Problem Set 1 (40 points)

Question 1 (Herstein 2.5 Q1). [10 points] If $H$ and $K$ are subgroups of $G$, show that $H \cap K$ is a subgroup of $G$. (Can you see that the same proof shows that the intersection of any number of subgroups of $G$, finite or infinite, is again a subgroup of $G$?)

Question 2 (Herstein 2.3 Q3). [10 points] If $G$ is a group such that $(a \cdot b)^2 = a^2 \cdot b^2$ for all $a, b \in G$, show that $G$ must be abelian.

Question 3 (Herstein 2.3 Q8). [20 points] If $G$ is a finite group, show that there exists a positive integer $N$ such that $a^N = e$ for all $a \in G$.

Question 4 (Herstein 2.3 Q9). [20 points] (a) If the group $G$ has three elements, show it must be abelian. (b) Do part (a) if $G$ has four elements. (c) Do part (a) if $G$ has five elements.

Question 5 (Zee I.1 Q3). [10 points] Show that $\mathbb{Z}_2 \otimes \mathbb{Z}_4 \neq \mathbb{Z}_8$.

Question 6 (Zee I.1 Q2). [10 points] Let $f(g)$ be a function of the elements in a finite group $G$, and consider the sum $\sum_{g \in G} f(g)$. Prove the identity $\sum_{g \in G} f(g) = \sum_{g \in G} f(gg') = \sum_{g \in G} f(g'g)$ for $g'$ an arbitrary element of $G$.

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