Text Classification and Naïve Bayes

The Task of Text Classification

(Reading: MR+S Ch13)
Is this spam?

Subject: Important notice!
From: Stanford University <newsforum@stanford.edu>
Date: October 28, 2011 12:34:16 PM PDT
To: undisclosed-recipients:;

Greats News!

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Who wrote which Federalist papers?

• 1787-8: anonymous essays try to convince New York to ratify U.S Constitution: Jay, Madison, Hamilton.
• Authorship of 12 of the letters in dispute
• 1963: solved by Mosteller and Wallace using Bayesian methods

James Madison  
Alexander Hamilton
Male or female author?

1. By 1925 present-day Vietnam was divided into three parts under French colonial rule. The southern region embracing Saigon and the Mekong delta was the colony of Cochin-China; the central area with its imperial capital at Hue was the protectorate of Annam...

2. Clara never failed to be astonished by the extraordinary felicity of her own name. She found it hard to trust herself to the mercy of fate, which had managed over the years to convert her greatest shame into one of her greatest assets...

Positive or negative movie review?

• unbelievably disappointing
• Full of zany characters and richly applied satire, and some great plot twists
• this is the greatest screwball comedy ever filmed
• It was pathetic. The worst part about it was the boxing scenes.
What is the subject of this article?

MEDLINE Article

MeSH Subject Category Hierarchy

- Antagonists and Inhibitors
- Blood Supply
- Chemistry
- Drug Therapy
- Embryology
- Epidemiology
- ...

Dan Jurafsky
Text Classification

• Assigning subject categories, topics, or genres
• Spam detection
• Authorship identification
• Age/gender identification
• Language Identification
• Sentiment analysis
• ...

Text Classification: definition

- **Input:**
  - a document \( d \)
  - a fixed set of classes \( C = \{c_1, c_2, \ldots, c_J\} \)

- **Output:** a predicted class \( c \in C \)
Classification Methods: Hand-coded rules

- Rules based on combinations of words or other features
  - spam: black-list-address OR (“dollars” AND “have been selected”)
- Accuracy can be high
  - If rules carefully refined by expert
- But building and maintaining these rules is expensive
Classification Methods:
Supervised Machine Learning

- **Input:**
  - a document $d$
  - a fixed set of classes $C = \{c_1, c_2, \ldots, c_J\}$
  - A training set of $m$ hand-labeled documents $(d_1, c_1), \ldots, (d_m, c_m)$

- **Output:**
  - a learned classifier $\gamma : d \to c$
Classification Methods: Supervised Machine Learning

• Any kind of classifier
  • Naïve Bayes
  • Logistic regression
  • Support-vector machines
  • k-Nearest Neighbors

• ...
Text Classification and Naïve Bayes

Naïve Bayes (I)
Naïve Bayes Intuition

• Simple ("naïve") classification method based on Bayes rule
• Relies on very simple representation of document
  • Bag of words
I love this movie! It's sweet, but with satirical humor. The dialogue is great and the adventure scenes are fun... It manages to be whimsical and romantic while laughing at the conventions of the fairy tale genre. I would recommend it to just about anyone. I've seen it several times, and I'm always happy to see it again whenever I have a friend who hasn't seen it yet.
The bag of words representation

I love this movie! It's sweet, but with satirical humor. The dialogue is great and the adventure scenes are fun... It manages to be whimsical and romantic while laughing at the conventions of the fairy tale genre. I would recommend it to just about anyone. I've seen it several times, and I'm always happy to see it again whenever I have a friend who hasn't seen it yet.
The bag of words representation: using a subset of words

\[ Y(\ ) = c \]

- love
- sweet
- satirical
- great
- fun
- whimsical
- romantic
- laughing
- recommend
- several
- happy
- again
- happy
- again
The bag of words representation

\[ Y() \Rightarrow C \]

<table>
<thead>
<tr>
<th>Word</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>great</td>
<td>2</td>
</tr>
<tr>
<td>love</td>
<td>2</td>
</tr>
<tr>
<td>recommend</td>
<td>1</td>
</tr>
<tr>
<td>laugh</td>
<td>1</td>
</tr>
<tr>
<td>happy</td>
<td>1</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>
Bag of words for document classification
Text Classification and Naïve Bayes

Formalizing the Naïve Bayes Classifier
Bayes’ Rule Applied to Documents and Classes

- For a document \( d \) and a class \( c \)

\[
P(c \mid d) = \frac{P(d \mid c)P(c)}{P(d)}
\]
Naïve Bayes Classifier (I)

\[ c_{MAP} = \arg\max_{c \in C} P(c | d) \]

\[ = \arg\max_{c \in C} \frac{P(d | c)P(c)}{P(d)} \]

\[ = \arg\max_{c \in C} P(d | c)P(c) \]

MAP is “maximum a posteriori” = most likely class
Bayes Rule
Dropping the denominator
Naïve Bayes Classifier (II)

\[ c_{MAP} = \arg\max_{c \in C} P(d | c) P(c) \]

\[ = \arg\max_{c \in C} P(x_1, x_2, \ldots, x_n | c) P(c) \]
Naïve Bayes Classifier (IV)

\[ c_{MAP} = \arg \max_{c \in C} P(x_1, x_2, \ldots, x_n | c) P(c) \]

- \( O(|X|^n \cdot |C|) \) parameters
- Could only be estimated if a very, very large number of training examples was available.

How often does this class occur?
We can just count the relative frequencies in a corpus.
Multinomial Naïve Bayes Independence Assumptions

\[ P(x_1, x_2, \ldots, x_n | c) \]

- **Bag of Words assumption**: Assume position doesn’t matter
- **Conditional Independence**: Assume the feature probabilities \( P(x_i | c_j) \) are independent given the class \( c \).

\[ P(x_1, \ldots, x_n | c) = P(x_1 | c) \cdot P(x_2 | c) \cdot P(x_3 | c) \cdot \ldots \cdot P(x_n | c) \]
Multinomial Naïve Bayes Classifier

\( c_{MAP} = \arg\max_{c \in C} P(x_1, x_2, \ldots, x_n \mid c) P(c) \)

\( c_{NB} = \arg\max_{c \in C} P(c) \prod_{x \in X} P(x \mid c) \)
Applying Multinomial Naive Bayes Classifiers to Text Classification

positions ← all word positions in test document

\[ c_{NB} = \arg \max_{c_j \in C} P(c_j) \prod_{i \in \text{positions}} P(x_i \mid c_j) \]
Text Classification and Naïve Bayes

Naïve Bayes: Learning
Learning the Multinomial Naïve Bayes Model

• First attempt: maximum likelihood estimates
  • simply use the frequencies in the data

\[
\hat{P}(c_j) = \frac{N(C = c_j)}{N}
\]

\[
\hat{P}(x_i \mid c_j) = \frac{N(X_i = x_i, C = c_j)}{N(C = c_j)}
\]
Parameter estimation

\[ \hat{P}(X_i = w \mid c_j) = \text{fraction of times word } w \text{ appears among all words in documents of topic } c_j \]

- Create mega-document for topic \( j \) by concatenating all docs in this topic
  - Use frequency of \( w \) in mega-document
Problem with Maximum Likelihood

- What if we have seen no training documents with the word *fantastic* and classified in the topic *positive* (*thumbs-up*)?

  \[
  \hat{P}(x_i = "fantastic" \mid C = pos) = \frac{N(x_i = "fantastic", C = pos)}{N(C = pos)} = 0
  \]

- Zero probabilities cannot be conditioned away, no matter the other evidence!

  \[
  c_{MAP} = \arg\max_c \hat{P}(c) \prod_i \hat{P}(x_i \mid c)
  \]
**Smoothing the Naïve Bayes estimates**

- **Laplace (add-1):**
  \[
  \hat{P}(x_i \mid c_j) = \frac{N(X_i = x_i, C = c_j) + 1}{N(C = c_j) + k}
  \]
  
  \# of values of \(X_i\)

- **Applied to text:**
  \[
  \hat{P}(w \mid c) = \frac{\text{count}(w, c) + 1}{\text{count}(c) + |V|}
  \]
Multinomial Naïve Bayes: Learning

- From training corpus, extract *Vocabulary*

- Calculate $P(c_j)$ terms
  - For each $c_j$ in $C$ do
    
    $\text{docs}_j \leftarrow$ all docs with class $= c_j$

    $P(c_j) \leftarrow \frac{|\text{docs}_j|}{|\text{total # documents}|}$

- Calculate $P(w_k \mid c_j)$ terms
  - $\text{Text}_j \leftarrow$ single doc containing all $\text{docs}_j$
  - For each word $w_k$ in *Vocabulary*
    
    $n_k \leftarrow$ # of occurrences of $w_k$ in $\text{Text}_j$

    $P(w_k \mid c_j) \leftarrow \frac{n_k + \alpha}{N + \alpha \mid V o c a b u l a r y |}$
Laplace (add-1) smoothing: unknown words

Add one extra word to the vocabulary, the “unknown word” $w_u$

$$\hat{P}(w_u | c) = \frac{\text{count}(w_u, c) + 1}{\left( \sum_{w \in \mathcal{V}} \text{count}(w, c) \right) + |V + 1|}$$

$$= \frac{1}{\left( \sum_{w \in \mathcal{V}} \text{count}(w, c) \right) + |V + 1|}$$
Text Classification and Naïve Bayes

Naïve Bayes: Relationship to Language Modeling
Generative Model for Multinomial Naïve Bayes

\[
c = \text{China} \\
X_1 = \text{Shanghai} \\
X_2 = \text{and} \\
X_3 = \text{Shenzhen} \\
X_4 = \text{issue} \\
X_5 = \text{bonds}
\]
Naïve Bayes and Language Modeling

• Naïve bayes classifiers can use any sort of feature
  • URL, email address, dictionaries, network features

• But if, as in the previous slides
  • We use only word features
    • we use all of the words in the text (not a subset)

• Then
  • Naïve bayes has an important similarity to language modeling.
Each class = a unigram language model

- Assigning each word: $P(\text{word} \mid c)$
- Assigning each sentence: $P(s \mid c) = \prod P(\text{word} \mid c)$

<table>
<thead>
<tr>
<th>Class pos</th>
<th>l</th>
<th>love</th>
<th>this</th>
<th>fun</th>
<th>film</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>l</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>love</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>this</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.05</td>
<td>fun</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>film</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$P(s \mid \text{pos}) = 0.0000005$
Naïve Bayes as a Language Model

- Which class assigns the higher probability to \( s \)?

<table>
<thead>
<tr>
<th>Model pos</th>
<th>Model neg</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>I</td>
<td>I</td>
</tr>
<tr>
<td>0.1</td>
<td>0.001</td>
</tr>
<tr>
<td>love</td>
<td>love</td>
</tr>
<tr>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>this</td>
<td>this</td>
</tr>
<tr>
<td>0.05</td>
<td>0.005</td>
</tr>
<tr>
<td>fun</td>
<td>fun</td>
</tr>
<tr>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>film</td>
<td>film</td>
</tr>
</tbody>
</table>

\[
P(s|\text{pos}) > P(s|\text{neg})
\]
Text Classification and Naïve Bayes

Multinomial Naïve Bayes: A Worked Example
\[ \hat{P}(c) = \frac{N_c}{N} \]

\[ \hat{P}(w | c) = \frac{\text{count}(w, c) + 1}{\text{count}(c) + |V|} \]

**Priors:**

\[ P(c) = \frac{3}{4} \quad P(j) = \frac{1}{4} \]

**Conditional Probabilities:**

\[
\begin{align*}
\text{P(Chinese | c)} &= \frac{5+1}{8+6} = \frac{6}{14} = \frac{3}{7} \\
\text{P(Tokyo | c)} &= \frac{0+1}{8+6} = \frac{1}{14} \\
\text{P(Japan | c)} &= \frac{0+1}{3+6} = \frac{1}{14} \\
\text{P(Chinese | j)} &= \frac{1+1}{3+6} = \frac{2}{9} \\
\text{P(Tokyo | j)} &= \frac{1+1}{3+6} = \frac{2}{9} \\
\text{P(Japan | j)} &= \frac{1+1}{3+6} = \frac{2}{9}
\end{align*}
\]

<table>
<thead>
<tr>
<th>Doc</th>
<th>Words</th>
<th>Class</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Training</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 Chinese Beijing Chinese</td>
<td>c</td>
</tr>
<tr>
<td></td>
<td>2 Chinese Chinese Shanghai</td>
<td>c</td>
</tr>
<tr>
<td></td>
<td>3 Chinese Macao</td>
<td>c</td>
</tr>
<tr>
<td></td>
<td>4 Tokyo Japan Chinese</td>
<td>j</td>
</tr>
<tr>
<td></td>
<td>Test</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5 Chinese Chinese Chinese Tokyo Japan</td>
<td>?</td>
</tr>
</tbody>
</table>

**Choosing a class:**

\[
\begin{align*}
P(c | d5) &\propto \frac{3}{4} \times \left(\frac{3}{7}\right)^3 \times \frac{1}{14} \times \frac{1}{14} \\
&\approx 0.0003
\end{align*}
\]

\[
\begin{align*}
P(j | d5) &\propto \frac{1}{4} \times \left(\frac{2}{9}\right)^3 \times \frac{2}{9} \times \frac{2}{9} \\
&\approx 0.0001
\end{align*}
\]
Naïve Bayes in Spam Filtering

- SpamAssassin Features:
  - Online Pharmacy
  - Mentions millions of (dollar) ((dollar) NN,NNN,NNN.NN)
  - From: starts with many numbers
  - Subject is all capitals
  - HTML has a low ratio of text to image area
  - One hundred percent guaranteed
  - Claims you can be removed from the list
  - 'Prestigious Non-Accredited Universities'
Summary: Naive Bayes is Not So Naive

- Very Fast, low storage requirements
- Robust to Irrelevant Features
  Irrelevant Features cancel each other without affecting results
- Very good in domains with many equally important features
  Decision Trees suffer from fragmentation in such cases – especially if little data
- Optimal if the independence assumptions hold: If assumed independence is correct, then it is the Bayes Optimal Classifier for problem
- A good dependable baseline for text classification
  - But we will see other classifiers that give better accuracy
Text Classification and Naïve Bayes

Text Classification: Evaluation
The 2-by-2 contingency table

<table>
<thead>
<tr>
<th></th>
<th>correct</th>
<th>not correct</th>
</tr>
</thead>
<tbody>
<tr>
<td>selected</td>
<td>tp</td>
<td>fp</td>
</tr>
<tr>
<td>not selected</td>
<td>fn</td>
<td>tn</td>
</tr>
</tbody>
</table>

Accuracy = (tp + tn) / (tp + fp + fn + tn)
Precision and recall

- **Precision**: % of selected items that are correct
- **Recall**: % of correct items that are selected

\[
P = \frac{tp}{tp + fp} \quad \text{and} \quad R = \frac{tp}{tp + fn}
\]

\[
F1 = \frac{2PR}{P + R}
\]

<table>
<thead>
<tr>
<th></th>
<th>correct</th>
<th>not correct</th>
</tr>
</thead>
<tbody>
<tr>
<td>selected</td>
<td>tp</td>
<td>fp</td>
</tr>
<tr>
<td>not selected</td>
<td>fn</td>
<td>tn</td>
</tr>
</tbody>
</table>
A combined measure: F

- A combined measure that assesses the P/R tradeoff is F measure (weighted harmonic mean):

\[
F = \frac{1}{\alpha \frac{1}{P} + (1-\alpha) \frac{1}{R}} = \frac{(\beta^2 + 1)PR}{\beta^2 P + R}
\]

- The harmonic mean is a very conservative average
- People usually use balanced F1 measure
  - i.e., with \( \beta = 1 \) (that is, \( \alpha = \frac{1}{2} \)):
    \[
    F1 = \frac{2PR}{(P+R)}
    \]
Text Classification and Naïve Bayes

Text Classification: Evaluation
More Than Two Classes: Sets of binary classifiers

- Dealing with *any-of* or *multivalue* classification
  - A document can belong to 0, 1, or >1 classes.

- For each class $c \in C$
  - Build a classifier $\gamma_c$ to distinguish $c$ from all other classes $c' \in C$

- Given test doc $d$,
  - Evaluate it for membership in each class using each $\gamma_c$
  - $d$ belongs to *any* class for which $\gamma_c$ returns true
More Than Two Classes: Sets of binary classifiers

• **One-of** or multinomial classification
  • Classes are mutually exclusive: each document in exactly one class

• For each class $c \in C$
  • Build a classifier $\gamma_c$ to distinguish $c$ from all other classes $c' \in C$

• Given test doc $d$,
  • Evaluate it for membership in each class using each $\gamma_c$
  • $d$ belongs to the one class with maximum score
Evaluation:
Classic Reuters-21578 Data Set

• Most (over)used data set, 21,578 docs (each 90 types, 200 tokens)
• 9603 training, 3299 test articles
• 118 categories
  • An article can be in more than one category
  • Learn 118 binary category distinctions
• Average document (with at least one category) has 1.24 classes
• Only about 10 out of 118 categories are large

Common categories (#train, #test)
• Earn (2877, 1087)
• Acquisitions (1650, 179)
• Money-fx (538, 179)
• Grain (433, 149)
• Crude (389, 189)
• Trade (369,119)
• Interest (347, 131)
• Ship (197, 89)
• Wheat (212, 71)
• Corn (182, 56)
**Confusion matrix c**

- For each pair of classes \(<c_1, c_2>\) how many documents from \(c_1\) were incorrectly assigned to \(c_2\)?
  - \(c_{3,2}: 90\) wheat documents incorrectly assigned to poultry

<table>
<thead>
<tr>
<th>Docs in test set</th>
<th>Assigned UK</th>
<th>Assigned poultry</th>
<th>Assigned wheat</th>
<th>Assigned coffee</th>
<th>Assigned interest</th>
<th>Assigned trade</th>
</tr>
</thead>
<tbody>
<tr>
<td>True UK</td>
<td>95</td>
<td>1</td>
<td>13</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>True poultry</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>True wheat</td>
<td>10</td>
<td>90</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>True coffee</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>34</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>True interest</td>
<td>-</td>
<td>1</td>
<td>2</td>
<td>13</td>
<td>26</td>
<td>5</td>
</tr>
<tr>
<td>True trade</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>14</td>
<td>5</td>
<td>10</td>
</tr>
</tbody>
</table>
Per class evaluation measures

Recall:
Fraction of docs in class \( i \) classified correctly:

\[
\frac{c_{ii}}{\sum_j c_{ij}}
\]

Precision:
Fraction of docs assigned class \( i \) that are actually about class \( i \):

\[
\frac{c_{ii}}{\sum_j c_{ji}}
\]

Accuracy:
Fraction of docs classified correctly:

\[
\frac{\sum_i c_{ii}}{\sum_j \sum_i c_{ij}}
\]
Micro- vs. Macro-Averaging

- If we have more than one class, how do we combine multiple performance measures into one quantity?
- **Macroaveraging**: Compute performance for each class, then average.
- **Microaveraging**: Collect decisions for all classes, compute contingency table, evaluate.
Micro- vs. Macro-Averaging: Example

<table>
<thead>
<tr>
<th>Class 1</th>
<th>Class 2</th>
<th>Micro Ave. Table</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Truth: yes</td>
<td>10 90 100</td>
</tr>
<tr>
<td></td>
<td>Truth: no</td>
<td>10 10 20</td>
</tr>
<tr>
<td>Classifier: yes</td>
<td>10 90</td>
<td>Classifier: yes</td>
</tr>
<tr>
<td>Classifier: no</td>
<td>10 10</td>
<td>Classifier: no</td>
</tr>
</tbody>
</table>

- Macroaveraged precision: \((0.5 + 0.9)/2 = 0.7\)
- Microaveraged precision: \(100/120 = .83\)
- Microaveraged score is dominated by score on common classes
**Development Test Sets and Cross-validation**

- **Metric:** P/R/F1 or Accuracy
- **Unseen test set**
  - avoid overfitting (‘tuning to the test set’)
  - more conservative estimate of performance
- **Cross-validation over multiple splits**
  - Handle sampling errors from different datasets
  - Pool results over each split
  - Compute pooled dev set performance
Text Classification and Naïve Bayes

Text Classification: Practical Issues
The Real World

- Gee, I’m building a text classifier for real, now!
- What should I do?
No training data?
Manually written rules

If (wheat or grain) and not (whole or bread) then
Categorize as grain

- Need careful crafting
  - Human tuning on development data
  - Time-consuming: 2 days per class
Very little data?

• Use Naïve Bayes
  • Naïve Bayes is a “high-bias” algorithm (Ng and Jordan 2002 NIPS)
• Get more labeled data
  • Find clever ways to get humans to label data for you
• Try semi-supervised training methods:
  • Bootstrapping, EM over unlabeled documents, ...
A reasonable amount of data?

• Perfect for all the clever classifiers
  • SVM
  • Regularized Logistic Regression
• You can even use user-interpretable decision trees
  • Users like to hack
  • Management likes quick fixes
A huge amount of data?

• Can achieve high accuracy!

• At a cost:
  • SVMs (train time) or kNN (test time) can be too slow
  • Regularized logistic regression can be somewhat better

• So Naïve Bayes can come back into its own again!
Accuracy as a function of data size

- With enough data
  - Classifier may not matter

Brill and Banko on spelling correction
Real-world systems generally combine:

- Automatic classification
- Manual review of uncertain/difficult/"new" cases
Multiplying lots of probabilities can result in floating-point underflow.

Since \( \log(xy) = \log(x) + \log(y) \)

- Better to sum logs of probabilities instead of multiplying probabilities.
- Class with highest un-normalized log probability score is still most probable.

\[
c_{NB} = \arg\max_{c_j \in C} \log P(c_j) + \sum_{i \in \text{positions}} \log P(x_i \mid c_j)
\]

- Model is now just max of sum of weights
How to tweak performance

- Domain-specific features and weights: *very* important in real performance
- Sometimes need to collapse terms:
  - Part numbers, chemical formulas, ...
  - But stemming generally doesn’t help
- Upweighting: Counting a word as if it occurred twice:
  - title words *(Cohen & Singer 1996)*
  - first sentence of each paragraph *(Murata, 1999)*
  - In sentences that contain title words *(Ko *et al*, 2002)*