

Problem Set 3(50 points)

Question 1 (Zee I.3 Q5). [30 points] *Count the number of elements with a given cycle structure.*

Question 2 (Zee I.3 Q6). [10 points] *List the possible cycle structures in S_5 and count the number of elements with each structure.*

Question 3 (Zee I.3 Q8). [10 points] *Show that A_4 is not simple.* †

Question 4 (Herstein 2.6 Q8). [10 points] *Give an example of a group G , subgroup H , and an element $a \in G$ such that $aHa^{-1} \subset H$ but $aHa^{-1} \neq H$.*

Question 5 (Herstein 2.6 Q18). [20 points] *Let G be a group in which, for some integer $n > 1$, $(ab)^n = a^n b^n$ for all $a, b \in G$. Show that*

(a) $G^{(n)} = \{x^n \mid x \in G\}$ *is a normal subgroup of G .*

(b) $G^{(n-1)} = \{x^{n-1} \mid x \in G\}$ *is a normal subgroup of G .*

Question 6 (Herstein 2.7 Q10). [10 points] *Prove that a group of order 9 is abelian.*

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¹ex:98203078Hw3