The First Part Of Exercises In String Theory Course

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I. ACTION OF A FREE MASSIVE PARTICLE IN MINKOWSKI SPACE-TIME

In this exercise, we are going to learn a bit more about the action of a free particle as a warm up.

Consider the below action

$$\mathcal{S}[x(t)] = -m \int_{\tau_0}^{\tau_1} d\tau \left[-\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}\eta_{\mu\nu} \right]^{1/2}. \tag{1}$$

1. Show this action is invariant under reparameterization. Find the infinitesimal of $x^\mu(\tau)$, under infinitesimal transformation $\tau \to \tau + \xi$.
2. Find equation of motion and all of primary and secondary constrains.
3. What is the form of Hamiltonian in this case? Is it consistent with Poisson bracket formalism which you have seen in classical mechanics? If there is any problem, solve it.
4. Show that every action with reparameterization symmetry, has a vanishing Hamiltonian.

II. ACTION OF FREE MASSLESS PARTICLE

In this exercise, our aim is to analyze the general action of causal particles.
Consider an action as follows

$$\mathcal{S}[x(t), \lambda] = \frac{1}{2}\int_{\tau_0}^{\tau_1} \lambda(\tau) \left[\lambda^{-2}(\tau)\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}\eta_{\mu\nu} - m^2 \right] d\tau \tag{2}$$

1. Under what condition for λ, this action would be reparameterizable?
2. Find equations of motion and all of primary and secondary constrains.
3. Check is the same as (1) in the massive case.
4. Work out equations of motion in the cases of $m = 0,\ \lambda = 1$ and also $m \neq 0,\ \lambda = 1/m$. Try to interpret these two gauge.

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