Exercise 2

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Redo the model that we did for SGE & take

* the $\vec{X}_+ = \begin{pmatrix}1\\0\end{pmatrix}$ & $\vec{X}_- = \begin{pmatrix}0\\1\end{pmatrix}$.

* Find the operators for M_x, M_y & M_z.

(The matrix representation based on $|x+\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ & $|x_-\rangle = \begin{pmatrix}0\\1\end{pmatrix}$)

Repeat the formalism that we did for a setting where the box has 3 outcomes.

Show that $\left(\vec{\nu}_{in}^{\,\dagger} \cdot \vec{z}_+\right)\left(\vec{z}_+^{\,\dagger}\ \vec{\nu}_{in}\right) =$

$$\vec{\nu}_{in}^{\,\dagger} \cdot \Pi_{z_+} \cdot \nu_{in} \quad \text{with } \Pi_{z_+} = \vec{z}_+ \cdot \vec{z}_+^{\,\dagger}$$

→ Check that for the state $|\psi\rangle = \begin{pmatrix}\alpha\\ \beta\end{pmatrix}$, $|\alpha|^2 + |\beta|^2 = 1$, guarantees the probability normalization in all bases. (For all different measurements)

Unitary operators are normal operators and therefore there exists

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$$U = \sum_i u_i \, |i\rangle\langle i|$$

Show that $u_i = e^{i\phi}$.

For A, B in $\mathcal{L}(\mathcal{H})$ show that
$$\mathrm{Tr}\left([A, B]\right) = 0$$
for finite dimensional $\mathcal{H}$.

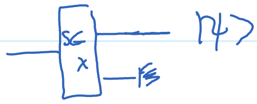
We showed that for $A \in \mathcal{L}(\mathcal{H})$

1. $A(t) = U^\dagger(t) \, A \, U(t)$ for $U(t) = e^{-iHt/\hbar}$.

(Assuming that H is time-independent.)

Show that
$$\frac{d\hat{A}(t)}{dt} = \frac{i}{\hbar}\left[A(t), H\right].$$

2. Show that this true for time-dependent Hamiltonians too (Hint: Consider $U(t \to t + \delta t)$ for $\delta t \to 0$).

Consider $|\psi\rangle$ from 

1) Calculate $\langle M_x \rangle$ & $\langle M_x^2 \rangle$ on $|\psi\rangle$.

Then calculate $\Delta M_x^2 = \langle M_x^2 \rangle - \langle M_x \rangle^2$.

2) Repeat the calculation in 1 for M_z.

Find an example of U such that
$UU^\dagger = \mathbb{1}$ but $UU^\dagger \neq U^\dagger U$.

Can the state be uniquely specified from the probability distribution? (see the example of the discrete position.)