Special relativity

Problem Set 1

October 24, 2025

Problem 1

Galilean Transformations as a Group

I. State the defining axioms of a group $(G, \circ)$, listing and briefly describing the four fundamental properties required for $(G, \circ)$ to form a group.

II. Define a *continuous group* and explain how it differs from a discrete group. Provide one physical example of each.

III. The Galilean transformation relates the space–time coordinates $(\vec{x}, t)$ in an inertial frame S to those $(\vec{x}', t')$ in another inertial frame S' moving with constant velocity $\vec{v}$ relative to S:

$$\vec{x}' = \vec{x} - \vec{v}t, \qquad t' = t.$$

Show that the set of all such transformations forms a group under composition. Identify explicitly the group parameters, the identity element, and the inverse transformation.

IV. The full Galilean group in three dimensions contains ten independent continuous parameters corresponding to spatial translations, time translations, spatial rotations, and boosts.

- List these ten parameters explicitly.
- Write the general Galilean transformation in terms of these parameters.

Problem 2

Galilean Transformation and the Non-Invariance of the Wave Equation

I. Consider a scalar field $u(x,t)$ defined as

$$u(x,t) = f(x - vt),$$

where v is a constant velocity. Show that this represents a wave propagating rigidly along the x-axis with speed v.

II. Show that $u(x,t)$ satisfies the first-order (transport) equation

$$\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial x}\right) u(x,t) = 0.$$

Now consider two inertial frames $S(x,t)$ and $S'(x',t')$ related by the Galilean transformation

$$x' = x - \mathscr{V}t, \qquad t' = t,$$

where $\mathscr{V}$ is the relative velocity between the two frames. Using the chain rule, show that

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t'} - \mathscr{V}\frac{\partial}{\partial x'}, \qquad \frac{\partial}{\partial x} = \frac{\partial}{\partial x'}.$$

Hence find an explicit expression for $\frac{\partial^2}{\partial t^2}$ in terms of derivatives with respect to x' and t'.

III. Consider the wave equation in S:

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2}.$$

Transform this equation to the S' frame using the relations above, and show explicitly that the resulting equation **in S' is not of the same form so classical wave equation is not form-invariant under the Galilean transformation.**

IV. Discuss whether this non-invariance poses a problem for classical physics. In particular, consider a wave representing sound propagation, and explain why it is reasonable that the equation is not form-invariant in this case. Then, contrast this with the case of light waves and explain why this led to a conceptual crisis that motivated the development of special relativity.

Problem 3

From the class lecture, identify any part that you find ambiguous or unclear. You may optionally include a suggested answer or comment, but it is sufficient to submit a single, clearly stated question.

Nature and Nature's laws lay hid in night:
God said "Let Newton be!" And all was light.
Alexander Pope (1688 – 1744)

It did not last: The Devil howling "Ho!
Let Einstein be!" restored the status quo.
John Collings Squire (1884 – 1958)