Special relativity

Problem Set 3

November 18, 2025

Problem 1

Accelerated Observer
A rocket in flat spacetime moves with constant proper acceleration a along the x-axis. An observer inside the rocket carries a clock measuring proper time τ.

I. show that with the additional condition that the magnitude of the 4-acceleration is constant, the worldline of the observer satisfies the differential equation $\eta_{\mu\nu}\frac{d^2x^\mu}{d\tau^2}\frac{dx^\nu}{d\tau} = 0$.

II. Show that the trajectory of the observer is given by the hyperbola

$$x^2 - t^2 = \frac{1}{a^2}.$$

III. Given an object with acceleration a_0 in its instantaneous rest frame S, find an expression for the acceleration in the inertial frame S', which is moving in the x-direction with velocity v relative to S. What is the maximal and minimal acceleration in S' depending on the direction of the acceleration based on your expression?

IV. Explain why an observer with constant proper acceleration never reaches or crosses the line $x = t$, even though at late times the observer's velocity approaches the speed of light.

V. Compute the proper time required for the occupants of a rocket ship to travel the ~ 30,000 light-years from the Earth to the center of the Galaxy. Assume that they maintain an acceleration of one "earth gravity" ($g \approx 10^3\,\mathrm{cm/s^2}$) for half the trip, and then decelerate at one earth gravity for the remaining half.

Problem 2

Velocity Addition

A spaceship S' moves relative to the Earth frame S along the x-axis with velocity $V = 0.6c$. Inside the spaceship, a probe moves with velocity $\vec{u}'$ relative to the spaceship. Its velocity components in S' are:

$$u'_x = 0.8c, \qquad u'_y = 0.5c.$$

I. Find the velocity components u_x and u_y of the probe in the Earth frame S using the relativistic velocity addition formulas.

II. Compute the magnitude $|\vec{u}|$ and the direction $\theta = \tan^{-1}(u_y/u_x)$ of the probe's velocity in the Earth frame.

III. Suppose now that another spaceship S'' moves relative to S' along the x-axis with velocity $W = 0.5c$. Compute the probe's velocity components in S'' and its speed $|\vec{u}''|$.

IV. why the velocities do not simply add as in Newtonian mechanics, and show numerically how $u_x < u'_x + V$.

Problem 3

Moving Light Source
A light source is moving at speed v and at an angle θ relative to the separation between the source and a stationary observer.

I. Consider a light pulse with frequency ω_0 in the rest frame of the source and determine the frequency ω measured by the observer.

II. Compute the angle θ for which $\omega = \omega_0$.

Problem 4

The Rindler Fast-Walker Paradox
A fast walker moves over a metal grid with spacing equal to the lRindler Fast Walkerength of his foot. From his frame, the grid is Lorentz contracted, so he should step safely. From the grid's frame, his foot is contracted, so he should fall. Explain what actually happens.

Problem 5

From the class lecture, identify any part that you find ambiguous or unclear. You may optionally include a suggested answer or comment, but it is sufficient to submit a single, clearly stated question.

The thinking man is a strange beast, always questioning even what is obvious.
from The Clouds, Aristophanes