Synaptic Plasticity and Synchronization

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Sharif Univ.

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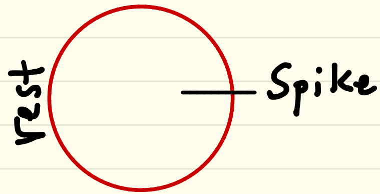
Kuramoto model: (Neuron as oscillator)



$$\dot{\theta}_i = \omega_i + \sum_{j=1}^{N} K_{ij} \sin(\theta_j - \theta_i)$$

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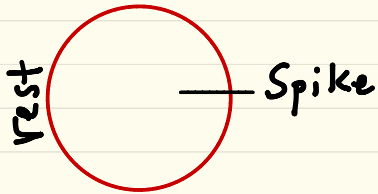

$$\dot{\theta}_i = \omega_i + \sum_{j=1}^{N} K_{ij} \sin(\theta_j - \theta_i)$$

K_{ij} : Connection weight, $K_{ij} \geq 0$

ω_i : natural frequency of neuron i

Synchronization Parameter

$$P = \frac{1}{N} \left| \sum_{j=1}^{N} e^{i\theta_j} \right|$$

$$0 \leq P \leq 1$$

Synaptic Plasticity

rule I: locality

rule II: type invariant

rule III: boundedness

rule IV: option of decreasing

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rule I: locality

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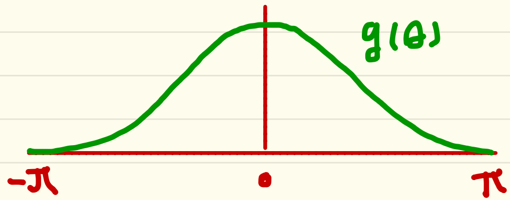
rule IV: option of decreasing



$$g(\theta) = \frac{1+\cos\theta}{2}$$

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Excitatory Network

$$\begin{cases} \dot{\theta}_i = \omega_i + \sum_j K_{ij} \sin(\theta_j - \theta_i) \\ \dot{K}_{ij} = \varepsilon(-K_{ij} + \mu\, g(\theta_i - \theta_j)) \end{cases}$$

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Identical Network: $\omega_1 = \omega_2 = \cdots = \omega_N$

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Gradient System $\quad V = \sum_{i<j} \frac{1}{2} K_{ij}^2 - \mu K_{ij}\, g(\varphi_i - \varphi_j)$

$$\frac{dV}{dt} \leq 0$$

V is bounded from below, then all trajectories converge to the local minimum of V.

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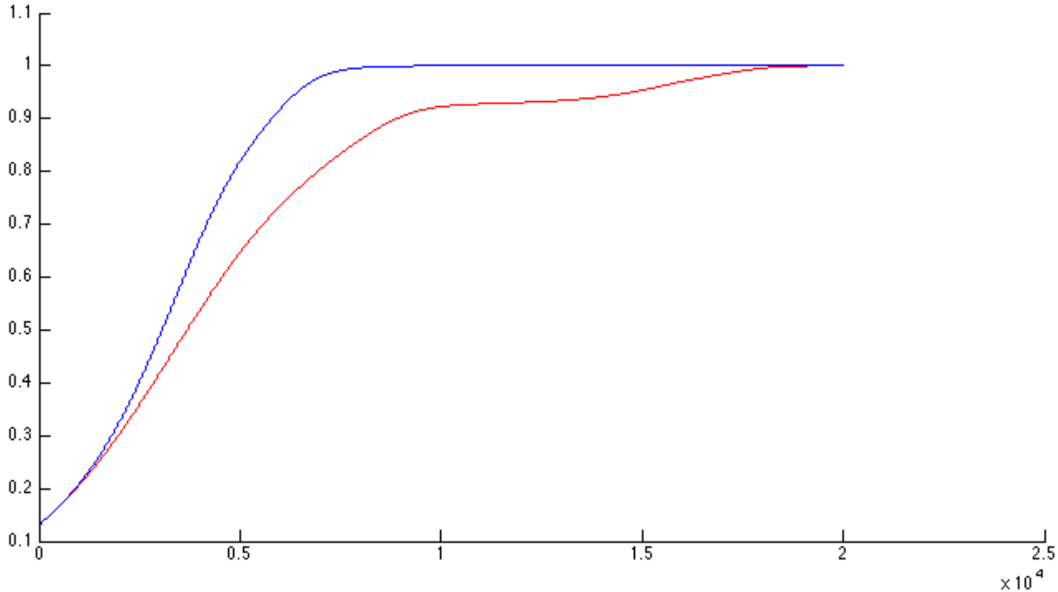
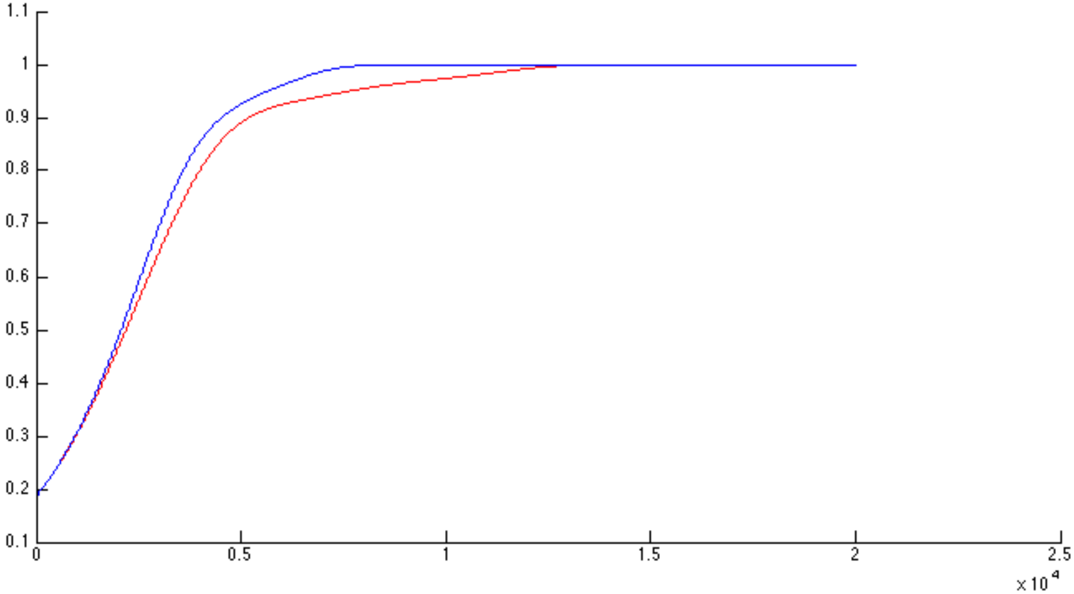
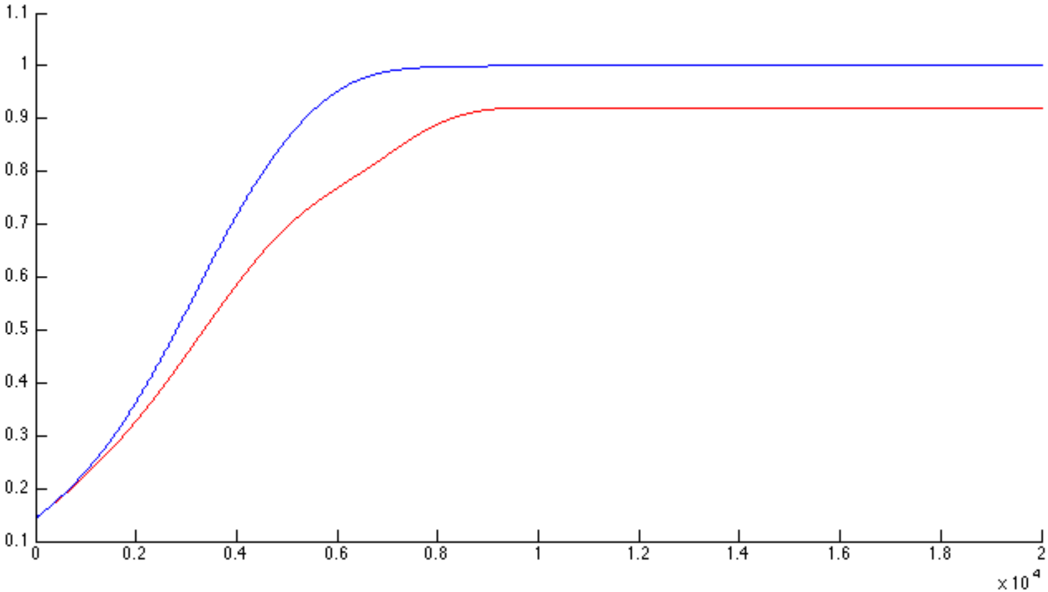
observe: network without plasticity gets synchronized faster.



N=50, epsilon=0.5



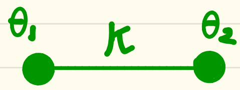
N=50, epsilon=0.2



N=50, epsilon=2

Inhomogeneuse Network

Two Neurons



$\omega_1 \neq \omega_2$

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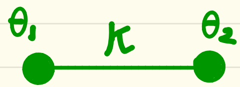

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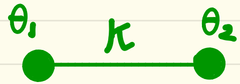

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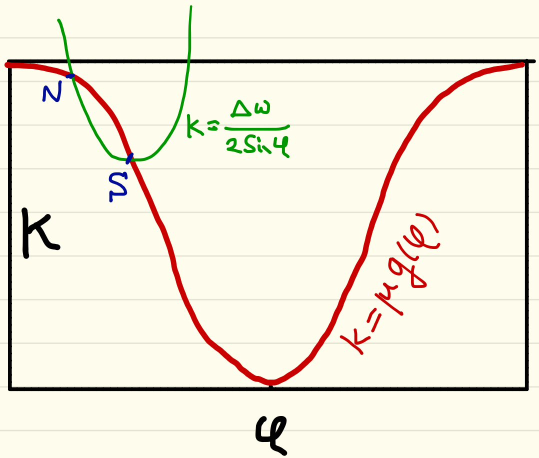
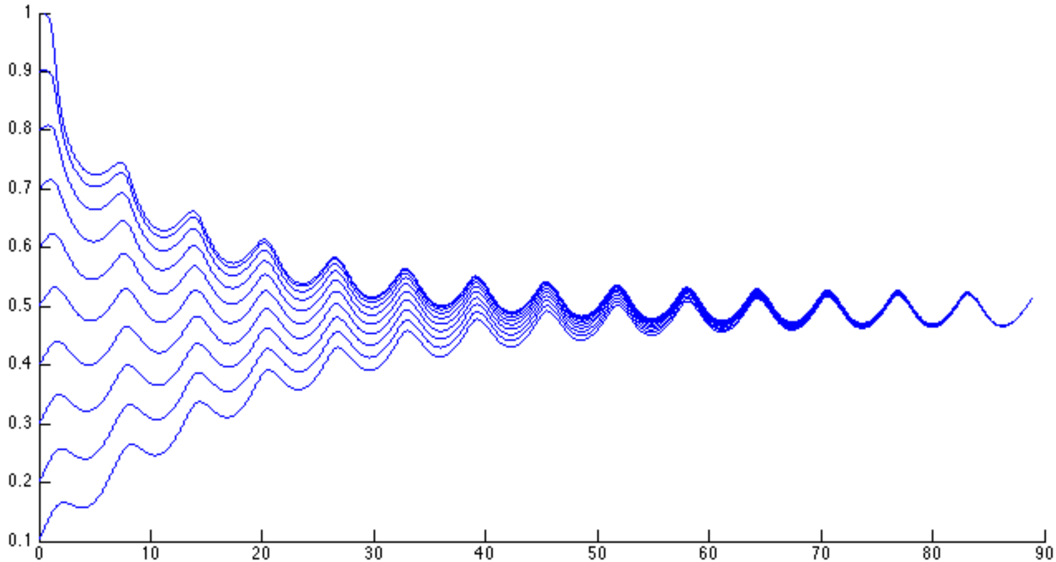


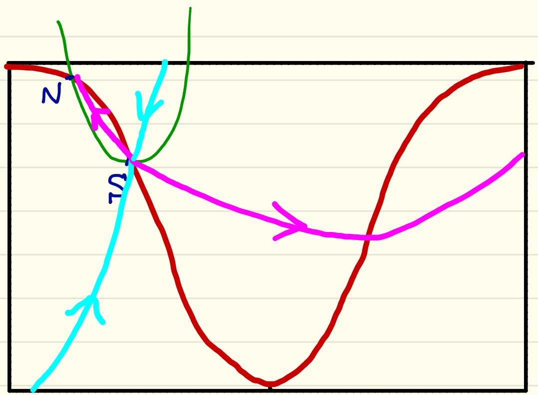
$$\varphi = \theta_1 - \theta_2 \longrightarrow \begin{cases} \dot\varphi = \Delta\omega - 2K\sin\varphi \\ \dot K = \varepsilon(-K + \mu g(\varphi)) \end{cases}$$

Result 1: If $|\Delta\omega| > M = \max \mu \sin\varphi (1+\cos\varphi)$, then there is a periodic global asymptotic solution.



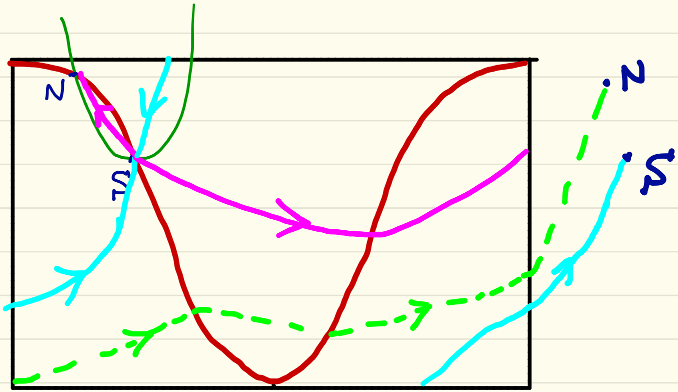
Delta=2, epsilon=0.2, mu=1

$\Delta w < M$?



lucky case

all solutions, except the stable curves, will converge to the stable nodes "N".



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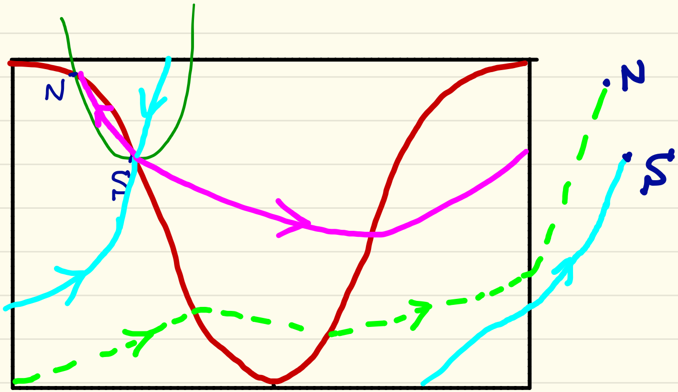
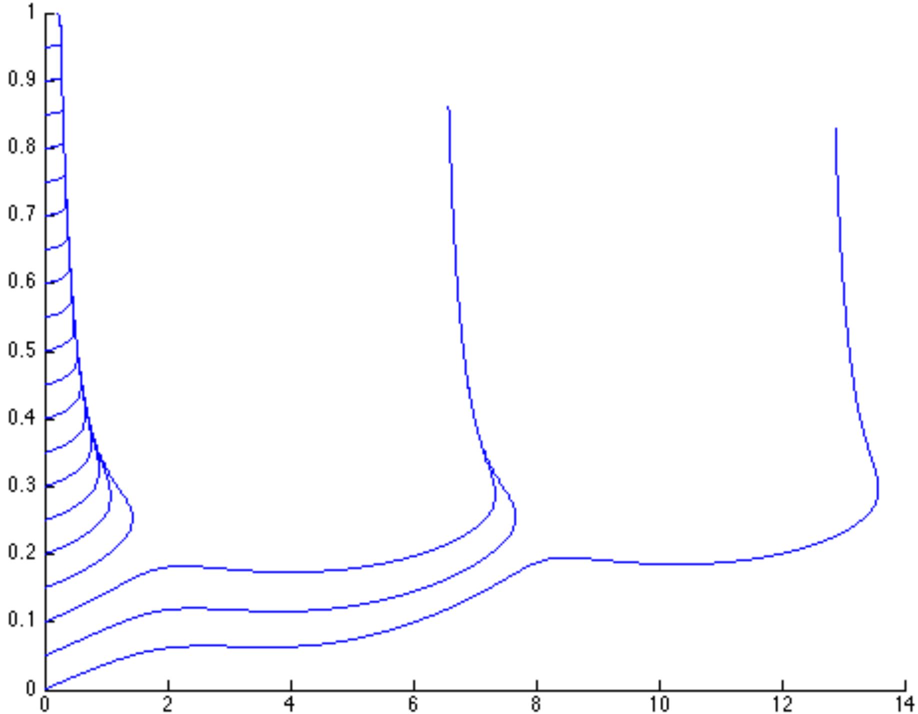


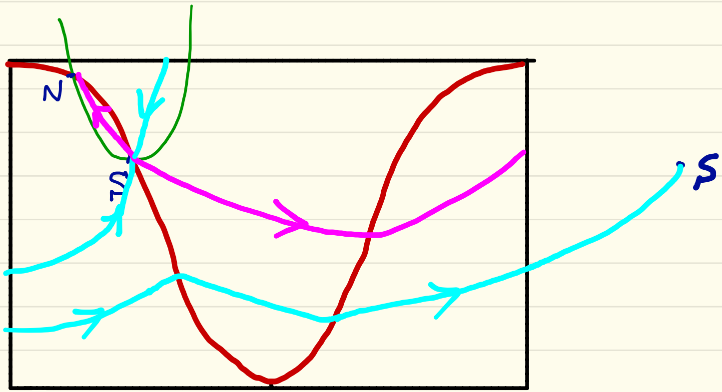
lucky case

Result 2: For $\Delta w < \mu$, we will have the lucky case.



Delta=0.5, epsilon=0.2, mu=1

$$\mu < \Delta\omega < M$$



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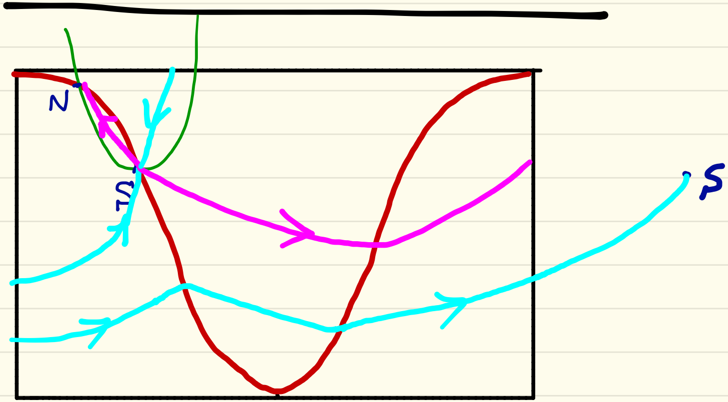
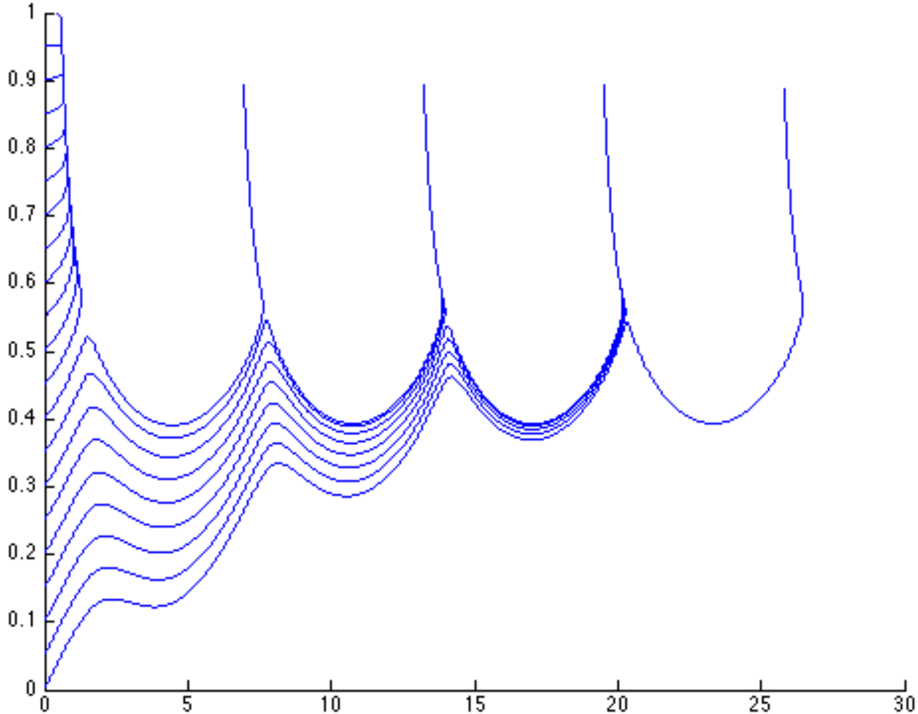
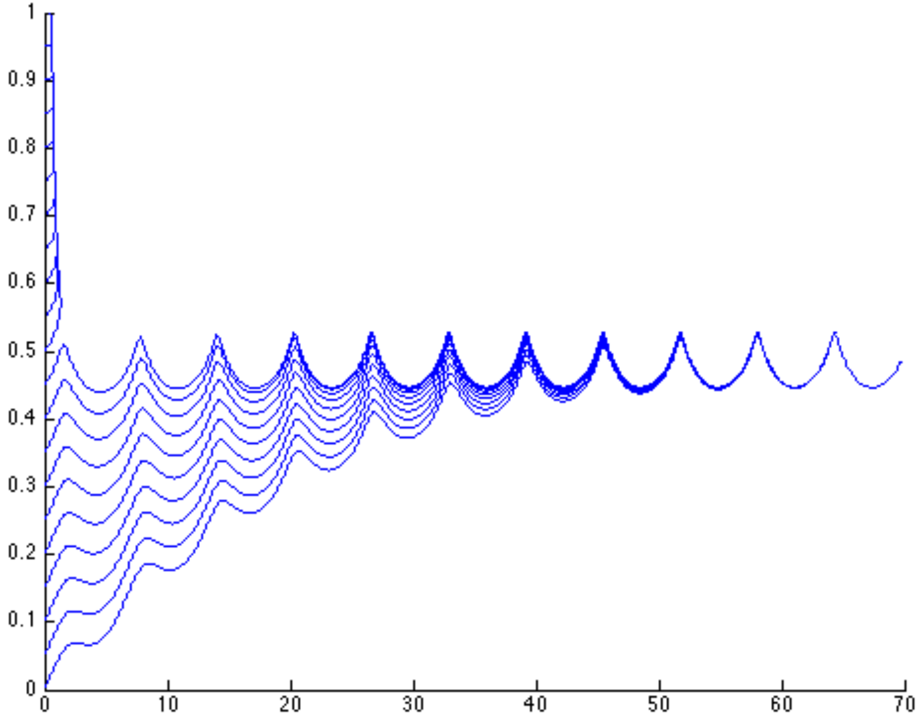


Result 3: There is $\varepsilon_0 > 0$, sth for $\varepsilon > \varepsilon_0$ the lucky case will be happened.

For $\varepsilon < \varepsilon_0$, there exists a periodic attractor solution



Delta=1.1, epsilon=0.1, mu=1



Delta=1.1, epsilon=0.05, mu=1

without plasticity

$\varphi = \theta_1 - \theta_2 \implies \dot{\varphi} = \Delta\omega - 2K\sin\varphi$ where 'K' is a constant.

$\exists$ stable node $0 < \varphi_* < \pi/2$, $\sin\varphi_* = \dfrac{\Delta\omega}{K}$

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If $K \nearrow \mu$ then $\varphi_* \searrow 0$

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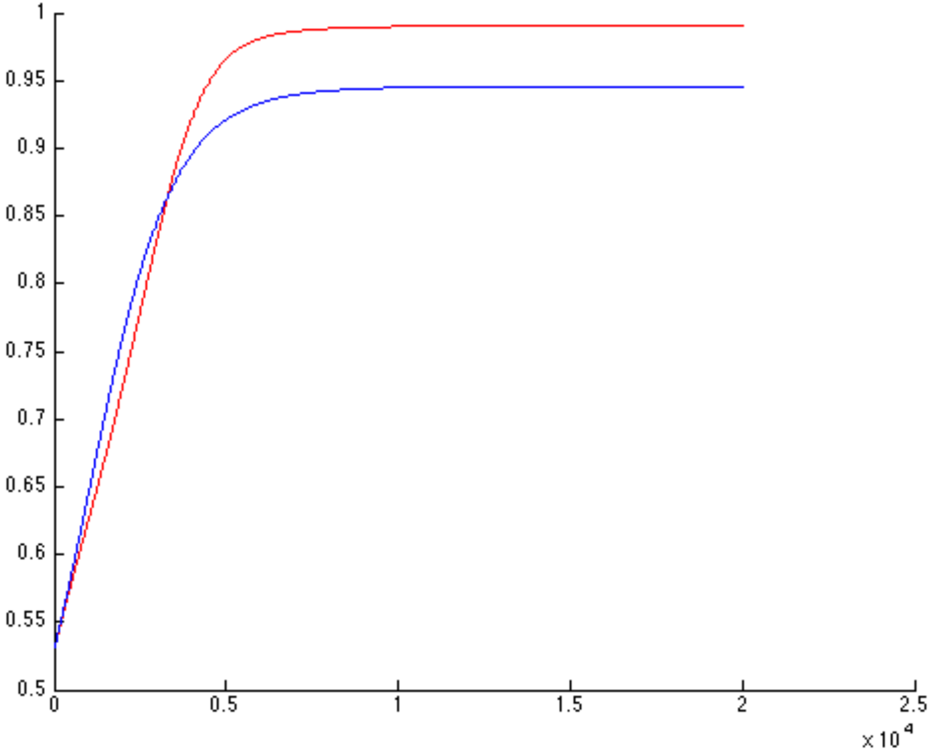
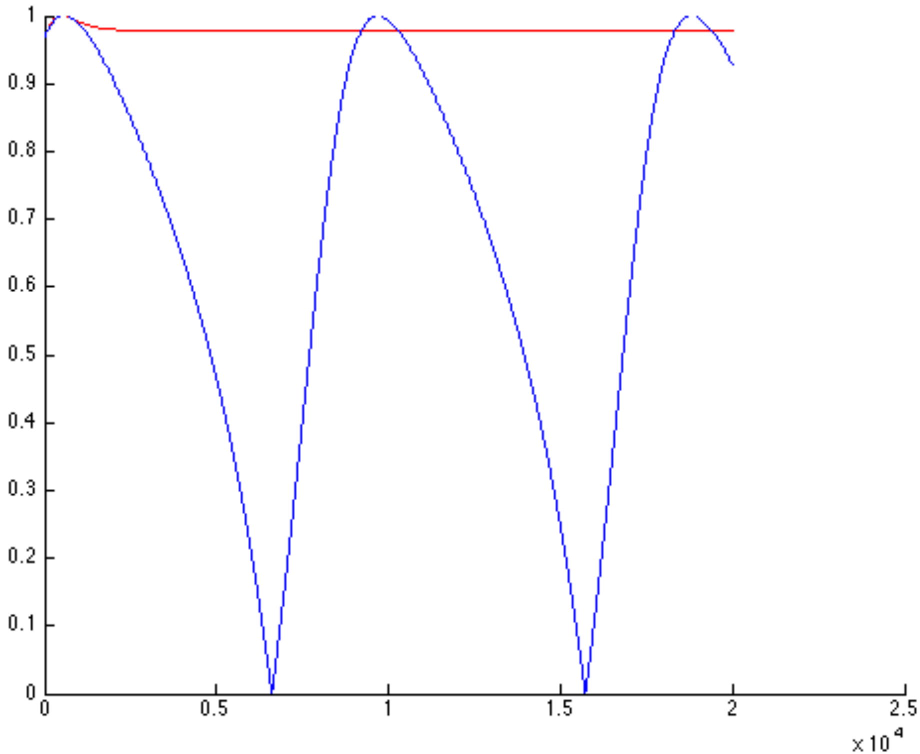
Question: Plasticity $\longrightarrow$ Synchrony?



1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0.5
0
0.5
1
1.5
2
2.5
x 10^4


1
0.9
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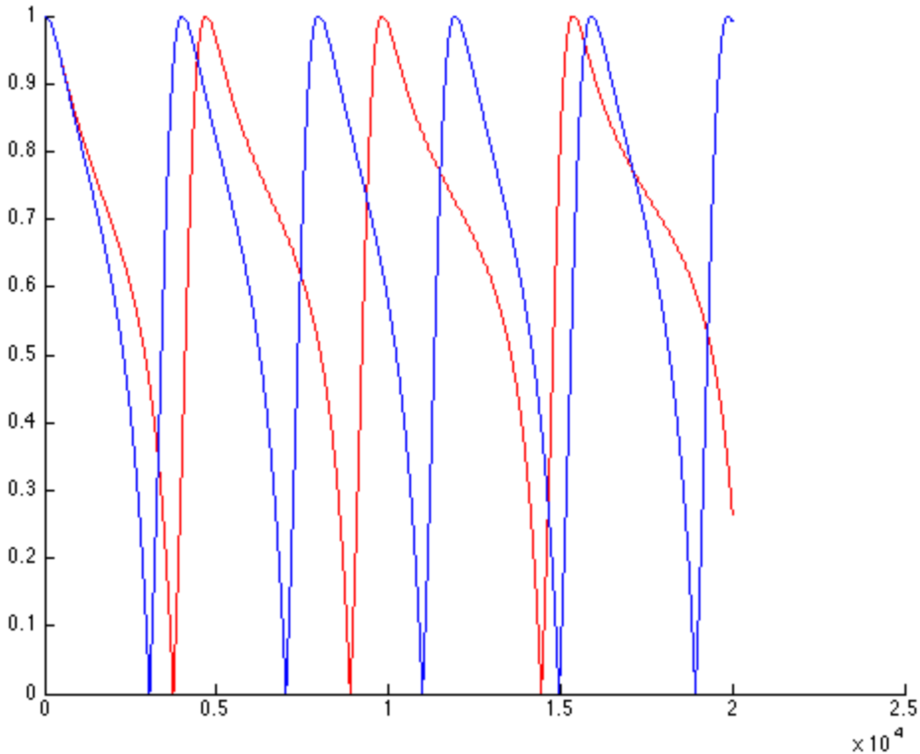

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