Dr.Memarzadeh Group Theory
Deadline: 1401/01/27 23:59

Problem Set 4(60 points)

Question 1 (Zee I.3 Q9). *[10 points] Show that A_4 is an invariant subgroup (in fact, maximal) of S_4.*

Question 2 (Zee I.3 Q11). *[10 points] Calculate the derived subgroup of the dihedral group.*

Question 3 (Zee I.3 Q12). *[10 points] Given two group elements f and g, show that, while in general $fg \neq gf$, fg is equivalent to gf (that is, they are in the same equivalence class).*

Question 4 (Zee I.3 Q14). *[10 points] Using Cayley's theorem, map V to a subgroup of S_4. List the permutation corresponding to each element of V. Do the same for Z_4.*

Question 5 (Zee I.3 Q16). *[10 points] In a Coxeter group, show that if $n_{ij} = 2$, then a_i and a_j commute.*

Question 6 (Zee I.2 Q19). *[10 points] Show that the derived subgroup of S_n is A_n. (In the text, with the remark about even permutations we merely showed that it is a subgroup of S_n.)*

Question 7 (Zee I.3 Q20). *[10 points] A set of real-valued functions f_i of a real variable x can also define a group if we define multiplication as follows: given f_i and f_j, the product $f_i \cdot f_j$ is defined as the function $f_i\left(f_j(x)\right)$. Show that the functions $I(x) = x$ and $A(x) = (1 - x)^{-1}$ generate a three-element group.* [12] *Furthermore, including the function $C(x) = x^{-1}$ generates a six-element group.*

Question 8 (Herstein 2.6 Q10). *[20 points] If H is a subgroup of G, let $N(H) = \{g \in G \mid gHg^{-1} = H\}$. Prove*
(a) $N(H)$ is a subgroup of G.
(b) H is normal in $N(H)$.
(c) If H is a normal subgroup of the subgroup K in G, then $K \subset N(H)$ (that is, $N(H)$ is the largest subgroup of G in which H is normal).
(d) H is normal in G if and only if $N(H) = G$.

Question 9 (Herstein 2.6 Q11). *[10 points] If N and M are normal subgroups*[1] *of G, prove that NM is also a normal subgroup of G.*

Question 10 (Herstein 2.6 Q20). *[20 points] Let G be a group such that $(ab)^p = a^p b^p$ for all $a, b \in G$, where p is a prime number. Let $S = \left\{ x \in G \mid x^{p^m} = e \text{for some } m \text{ depending on } x \right\}$. Prove*
(a) S is a normal subgroup of G.
(b) If $\bar{G} = G/S$ and if $\bar{x} \in \bar{G}$ is such that $\bar{x}^p = \bar{e}$ then $\bar{x} = \bar{e}$.

Question 11 (Herstein 2.7 Q6). *[20 points] If N, M are normal subgroups of G, prove that $NM/M \approx N/N \cap M$*

Send to groupTheory.sut@gmail.com with subject ID+Hw#n [2]

[1] normal subgroup = invariant subgroup
[2] ex:98203078Hw3