(A) Prove that

$$e^{iA\lambda} B e^{-iA\lambda} = B + i[A, B] + \frac{i}{2!}[A,[A,B]] + \cdots$$

(A) Prove that

$$e^{i\hat{P}a/\hbar} X e^{-i\frac{Pa}{\hbar}} = \hat{X} + a\mathbb{1}$$

(A) Calculate

$$e^{i\theta\sigma_z} \sigma_x e^{-i\theta\sigma_z}$$ for the Pauli operators

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

What does the result tell you?

(A) $\hat{X}|\phi\rangle = a|\phi\rangle \Longrightarrow |\phi\rangle = |a\rangle$

Can we always do this? What's special about $\hat{X}$?

Is this true for

$$A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & b \end{bmatrix} ?$$

$$\begin{bmatrix} 0 & 0 & 6 \end{bmatrix} \; :$$

A Find the factor A in transformation of $\psi(x) \to \tilde{\psi}(p)$.

A Show that $V(\bar{X})\,\psi(x) = \langle x | V(\hat{X}) | \psi \rangle = V(x)\,\psi(x)$

A Find $L\,\psi(x)$ for $\hat{L} = \hat{R} \times \hat{P}$

A " X in the P basis :

$$X\,\tilde{\psi}(p) = ?$$

A Find the evolution eq in the momentum space, i.e the differential eq. for $\tilde{\psi}(p)$.

A What's the dispersion relation correspo-ding to the Schrödinger's eq. ?

A For $|\psi(t=0)\rangle = |p\rangle$ find $\tilde{\psi}(p,t)$.

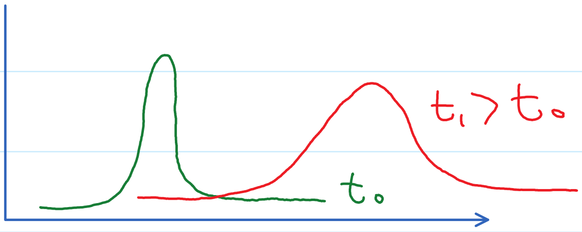
What's the $Pr(x)$ for this state? What's wrong with it?
Does it change with time?
What does such a wave function mean (say for electrons)?

(A) Take $\psi(x) = e^{-(x-x_0)^2/2\sigma^2}$ for $t=0$ and find

a) $\tilde{\psi}(p,0)$ b) $\psi(x,t)$

c) Does this have the normalization problem of plane waves?

d) Show that the probability $Pr(x)$ spreads out in time.



(A) Uncertainty Relation

Show that for $[\hat{X}, \hat{P}] = i\hbar \mathbb{1}$ we get

$$\Delta\hat{X}\,\Delta\hat{P} \geq \hbar/2$$

with

$$\Delta\hat{X} = \sqrt{\langle\hat{X}^2\rangle - \langle\hat{X}\rangle^2} \quad \& \quad \Delta\hat{P} = \sqrt{\langle\hat{P}^2\rangle - \langle P\rangle^2}.$$

See if you can generalize this for any two operators $\hat{A}, \hat{B}$ such that

$$[\hat{A}, \hat{B}] = \hat{C}$$ with $\hat{C}$ some operator.