Filtering Noisy ECG Signals Using the Extended Kalman Filter Based on a Modified Dynamic ECG Model

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Abstract

In this paper an Extended Kalman Filter (EKF) has been proposed for the filtering of noisy ECG signals. The method is based on a modified nonlinear dynamic model, previously introduced for the generation of synthetic ECG signals. An automatic parameter selection method has also been suggested, to adapt the model with a vast variety of normal and abnormal ECG signals. The results show that the EKF output is able to track the original ECG signal shape even in the most noisiest epochs of the ECG signal. The proposed method may serve as an efficient filtering procedure for applications such as the noninvasive extraction of fetal cardiac signals from maternal abdominal signals.

1. Introduction

The extraction of high resolution ECG signals from noisy measurements is among the most tempting open problems of biomedical signal processing. Specifically, the extraction of ECG signals from low SNR measurements is the state of the art in applications such as the noninvasive extraction of fetal ECG signals, recorded from an array of electrodes placed on the maternal abdomen [1].

On the other hand, in recent years some research has been conducted towards the generation of synthetic ECG signals. Regarding the physiological bases of ECG signals, a true ECG model should consider the morphology of the PQRST complex, together with the RR-wave timing. In a previous work [2], a synthetic model has been proposed which has unified the morphology and pulse timing of the ECG signal in a single nonlinear dynamic model. Concerning the simplicity and flexibility of this model it is believed that it can be easily adapted to a broad class of normal and abnormal ECG signals. This model may be further used in dynamic adaptive filters, such as the *Kalman Filter*, for ECG filtering applications. Meanwhile, the dynamic model of [2] is nonlinear and requires the nonlinear counterparts of the conventional Kalman Filter.

In a recent work [3], the authors have developed an *Extended Kalman Filter* (EKF) based on the mentioned dynamic model for noisy ECG filtering. In this paper, the synthetic ECG model has been further modified to fulfill the requirements of the EKF filter. The EKF model parameter selection has also been automated in order to adapt the method to different normal and abnormal ECG signals. The results show that the proposed method can fully track the ECG signal even in the noisy epochs, where the observed ECG signal is almost lost in noise.

2. Theory

2.1. Extended Kalman filter review

The *Extended Kalman Filter (EKF)* is a nonlinear extension of conventional Kalman Filter that has been specifically developed for systems having nonlinear dynamic models [4]. For a discrete nonlinear system with the state vector $\underline{x}_k$ and observation vector $\underline{y}_k$, the dynamic model may be formulated as follows:

$$\begin{cases} \underline{x}_{k+1} = f(\underline{x}_k, \underline{w}_k, k) \\ \underline{y}_k = g(\underline{x}_k, \underline{v}_k, k), \end{cases} \tag{1}$$

where $\underline{w}_k$ and $\underline{v}_k$ are the process and measurement noises respectively with covariance matrices $Q_k=E\{\underline{w}_k\underline{w}_k^T\}$ and $R_k=E\{\underline{v}_k\underline{v}_k^T\}$.

The initial state estimate of the state $\underline{x}_0$ is defined as $\overline{\underline{x}}_0 = E\{\underline{x}_0\}$ with $P_0 = E\{(\underline{x}_0 - \overline{\underline{x}}_0)(\underline{x}_0 - \overline{\underline{x}}_0)^T\}$.

In order to use a Kalman filter formalism for this system, it is necessary to derive a linear approximation of (1) near a desired reference point $(\hat{\underline{x}}_k, \hat{\underline{w}}_k, \hat{\underline{v}}_k)$. This

$$\begin{cases} \underline{x}_{k+1} \approx f(\hat{\underline{x}}_k, \hat{\underline{w}}_k, k) + A_k(\underline{x}_k - \hat{\underline{x}}_k) + F_k(\underline{w}_k - \hat{\underline{w}}_k) \\ \underline{y}_k \approx g(\hat{\underline{x}}_k, \hat{\underline{v}}_k, k) + C_k(\underline{x}_k - \hat{\underline{x}}_k) + G_k(\underline{v}_k - \hat{\underline{v}}_k) \end{cases} \tag{2}$$

approximation will lead to the following linear estimate:
where

$$A_k = \left.\frac{\partial f(\underline{x}, \hat{\underline{w}}_k, k)}{\partial \underline{x}}\right|_{\underline{x}=\hat{\underline{x}}_k} \qquad F_k = \left.\frac{\partial f(\hat{\underline{x}}_k, \underline{w}, k)}{\partial \underline{w}}\right|_{\underline{w}=\hat{\underline{w}}_k}$$
$$C_k = \left.\frac{\partial g(\underline{x}, \hat{\underline{v}}_k, k)}{\partial \underline{x}}\right|_{\underline{x}=\hat{\underline{x}}_k} \qquad G_k = \left.\frac{\partial g(\hat{\underline{x}}_k, \underline{v}, k)}{\partial \underline{v}}\right|_{\underline{v}=\hat{\underline{v}}_k} \tag{3}$$

In order to implement the EKF, the time propagation is done using the original nonlinear equation, while the Kalman filter gain and the covariance matrix are calculated from the linearized equations. Further issues concerning the implementation of the EKF may be followed in [4] and [5].

2.2. Synthetic electrocardiogram

MCSharry *et al.* [2] have proposed a synthetic ECG generator, which is based on a nonlinear dynamic model. This model has several parameters, which makes it adaptable to many normal and abnormal ECG signals. As it may be seen in (4) the dynamic model consists of a three dimensional state equation, which generates a trajectory with the coordinate (*x*,*y*,*z*).

$$\begin{aligned} x' &= \alpha x - \omega y \\ y' &= \alpha y + \omega x \\ z' &= -\sum_{i\in\{P,Q,R,S,T\}} a_i \Delta\theta_i \exp(-\frac{\Delta\theta_i^2}{2b_i^2}) - (z - z_0) \end{aligned} \quad (4)$$

In this equation $\alpha = 1 - \sqrt{x^2 + y^2}$, $\Delta\theta_i = (\theta - \theta_i) \bmod (2\pi)$, $\theta = \operatorname{atan2}(y, x)$ (the four quadrant arctangent of the real parts of the elements of *x* and *y*, with $-\pi \le \operatorname{atan2}(y,x) \le \pi$, and ω is the angular velocity of the trajectory as it moves around the limit cycle [2]. The baseline wander of the ECG signal has been modeled with z_0, which is coupled with the respiratory frequency f_2:

$$z_0 = A\sin(2\pi f_2 t), \qquad A = 0.15mV, \quad f_2 = 0.25Hz \quad (5)$$

Some typical values of the parameters of (4) taken from [2] are listed in Table I. As it will be later seen, these variables may be assumed as random process noises for the proposed EKF model.

TABLE I. Typical parameters of the synthetic ECG model [2]

Index (i)	P	Q	R	S	T
Time (Sec.)	-0.2	-0.05	0	0.05	0.3
θ_i (rads.)	-π/3	-π/12	0	π/12	π/2
a_i	1.2	-5.0	30.0	-7.5	0.75
b_i	0.25	0.1	0.1	0.1	0.4

The three dimensional trajectory which is generated by (4), consists of a circular limit cycle which is pushed up and down when it approaches one of the P, Q, R, S or T points. In fact, each of the components of the ECG waveform has been modeled with a Gaussian function, which is located at a specific angle. This may easily be viewed from (4), by neglecting the baseline wander term (z-z_0) and integrating the z' equation. The projection of the three dimensional trajectory on the *z* axis gives a synthetic ECG signal.

3. Method

3.1. Modification of the dynamic model

The polar form of the dynamic equations (4) is:

$$\begin{aligned} r' &= r(1 - r) \\ \theta' &= \omega \\ z' &= -\sum_{i\in\{P,Q,R,S,T\}} a_i \Delta\theta_i \exp(-\frac{\Delta\theta_i^2}{2b_i^2}) - (z - z_0) \end{aligned} \quad (6)$$

These new set of equations have some benefits compared to the original equations. First of all, the polar form is much simpler and its interpretation is straightforward. Accordingly, the first equation will reach the limit cycle of $r = 1$ with any initial value of r; but as it is further seen the second and third equations are independent from r. This means that the first differential equation may be totally omitted, since it doesn't affect the synthetic ECG signal (the *z* state variable). Another benefit of this representation is that the phase parameter is an explicit state-variable; noting that this parameter indicates the angular location of the P, Q, R, S and T waves. This point is further used in the implementation of the EKF. Meanwhile the simplified discrete form of (6) is:

$$\begin{cases} \theta[k+1] = \theta[k] + \omega \cdot \Delta \\ z[k+1] = -\sum_{i\in\{P,Q,R,S,T\}} \Delta \cdot a_i \cdot \Delta\theta_i \exp(-\frac{\Delta\theta_i^2}{2b_i^2}) + z[k] + N \cdot \Delta \end{cases} \quad (7)$$

where Δ is the sampling time and $\Delta\theta_i = (\theta - \theta_i) \bmod (2\pi)$. *N* is a random additive noise which has been placed instead of the baseline wander of (4) and to model other additive sources of process noise.

3.2. Linearization of the nonlinear dynamic model

In order to set up an EKF model based on the nonlinear synthetic model of (7), it is necessary to have a linearized version of the nonlinear model. For this, the state-equation of (7) needs to be linearized using (2) and (3). In this procedure θ and *z* are the state variables, and ω, a_i, θ_i, b_i and *N* are assumed to be the process noises. By defining:

$$\begin{cases} \theta[k+1] = F(\theta, z, \omega, k) \\ z[k+1] = G(\theta, z, a_i, \theta_i, b_i, N, k), \end{cases} \quad (8)$$

the following equations represents the linearized model with respect to the state variables θ and *z*.

$$\begin{aligned} &\frac{\partial F}{\partial z} = 0 \qquad \frac{\partial F}{\partial \theta} = \frac{\partial G}{\partial z} = 1 \\ &\frac{\partial G}{\partial \theta} = \sum_{i\in\{P,Q,R,S,T\}} -\Delta \cdot a_i [1 - \frac{\Delta\theta_i^2}{b_i^2}] \exp(-\frac{\Delta\theta_i^2}{2b_i^2}) \end{aligned} \quad (9)$$

Identically (10) is the linearization of (7) respecting the process noises.

$$\begin{aligned} &\frac{\partial F}{\partial a_i} = \frac{\partial F}{\partial b_i} = \frac{\partial F}{\partial \theta_i} = \frac{\partial F}{\partial N} = 0 \qquad \frac{\partial F}{\partial \omega} = \Delta \\ &\frac{\partial G}{\partial a_i} = -\Delta \cdot \Delta\theta_i \exp(-\frac{\Delta\theta_i^2}{2b_i^2}), \quad i \in \{P,Q,R,S,T\} \\ &\frac{\partial G}{\partial b_i} = -\Delta \cdot a_i \frac{\Delta\theta_i^3}{b_i^3} \exp(-\frac{\Delta\theta_i^2}{2b_i^2}) \qquad \frac{\partial G}{\partial \omega} = 0 \\ &\frac{\partial G}{\partial \theta_i} = \Delta \cdot a_i [1 - \frac{\Delta\theta_i^2}{b_i^2}] \exp(-\frac{\Delta\theta_i^2}{2b_i^2}) \qquad \frac{\partial G}{\partial N} = \Delta \end{aligned} \quad (10)$$

In accordance with the notation of (1), the process noise vector may be defined as follows:

$$\underline{w}_k = [a_P,...,a_T, b_P,...,b_T, \theta_P,...,\theta_T, \omega, N], \qquad (11)$$

with $Q_k = E\{\underline{w_k}\underline{w_k}^T\}$.

The dynamic model is now ready to be used in an EKF.

3.3. Construction of the EKF model

Having linearized the ECG dynamic model, an EKF may be developed. The relationship between the state variables and measurements of the proposed EKF depends on the location of the electrodes and the origin of the measurement noise. For example motion artifacts, environmental noises or bioelectrical artifacts such as the Electromyogram (EMG) or the Electrogastrogram (EGG), may be assumed as the measurement noises. While the measurement noise can generally take a complex and nonlinear form, the results of this paper are based on a simple additive Gaussian noise.

In addition to the ECG observations, the phase θ may also be added as a second observation. In fact by studying the values of Table I, it is seen that the R-peak is assumed to occur at θ=0. So by simply detecting the R-Peaks an additional observation is achieved.

Hence the phase observations (φ_k) and the noisy ECG measurements (s_k) may be related to the state vector as follows:

$$\begin{bmatrix} \varphi_k \\ s_k \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \theta_k \\ z_k \end{bmatrix} + \begin{bmatrix} v1_k \\ v2_k \end{bmatrix} \qquad (12)$$

with $R_k = E\{[v1_k, v2_k].[v1_k, v2_k]^T\}$.

In the context of estimation theory, the variance of the observation noise in (12) represents the degree of reliability of a single observation. In other words, when a rather precise measurement of the states of a system is valid the value of R_k is low, and the Kalman filter gain is adapted such as to rely on that specific measurement. While for the epochs that the measurements are too noisy or there are no measurements available, the value of R_k is high and the Kalman filter tries to follow its underlying dynamics rather than relying on the observations. This point may be used for adding additional measurements for the angle θ. In fact, θ is a periodic value that starts from θ=0 at the R-peak and ends with the next R-peak. This means that even for the angles other than θ=0, it is possible to assign a phase measurement between 0 and 2π to each angle. Of course for these angles the variance of the measurement phase noise may be increased to encounter the more uncertainty in the phase value. This intuition about the observation noise will be further referred to in the following section.

3.4. Automatic estimation of the EKF parameters

Prior to the implementation of the EKF model, it is necessary to have an estimate of the values of the process and measurement noise covariance matrices. Generally, for the 17 noise parameters of (11) a 17x17 process noise covariance matrix (Q_k) should be found; but if the noises are uncorrelated with each other, the matrix is simplified to a diagonal matrix. The measurement noise covariance matrix (R_k) has a similar case.

In order to automate the parameter selection procedure for any given ECG signal, the parameters should be estimated from the signal itself. For this, the given noisy ECG signal is transformed to a three dimensional plot using a phase wrapping technique. In order to do so, the locations of the R-peaks are detected from the ECG signal by using a peak detector. These points correspond to θ=0 in the synthetic dynamic model. Then the points lying between two R-peaks are linearly assigned a phase between 0 and 2π. The ECG signal may be plotted versus the assigned phases in polar coordinates on the unit circle ($r=1$). Figure (1) represents a typical phase-wrapped noisy ECG signal, using a sample signal taken from the PhysioNet database.

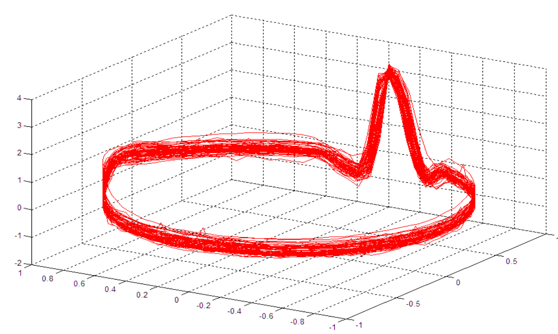

Figure 1. A phase-wrapped noisy ECG signal

It is now possible to estimate the dynamic model parameters for the given ECG signal. The values of Table I, which needed to be selected manually, may now be chosen as follows: The mean of the phase-wrapped ECG is calculated and five Gaussian signals, corresponding with the P, Q, R, S and T shapes, are fitted on this average. This gives the mean values of the process noises a_i, b_i and θ_i. Further more, by calculating the deviation of the signal around the mean phase-wrapped ECG, the covariance of each parameter may also be estimated.

In order to estimate the angular frequency ω, a simple estimate would be $\omega=2\pi/T$; where T is the R-R peak period in each ECG cycle. More generally, ω is related to the *Heart Rate Variability* (HRV) of the ECG signal and is known to be influenced by other physiological systems of the body. Some authors have worked on the spectral specifications of the HRV [2]; this suggests that ω itself may be assigned a dynamic model. Meanwhile the results of the present paper are based on the first method.

The variance of the process noise N should also be estimated. Note that N is a parameter that represents the imprecision of the dynamic model, or the negligence of other physiological sources that influence the ECG signal. A simple estimate for this parameter would be a zero mean Gaussian noise, with an appropriate variance.

There are also several ways to estimate the variance of the measurement noises of (12). One way is to estimate the noise power from the deviations of the whole signal around the phase-wrapped ECG, or from the portions of the ECG signal between two successive T and P waves; which corresponds to the bottom part of Figure (1). Another way is to use a frequency domain estimate of the noise. Apparently the selection of the method depends on the origin of the expected noise. The reported results of the following section are based on first approach.

4. Results

The proposed EKF model was finally implemented in Matlab® based on an approach previously reported in [6].

The noisy ECG signal consisted of a normal sinus rhythm, taken from the PhysioNet ECG database [7]. The sampling frequency of the dataset was 125Hz. The mean ECG pulse rate was 1.2Hz or 72 beats per minute (BPM), which is modeled with the ω parameter in the ECG model of (6) (divided by 2π).

Figure (2) represents a sample portion of the ECG signal together with the output of the EKF model. As it may be seen, the EKF has followed the dynamics of the ECG signal, and has suppressed the noise.

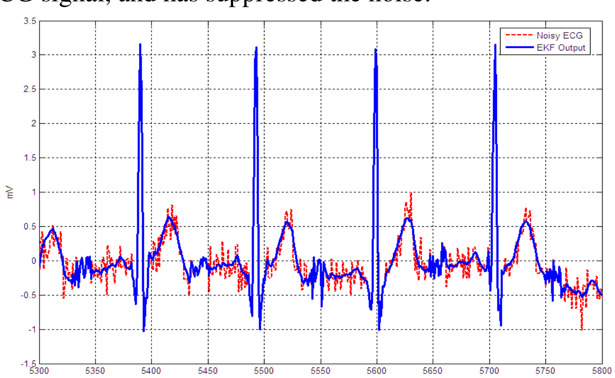

Figure 2. EKF output for ECG signals with erased

5. Discussion and conclusions

In this paper an *Extended Kalman Filter (EKF)* was designed for the filtering of ECG signals. The EKF's dynamic model was based on a modified three dimensional nonlinear dynamic model previously introduced for the generation of synthetic ECG signals. This nonlinear model was linearized in order to be used in an EKF. The designed filter was later applied to noisy ECG signals, and the results show the filter's capability in tracking and filtering noisy ECG signals.

The evaluation of the EKF implemented in this paper was quite qualitative. In practical applications it is necessary to represent more quantitative measures, together with issues concerning the stability and convergence of the Kalman filter.

The filtering performance is highly reliant on the underlying dynamics assumed for the ECG signal. It was shown that by using a flexible nonlinear dynamical model, together with the EKF, it is possible to construct a filter which can remove environmental noises and artifacts. The proposed method can serve as a base for the design of a robust ECG filter, with vast applications for low SNR ECG signals such as the noninvasive fetal cardiac signal extraction.

Future works include the combination of the proposed EKF model with source separation techniques, for the extraction of maternal and fetal cardiac signals from multi-channel surface electrode recordings.

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