به نام خدا

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1. An influential idea in inflationary model-building is that the inflaton could be a pseudoscalar axion. At the perturbative level, an axion enjoys a continuous shift symmetry, but this is broken nonperturbatively to a discrete symmetry, leading to a potential of the form

$$V(\phi) = \frac{V_0}{2}[1 - cos(\frac{\phi}{f})]$$

where f is the axion decay constant. At what value ϕ_i close to $\phi = \pi f$ does the field have to start in order for the evolution to give more than 50 e-folds of inflation? The model is called natural inflation.

2. Consider single-field inflation with a more general kinetic term

$$S = \int d^4x\sqrt{-g}P(X, \phi)$$

where $P(X, \phi)$ is an arbitrary function of $X \equiv \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ and ϕ.

a) By varying the action with respect to the metric, show that this corresponds to a perfect fluid with pressure P and energy density.

$$\rho = 2XP_{,X} - P$$

where $P_{,X} \equiv dP/dX$. You may easily check that this gives the expected result for the case of slow-roll inflation, $P = X - V(\phi)$, namely $\rho = X + V$.

b) By varying the action with respect to ϕ, show that the equation of motion for the inflaton is

$$-\frac{d}{dt}(a^3P_{,X}\dot{\phi}) + a^3P_{,X} = 0$$

For slow-roll inflation, this gives the Klein-Gordon equation.

c) Show that the slow-roll parameter is

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{3XP_{,X}}{2XP_{,X} - P}$$

For suitable $P(X)$ this may lead to inflation even without a flat potential.