1. Changes in temperature variation within a plane rectangular element is assumed to be linear (i.e. nodal temperature changes $T_1^0$, $T_2^0$, $T_3^0$, and $T_4^0$ are known). How would you incorporate this changes in a thermal elastic analysis? Derive the load vector required to incorporate a temperature change. How would you calculate the element stresses?

2. Compute the plane strain stiffness matrix in terms of the ration $r = a/b$ for the rectangular element shown. The element has unit thickness. Use the following displacement model. Specialize the matrix for $\nu = 0.2$, $r=1$

$$u(x, y) = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 xy$$
$$v(x, y) = \alpha_5 + \alpha_6 x + \alpha_7 y + \alpha_8 xy$$

$$[k] = \begin{bmatrix}
22 & -13 & 2 & -11 & 7.5 & -1.5 & 1.5 & -7.5 \\
22 & -11 & 2 & 1.5 & -7.5 & 7.5 & -1.5 \\
22 & -13 & -1.5 & 7.5 & -7.5 & 1.5 \\
22 & -7.5 & 1.5 & -7.5 & 1.5 \\
22 & 2 & -13 & -11 \\
Symmetric & 22 & -11 & -13 \\
Symmetric & 22 & 2 \\
Symmetric & 22 & 2 \\
\end{bmatrix} \quad v=0 \text{ and } r=1$$
3. Introduce four secondary external nodes (one per side) to the rectangular element in problem no.2 above and obtain its stiffness equations. Adopt the following quadratic displacement model:

\[ u(x, y) = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 x^2 + \alpha_5 xy + \alpha_6 y^2 + \alpha_7 x^2 y + \alpha_8 xy^2 \]

\[ \nu(x, y) = \alpha_9 + \alpha_{10} x + \alpha_{11} y + \alpha_{12} x^2 + \alpha_{13} xy + \alpha_{14} y^2 + \alpha_{15} x^2 y + \alpha_{16} xy^2 \]