlier, the cascade model has also been shown to provide an accurate representation of the PDF of local velocity fluctuations in turbulent flow. The natural question that arises is whether there is any similarity between pressure fluctuations in fluidized beds and velocity fluctuations in turbulent flow. Turbulent flows are characterized by a constant energy flux that is dissipated at very small scales by the fluid viscosity, representing the dominant physics of turbulent flow. On the other hand, in FBs the energy is dissipated by the collapse of the bubbles. Though it is not yet completely understood, such a dissipation mechanism might dominate the short-time dynamics of the pressure fluctuations, in which case the dissipation mechanisms of the two phenomena are completely different.

On the other hand, the apparent similarity between the pressure fluctuations in a FB and velocity fluctuations in turbulence might in part be due to the existence of the cascades of information contained within the large-to-small scale fluctuations that exist in both phenomena. The cascade exhibits itself in the deformation of the PDF as a function of the time scale τ, the key parameters for fitting the data for both phenomena to the cascade model. Such a similarity between the two phenomena clearly warrants further investigation.

6. Summary

The results presented in this paper indicate that pressure fluctuations in fluidized beds may be analyzed accurately by the cascade model that has been proposed for the fluctuations of the local velocities in turbulent flows. In particular, the probability density function (PDF) of the successive increments of the pressure fluctuation data is shown to follow the cascade equation used in turbulent flow for modeling of the PDF of the successive increments of local velocity fluctuations. The PDF evolves, however, towards a Gaussian distribution at large times. The surrogate and shuffled time series for the pressure fluctuations were also analyzed. The PDF for the surrogate time series and its evolution over various time scales are also well represented by the same type of the PDF. Thus, that the cascade equation provides an accurate representation of the PDF of the successive increments of the pressure data is due to the long-range correlations that exist in the data for fluidized beds.

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