Optimal sensor placement for source separation with noisy measurements

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***Abstract*—Optimal sensor placement for a single source extraction has been studied recently. In the current paper, a criterion to optimally placing the sensors for source separation of the noisy mixtures is proposed. Moreover, it is described how a blind source separation (BSS) method can be used to estimate the spatial gains from the measurements of the already placed sensors to enhance the placement of the remaining sensors. Numerical simulations show that the proposed criterion outperforms the previous criterion in source extraction from the noisy mixtures, and illustrates its efficiency in sensor placement for source separation.**

***Index Terms*—source separation, optimal sensor placement**

I. Introduction

Optimal sensor placement can be relevant in any application of using sensors to collect data, and for which the spatial positions of the sensors affect the performance of the measurement system or the cost of the sensor placement. It has drawn attentions in areas such as structural health monitoring [1], [2], source localization [3], [4], municipal water networks [5], [6] and wireless sensor networks [7], [8].

In a source separation problem, propagated signals from several sources are measured by sensors, and the goal is to estimate the latent source signals from the measured data. In linear source separation with instantaneous mixtures, the measured signals are linear combinations of the source signals. The coefficients of this combination typically depends on the signal attenuation between the sources and the sensors. Consequently, the spatial positions of the sensors determine the coefficients, which we refer to as spatial gains. Various methods have been developed to address this type of source separation problem [9]. Even when the spatial gains are completely unknown and the source separation is performed blindly, there are methods called equivariant blind source separation (BSS) methods, which lead to the performance independent of the spatial gains. Examples of equivariant BSS methods include equivariant adaptive separation via independence (EASI) [10] and equivariant nonstationary source separation [11]. Therefore, if an equivariant method is employed, the sensor positions will not affect the separation performance. However, such methods are limited to noiseless measurements.

In many real-life applications, the signals measured by sensors are subject to additive noise. For example, in electroencephalography (EEG), the recorded signals by the electrodes contain not only the sources of interest, related to the special brain events, but also non-event-driven ongoing brain activities as well as the artifacts caused by irrelevant activities such as eye blinking, which can be treated as additive noise [12]. In such cases, by assuming the noise spatial covariance matrix to be full rank, the sensor positions affect the performance of source separation, regardless of whether the problem is blind or not. In fact, considering the noise components as additional sources, the number of source components exceeds the number of sensors, violating the assumption required for equivariant methods. Additionally, it makes the perfect extraction of the sources impossible, even if the spatial gains are known. Consequently, finding the optimal sensor positions to improve the quality of source separation becomes interesting when dealing with noisy measurements. This paper addresses precisely this problem.

Recently, research has been conducted on sensor placement for extracting a single source from noisy measurements. This work introduces new criteria, based on the signal-to-noise ratio (SNR) of the linearly extracted signal, that are optimized to determine the optimal sensor positions [13], [14]. The results demonstrate that the proposed criteria outperform classical Kriging based sensor placement approaches in terms of the output SNR of the extracted signal [14]. However, to the best of our knowledge, the problem of sensor placement for source separation of noisy mixtures has not been previously studied.

In this paper, by assuming the prior information about the spatial gains of the sources to be given by a stochastic Gaussian Process (GP) model, an optimization criterion based on the expected value of the signal-to-interference-plus-noise ratio (SINR) of the separated signals is proposed. Numerical simulations demonstrate the efficiency of the proposed criterion in sensor placement for source separation. In addition, the paper illustrates how the recorded data from already placed sensors can be utilized in the placement procedure to reduce uncertainty regarding the spatial gains and help the placement of the remaining sensors.

This paper is organized as follows. In Section II, we present our proposed method for optimizing sensor placement and describe the estimation method for the spatial gains using sensor data. Section III provides numerical results and compares the performance of the proposed method with another sensor placement approach. Finally, Section IV concludes the paper.

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II. Proposed method

In this section, the problem statement, the proposed criterion for the sensor placement and its optimization approach are presented. Moreover, a method to estimate the spatial gains using sensor data to help the placement procedure is described.

A. Linear source estimation model

Assume that P independent sources propagate their signals, $\mathbf{s}(t) = [s_1(t), s_2(t), ..., s_P(t)]^T$, into a D-dimensional space $\mathcal{X} \subset \mathbb{R}^D$, and let $a^p(\mathbf{x})$ denote the spatial gain from the p-th source to the sensor whose coordinates are $\mathbf{x} \in \mathcal{X}$. The measured signal by a sensor at time t and coordinates $\mathbf{x}$ can be written as

$$y(\mathbf{x}, t) = \sum_{p=1}^{P} a_p(\mathbf{x}) s_p(t) + n(\mathbf{x}, t), \tag{1}$$

where $n(\mathbf{x}, t)$ is additive noise. The measurements are assumed to be an instantaneous noisy mixture of the sources, meaning that the propagation delay from the sources to the sensors is negligible, which is a realistic assumption in a large number of applications, such as electroencephalography. Let us assume that M sensors are placed at the positions $\mathbf{X}_M = \{\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_M\}$. The measured signals $\mathbf{y}(\mathbf{X}_M, t) = [y(\mathbf{x}_1, t), y(\mathbf{x}_2, t), ..., y(\mathbf{x}_M, t)]^T$ can be rewritten as

$$\mathbf{y}(\mathbf{X}_M, t) = \sum_{p=1}^{P} \mathbf{a}_p(\mathbf{X}_M) s_p(t) + \mathbf{n}(\mathbf{X}_M, t), \tag{2}$$

where $\mathbf{a}_p(\mathbf{X}_M) \triangleq [a_p(\mathbf{x}_1), a_p(\mathbf{x}_2), ..., a_p(\mathbf{x}_M)]^T$ is the vector of the spatial gains of the p-th source at the corresponding positions, and $\mathbf{n}(\mathbf{X}_M, t) \triangleq [n(\mathbf{x}_1, t), n(\mathbf{x}_2, t), ..., n(\mathbf{x}_M, t)]^T$ is the vector of additive noise.

We assume that the l-th source is estimated linearly using a vector $\mathbf{f}_l \in \mathbb{R}^M$, that is,

$$\hat{s}_l(t) = \mathbf{f}_l^T \mathbf{y}(\mathbf{X}_M, t) = \sum_{p=1}^{P} \mathbf{f}_l^T \mathbf{a}_p(\mathbf{X}_M) s_p(t) + \mathbf{f}_l^T \mathbf{n}(\mathbf{X}_M, t). \tag{3}$$

To choose the vector $\mathbf{f}$ optimally, one way is to find it such that the SINR of the estimated source is maximized. The SINR of the l-th estimated source is given by

$$\text{SINR}_l(\mathbf{f}_l, \mathbf{X}_M) = \frac{\mathbb{E}\left[(\mathbf{f}_l^T \mathbf{a}_l(\mathbf{X}_M) s_l(t))^2\right]}{\mathbb{E}\left[\left(\mathbf{f}_l^T (\sum_{p=1, p\neq l}^{P} \mathbf{a}_p(\mathbf{X}_M) s_p(t) + \mathbf{n}(\mathbf{X}_M, t))\right)^2\right]}. \tag{4}$$

The additive noise is assumed to have zero mean and be independent of the sources. The variances of the sources and the covariance matrix of the noise vector (denoted by $\mathbf{C}^n_{MM}$) are assumed to be constant over time. Moreover, without the loss of generality, the sources are assumed to have unit variances ($\mathbb{E}[(s_l(t))^2] = 1$), so their power is assumed to be embedded in the spatial gains. Then, the SINR in equation (4) can be simplified as

$$\text{SINR}_l(\mathbf{f}_l, \mathbf{X}_M) = \frac{\mathbf{f}_l^T \mathbf{a}_l(\mathbf{X}_M) \mathbf{a}_l(\mathbf{X}_M)^T \mathbf{f}_l}{\mathbf{f}_l^T \left(\sum_{p=1, p\neq l}^{P} \mathbf{a}_p(\mathbf{X}_M) \mathbf{a}_p(\mathbf{X}_M)^T + \mathbf{C}^n_{MM}\right) \mathbf{f}_l}. \tag{5}$$

Assuming that the noise covariance matrix is full rank, the vector $\mathbf{f}_l^* = (\sum_{p=1, p\neq l}^{P} \mathbf{a}_p(\mathbf{X}_M) \mathbf{a}_p(\mathbf{X}_M)^T + \mathbf{C}^n_{MM})^{-1} \mathbf{a}_l$ maximizes (5), and the maximum achievable SINR for the l-th linearly estimated source is given by

$$\text{SINR}_l(\mathbf{f}_l^*, \mathbf{X}_M) = \mathbf{a}_l(\mathbf{X}_M)^T (\sum_{p=1, p\neq l}^{P} \mathbf{a}_p(\mathbf{X}_M) \mathbf{a}_p(\mathbf{X}_M)^T + \mathbf{C}^n_{MM})^{-1} \mathbf{a}_l(\mathbf{X}_M). \tag{6}$$

B. Sensor placement criterion

To optimally place the sensors, one can search for the sensor positions that maximize the sum of the SINRs of the sources, assuming the spatial gains are known. However, in practice, perfect knowledge about the spatial gains may not be available. To address this issue, similar to [14], we model the spatial gains of each source as a realization of a stochastic Gaussian Process (GP), that is,

$$\hat{a}_p(\mathbf{x}) \sim \mathcal{GP}\left(m^{a_p}(\mathbf{x}), C^{a_p}(\mathbf{x}, \mathbf{x}')\right). \tag{7}$$

Here, $m^{a_p}(\mathbf{x})$ represents the mean of the Gaussian random variable $\hat{a}_p(\mathbf{x})$, and $C^{a_p}(\mathbf{x}, \mathbf{x}')$ is a symmetric positive-definite kernel function that specifies the covariance between $\hat{a}_p(\mathbf{x})$ and $\hat{a}_p(\mathbf{x}')$. It is assumed that the spatial gains for different sources are independent of each other. Modelling both prior information and uncertainty about the spatial gains, and the ability to generate various signal shapes are from the powerful properties of the GP. The mean and covariance function of the GP model, along with the covariance function of the noise, should be known in advance to utilize this model for sensor placement. The noise covariance function can be learned using measurements taken when the sources are not active, and the GP parameters can be estimated either using the prior information about the propagation properties of the environment, or using some rough measurements combined with the BSS methods and regression techniques.

By assuming a stochastic model for the spatial gains, the SINR of (6) also becomes stochastic. Therefore, we use the sum of the expected values of the SINRs of all the sources as a sensor placement criterion, that is,

$$J(\mathbf{X}_M) = \sum_{l=1}^{P} \mathbb{E}\left[\hat{\text{SINR}}_l(\mathbf{f}_l^*, \mathbf{X}_M)\right] = \sum_{l=1}^{P} \mathbb{E}\left[\hat{\mathbf{a}}_l^T (\sum_{p=1, p\neq l}^{P} \hat{\mathbf{a}}_p(\mathbf{X}_M) \hat{\mathbf{a}}_p(\mathbf{X}_M)^T + \mathbf{C}^n_{MM})^{-1} \hat{\mathbf{a}}_l\right]. \tag{8}$$

Even with the probability distributions of the spatial gains, obtaining a closed-form expression for the expected values of the SINR for each source is not straightforward. Therefore, in this paper, averaging over the Monte Carlo realizations of the SINR is used to numerically calculate its expected value. In other words, using the probability distributions of the spatial gains, L independent samples of them are generated, and for each sample of the spatial gains, a sample of the SINR is calculated using (6). Finally, the expected value of the SINR is estimated by averaging these L independent samples.

C. Optimization approach

In order to find the points that optimize the criterion, a practical approach is to perform a grid search over the available space. Let us assume a grid of T points in the space, $\mathbf{X}_T = \{\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_T\}$, as candidate positions to place M sensors. The set of M points that maximize the criterion (8) should be found, that is,

$$\mathbf{X}_M^* = \underset{\mathbf{X}_M \subset \mathbf{X}_T}{\operatorname{argmax}} J(\mathbf{X}_M). \tag{9}$$

Solving (9) requires a combinatorial search over $\binom{T}{M} = \frac{T!}{M!(T-M)!}$ possibilities, resulting in a significant computational cost. To address this issue, the greedy method introduced in [14] can be employed. This method breaks down the optimization into smaller sub-problems. In each sub-problem, the previously placed sensors are assumed to be fixed, and N new sensors are placed at the points that maximize the criterion. While this greedy approach does not guarantee an exact optimal solution, it makes the problem practically solvable in terms of computational cost. In each small optimization problem, the number of possibilities to be explored is less than $\binom{T}{N}$, which becomes feasible computationally for sufficiently small values of N.

This step by step sensor placement method can be improved by another idea, which is called sequential approach in [14]. In this approach, at each step, the measurements obtained from the already placed sensors are utilized to estimate the spatial gains in the placed positions. This reduces the uncertainty about the spatial gains in the whole available space, and improves the placement of the remaining sensors. For the remainder of this section, we assume that an estimation of the spatial gains at the placed sensor positions is available. A method for estimating the spatial gains is described in Section II-D.

We begin with an empty set of the points, and at each iteration, $N < M$ new positions are selected to place the sensors until the number of the placed sensors reaches M. Assume that K sensors are placed at the positions $\mathbf{X}_K = \{\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_K\}$ in previous iterations. We have an estimation $\mathbf{z}_p(\mathbf{X}_K)$ of the spatial gains for the p-th source obtained from the data measured by the placed sensors. This estimation can be expressed as

$$\mathbf{z}_p(\mathbf{X}_K) = \mathbf{a}_p(\mathbf{X}_K) + \mathbf{v}_p(\mathbf{X}_K), \tag{10}$$

where $\mathbf{a}_p(\mathbf{X}_K)$ is the vector of the true spatial gains and $\mathbf{v}_p(\mathbf{X}_K)$ is the measurement error. The error term is assumed to be Gaussian with zero mean and independent of the spatial gains. Therefore, for any N new points $\mathbf{X}_N$, the entries of the vectors $\hat{\mathbf{a}}_p(\mathbf{X}_M) = [\hat{\mathbf{a}}_p(\mathbf{X}_K)^T, \hat{\mathbf{a}}_p(\mathbf{X}_N)^T]^T$ and $\mathbf{z}_p(\mathbf{X}_K)$ are jointly Gaussian random variables all together. In order to exploit the information given by the estimation $\mathbf{z}_p(\mathbf{X}_K)$, the conditional distribution of $\hat{\mathbf{a}}_p(\mathbf{X}_M)$ given $\mathbf{z}_p(\mathbf{X}_K)$ should be determined. The conditional distribution is also a multivariate Gaussian and can be fully characterized by the conditional mean vector and covariance matrix, given by

$$\begin{aligned}\mathbf{m}_{M|K}^{a_p} &= \mathbb{E}[\hat{\mathbf{a}}_p(\mathbf{X}_M)|\mathbf{z}_p(\mathbf{X}_K)] = \\ &\mathbf{m}_M^{a_p} + \mathbf{C}_{MK}^{a_p}(\mathbf{C}_{KK}^{a_p} + \mathbf{C}_{KK}^{v})^{-1}(\mathbf{z}_p(\mathbf{X}_K) - \mathbf{m}_M^{a_p})\end{aligned} \tag{11}$$

and

$$\begin{aligned}\mathbf{C}_{M|K}^{a_p} &= \mathbb{E}\left[||\hat{\mathbf{a}}_p(\mathbf{X}_M) - \mathbf{m}_{M|K}^{a_p}||_2^2 \;\; |\mathbf{z}_p(\mathbf{X}_K)\right] = \\ &\mathbf{C}_{MM}^{a_p} - \mathbf{C}_{MK}^{a_p}(\mathbf{C}_{KK}^{a_p} + \mathbf{C}_{KK}^{v})^{-1}(\mathbf{C}_{MK}^{a_p})^T\end{aligned} \tag{12}$$

Here, $\mathbf{m}_M^{a_p}$ denotes the mean of $\hat{\mathbf{a}}_p(\mathbf{X}_M)$, and $\mathbf{C}_{KK}^{a_p}$, $\mathbf{C}_{MM}^{a_p}$ and $\mathbf{C}_{KK}^{v}$ are the covariance matrices of $\hat{\mathbf{a}}_p(\mathbf{X}_K)$, $\hat{\mathbf{a}}_p(\mathbf{X}_M)$ and $\mathbf{v}_p(\mathbf{X}_K)$, respectively. Furthermore, $\mathbf{C}_{MK}^{a_p}$ represents the cross-covariance matrix between $\hat{\mathbf{a}}_p(\mathbf{X}_M)$ and $\hat{\mathbf{a}}_p(\mathbf{X}_K)$.

In the same manner, the criterion of (8) can be calculated using Monte Carlo realization, but with the utilization of conditional distributions to generate samples of the spatial gains. Therefore, in each iteration, the optimization problem to be solved is

$$\mathbf{X}_N^* = \underset{\mathbf{X}_N \subset \mathbf{X}_T \setminus \mathbf{X}_K}{\operatorname{argmax}} \; J(\mathbf{X}_K \cup \mathbf{X}_N | \mathbf{z}_{\{P\}}(\mathbf{X}_K)), \tag{13}$$

where $J(\mathbf{X}_K \cup \mathbf{X}_N | \mathbf{z}_{\{P\}}(\mathbf{X}_K))$ represents the criterion at the points $\mathbf{X}_K \cup \mathbf{X}_N$, which is computed using the obtained conditional distributions.

D. Estimating spatial gains using sensor data

In this section, we present a method for estimating the spatial gains to help the placement procedure as described in the previous section. To estimate the spatial gains of a source, one can suggest inactivating all the sources except the desired one, but controlling the activation of the sources is often not available in many applications, *e.g.* in electroencephalography. Moreover, it is a hard or inaccurate procedure to estimate the spatial gains using the signal propagation properties of the environment. In such applications, the only practical way is to use sensors measured data. BSS techniques offer the advantage of not only separating sources from an unknown mixing model but also estimating the mixing coefficients. Hence, they can serve as a powerful tool for estimating spatial gains using sensor data.

The employed estimation approach consists of two stages. Firstly, a method similar to the principal component analysis (PCA) [15] is applied to reduce the dimensionality of the measurements to match the number of sources while minimizing the impact of noise. Secondly, a BSS method is used to estimate the spatial gains.

Assume that K placed sensors are used to measure N_s samples. Similar to (2), the measurements can be written as

$$\mathbf{y}(\mathbf{X}_K, t) = \mathbf{A}_K \mathbf{s}(t) + \mathbf{n}(\mathbf{X}_K, t), \quad t = 1, 2, ..., N_s, \tag{14}$$

where $\mathbf{A}_K = [\mathbf{a}_1(\mathbf{X}_K), \mathbf{a}_2(\mathbf{X}_K), ..., \mathbf{a}_P(\mathbf{X}_K)] \in \mathbb{R}^{K\times P}$ is the mixing matrix. This estimation is performed where the number of the placed sensors is greater than or equal to the number of the sources. In the first stage, our objective is to find a transformation matrix $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, ..., \mathbf{h}_P]^T \in \mathbb{R}^{P\times K}$ with orthonormal row vectors, in order to transform the measurements into a P dimensional space, that is, $\hat{\mathbf{y}}(t) = \mathbf{H}\mathbf{y}(t, \mathbf{X}_K)$. To obtain the optimum $\mathbf{H}$, the mean square error (MSE) criterion,

$$J_{MSE}(\mathbf{H}) = \mathbb{E}\left[\left\|\mathbf{A}_M\mathbf{s}(t) - \sum_{i=1}^{P}(\mathbf{h}_i^T\mathbf{y}(\mathbf{X}_M, t))\mathbf{h}_i\right\|^2\right], \tag{15}$$

is minimized, where $\sum_{i=1}^{P}(\mathbf{h}_i^T\mathbf{y}(\mathbf{X}_M, t))\mathbf{h}_i$ is the projection of $\mathbf{y}(\mathbf{X}_M, t)$ onto the subspace spanned by the basis vectors $\mathbf{h}_1, \mathbf{h}_2, ..., \mathbf{h}_P$. Assuming the identity covariance matrix for the sources $\mathbf{s}(t)$, the MSE can be simplified as $J_{MSE}(\mathbf{H}) = \mathrm{Tr}[\mathbf{A}_M\mathbf{A}_M^T] - \sum_{i=1}^{P}(\mathbf{h}_i^T(\mathbf{C}_{KK}^y - 2\mathbf{C}_{KK}^n)\mathbf{h}_i)$, where $\mathbf{C}_{KK}^y$ is the covariance matrix of the measurements, and can be calculated using measured samples. Therefore, minimizing the MSE leads to the maximizing $\sum_{i=1}^{P}(\mathbf{h}_i^T(\mathbf{C}_{KK}^y - 2\mathbf{C}_{KK}^n)\mathbf{h}_i)$ over the orthonormal vectors $\mathbf{h}_i,\ i = 1, 2, ..., P$. A classically known solution for this problem is the eigenvectors of $(\mathbf{C}_{KK}^y - 2\mathbf{C}_{KK}^n)$ that correspond to its P largest eigenvalues.

In the second stage, *i.e.* BSS, the modified fastICA algorithm for noisy measurements [15] is applied to the transformed measurements in order to separate the sources. For successful separation, the sources should be independent of each other and exhibit non-Gaussian distributions. Assume that $\hat{\mathbf{s}}(t)$ denote the separated source signals. Knowing that the spatial gains of the p-th source can be written as $\mathbf{a}_p(\mathbf{X}_M) = \mathbb{E}[\mathbf{y}(\mathbf{X}_M, t)s_p(t)]$, an estimate of the spatial gains of the p-th source, $\hat{\mathbf{a}}(\mathbf{X}_M)$, is obtained using the sample mean of $\mathbf{y}(\mathbf{X}_M, t)\hat{s}_p(t)$. This estimation can be modelled as $\mathbf{z}_p(\mathbf{X}_K)$ in (10) to be used in the placement procedure. In this paper, we assume that the estimation error $\mathbf{v}_p(\mathbf{X}_K)$ is negligible. However, we know that it can be a rough assumption in general, and obtaining the estimation error and studying its effect in modelling is left as a future work.

III. Numerical Simulations

In this section, the proposed criterion will be compared with the expected SNR criterion of [13] in a single source extraction task in Fig. 1. The performance of the proposed method will be studied with two different estimation approaches and two levels of the uncertainty in Fig. 2.

A. Simulation setup

The simulations are performed on either a 1D space with a grid of 200 points in the interval $[0, 1]$, or a 2D space with a 40×40 grid of the points within a unit square. The covariance functions of the noise and the GP model of the spatial gains are assumed to have the form $C(\mathbf{x}, \mathbf{x}') = \sigma^2 \exp(-||\mathbf{x} - \mathbf{x}'||^2/(2\rho^2))$. The GPs for the spatial gains of each source, $\mathcal{GP}\left(m^{a_p}(\mathbf{x}), C^{a_p}(\mathbf{x}, \mathbf{x}')\right)$, share the same parameters. The mean function $m^{a_p}(\mathbf{x})$ is generated using a GP with

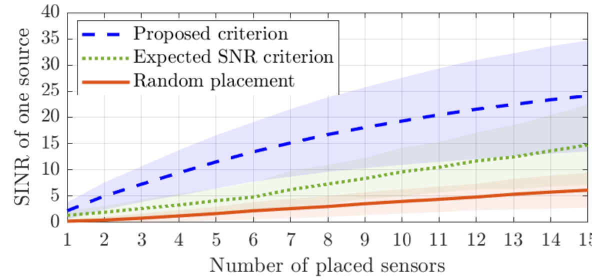


Fig. 1: Comparing the performance of the proposed criterion with expected SNR criterion and random placement according to the improvement of the SINR of a single source versus the number of the placed sensors, using sequential approach and perfect estimation of the spatial gains in 1D space.

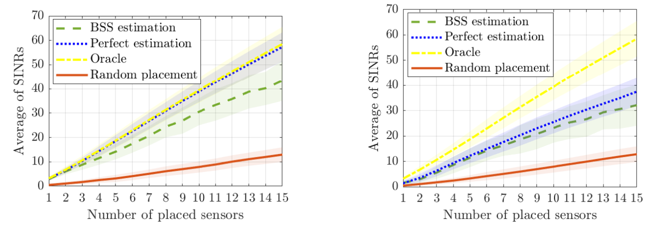
zero mean, variance parameter $\sigma_a = 1$, and the smoothness parameter $\rho_a = 0.05$ which matches the smoothness parameter of $C^{a_p}(\mathbf{x}, \mathbf{x}')$. The variance parameter of $C^{a_p}(\mathbf{x}, \mathbf{x}')$, which determines the level of uncertainty, is denoted as σ_u and varies across different simulations. The noise is assumed to have zero mean, and its covariance function parameters are set as $\sigma_n = 1$ and $\rho_n = 0.2\rho_a$. In each iteration of the sequential sensor placement, $N = 1$ new sensor is added. To calculate the expected value of the SINRs, averaging over $L = 20$ Monte Carlo realizations is used. Moreover, for each plot, 100 Monte Carlo simulations are repeated to obtain the mean and standard deviation of the desired values.

B. One source extraction

In the first part of the simulations, proposed criterion is compared with the expected SNR criterion of [13], which is designed for the optimal sensor placement for a single source extraction. The scenario assumes the extraction of one source from a mixture of 5 sources plus additive noise. The criterion of (8) is modified for the source extraction task, such that only the expected SINR of the extracted source is used instead of the sum of them. On the other hand, the expected SNR criterion of [13] uses the SNR of the extracted source as a criterion, neglecting the impact of other sources on the extracted signal. Assuming the extraction of the first source, the expected SNR criterion can be written as $J_{SNR} = \mathbb{E}\ \left[\hat{\mathbf{a}}_1^T(\mathbf{C}_{MM}^n)^{-1}\hat{\mathbf{a}}_1\right]$. Fig. 1 illustrates the output SINR of the extracted source, obtained from the sensors placed by these two criteria, and also random placement of the sensors. In random placement, the sensor positions are selected uniformly and independently from the available space. The variance parameter of the covariance function for the spatial gains is set to $\sigma_u = 0.1$. The sensors are placed in a 1D space, the sequential approach is used and the estimation of the spatial gains at the placed positions is assumed perfect, without any error. The figure demonstrates that the performance of the both criteria is better than the random sensor placement. For 15 placed sensors, proposed criterion and the expected SNR criterion of [13] yield mean SINR of 25 and 15, respectively, indicating that the proposed criterion outperforms the expected SNR criterion in source extraction. That is because the proposed criterion uses the information of the spatial gains of all the sources, instead of just the desired one.

C. Performance of the proposed method in source separation

In this simulation, the performance of the proposed method is studied in terms of the average of the output SINRs of the separated sources. Two cases are considered: perfect estimation of the spatial gains and estimation using the BSS method, both employing the sequential optimization approach. In the perfect estimation case, it is assumed that the spatial gains are estimated perfectly without any error at the position of the placed sensors. Whereas, in BSS estimation, the method of Section II-D is used to update the estimation of the spatial gains using the measurements of the placed sensors. For this estimation, the sources and the noise are assumed to have the uniform and Gaussian distributions, respectively, and 500 measured samples are used. The number of the sources is 3 and the placement is performed over a 2D space. Fig. 2 presents the improvement in the average output SINRs of the separated sources as the number of placed sensors increases, considering two levels of uncertainty: a) $\sigma_u = 0.1$, and b) $\sigma_u = 0.8$. The blue and green curves show the performance of the proposed method using perfect estimation and BSS estimation, respectively. Two additional curves are included: The red curve shows the average output SINRs obtained from the random placement of the sensors, as explained in the previous section, and the yellow curve indicates the performance of the oracle experiment. In the oracle experiment, the spatial gains are assumed to be deterministic and known throughout the space, and the SINRs of the sources are not stochastic and can be directly obtained using (6). This curve serves as an upper bound for the performance of the proposed placement method. As illustrated in Fig. 2a, the performance of the placement using the proposed method is significantly better than the randomly placing the sensors. For 15 placed sensors, the average SINR of the proposed method using the BSS estimation in a low level of the uncertainty ($\sigma_u = 0.1$) is 43, whereas it is 13 for the random placement. Because of the estimation error of the BSS estimation method, the green curve is below the blue curve. In Fig. 2b, as the uncertainty level increases from $\sigma_u = 0.1$ to $\sigma_u = 0.8$, the average SINR of the proposed method decreases from 57 to 38 when using the perfect estimation, and from 43 to 32 when using the BSS estimation, for 15 placed sensors.



(a) $\sigma_u = 0.1$ (b) $\sigma_u = 0.8$

Fig. 2: The performance of the proposed criterion with sequential optimization approach, in two case of the BSS estimation method and perfect estimation of the spatial gains is illustrated, and compared with random sensor placement and Oracle (completely known spatial gains). Two situations are studied: (a) $\sigma_u = 0.1$, (b) $\sigma_u = 0.8$.

IV. Conclusion

In this paper, to tackle with the problem of optimal sensor placement for linear source separation of the noisy mixtures, we proposed an optimization criterion based on the expected values of the SINRs of the separated signals. Moreover, to improve the prior knowledge of the spatial gains given by the stochastic GP models, we described a BSS method to update the estimation of the spatial gains using the measurements of the placed sensors. Numerical simulations show the efficiency of the proposed method in source separation, and its outperforming results in source extraction compared to the criterion proposed in [13].

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