Dr.Memarzadeh Group Theory
Deadline: 1400/12/07 23:59

Problem Set 1(40 points)

Question 1 (Herstein 2.5 Q1)**.** *[10 points] If H and K are subgroups of G, show that $H \cap K$ is a subgroup of G. (Can you see that the same proof shows that the intersection of any number of subgroups of G, finite or infinite, is again a subgroup of G?)*

Question 2 (Herstein 2.3 Q3)**.** *[10 points] If G is a group such that $(a \cdot b)^2 = a^2 \cdot b^2$ for all $a, b \in G$, show that G must be abelian.*

Question 3 (Herstein 2.3 Q8)**.** *[20 points] If G is a finite group, show that there exists a positive integer N such that $a^N = e$ for all $a \in G$.*

Question 4 (Herstein 2.3 Q9)**.** *[20 points] (a) If the group G has three elements, show it must be abelian. (b) Do part (a) if G has four elements. (c) Do part (a) if G has five elements.*

Question 5 (Zee I.1 Q3)**.** *[10 points] Show that $Z_2 \otimes Z_4 \neq Z_8$.*

Question 6 (Zee I.1 Q2)**.** *[10 points] Let $f(g)$ be a function of the elements in a finite group G, and consider the sum $\sum_{g \epsilon G} f(g)$. Prove the identity $\sum_{g \epsilon G} f(g) = \sum_{g \epsilon G} f(gg') = \sum_{g \epsilon G} f(g'g)$ for g' an arbitrary element of G.*

SEND TO GROUPTHEORY.SUT@GMAIL.COM WITH SUBJECT ID+HW#N [1]

[1] ex:98203078Hw1