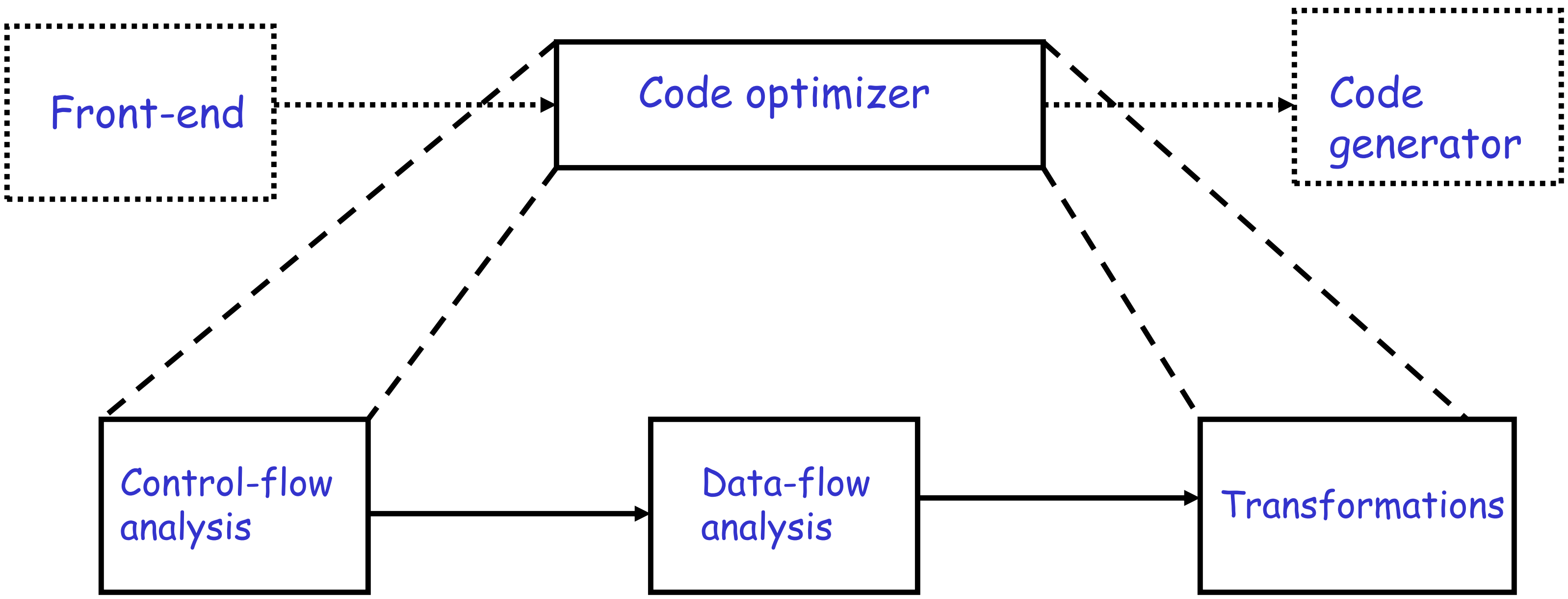
40-414 Compiler Design

Global Optimizations

Lecture 12

Organization of a Code Optimizer (Revisited)



Local Optimization

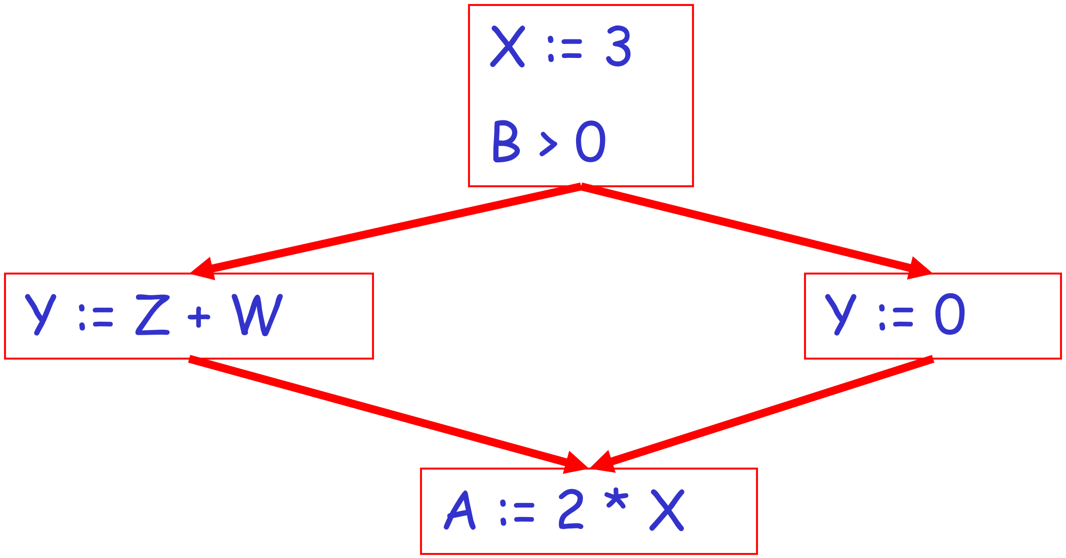
Recall the simple basic-block optimizations

- Constant propagation
- Dead code elimination

X := 3 Y := Z * W Q := X + Y	→	X := 3 Y := Z * W Q := 3 + Y	→	Y := Z * W Q := 3 + Y

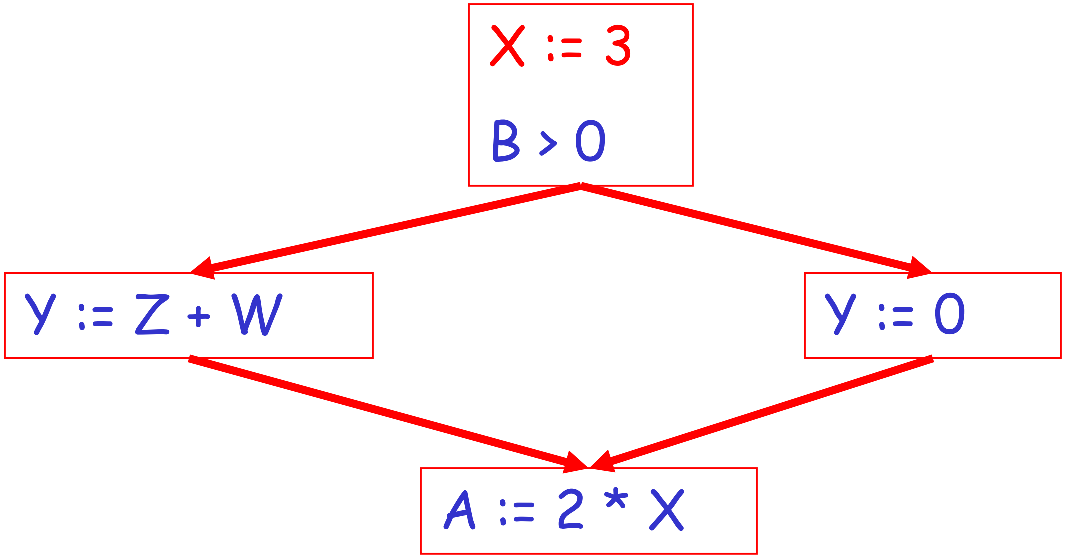
Global Optimization

These optimizations can be extended to an entire control-flow graph



Global Optimization

These optimizations can be extended to an entire control-flow graph



Global Optimization

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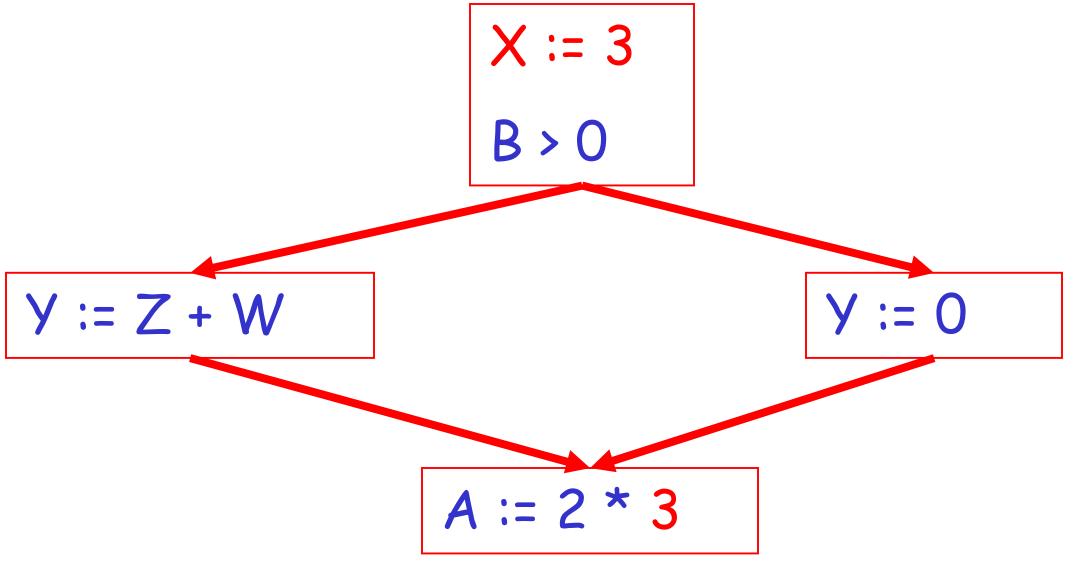
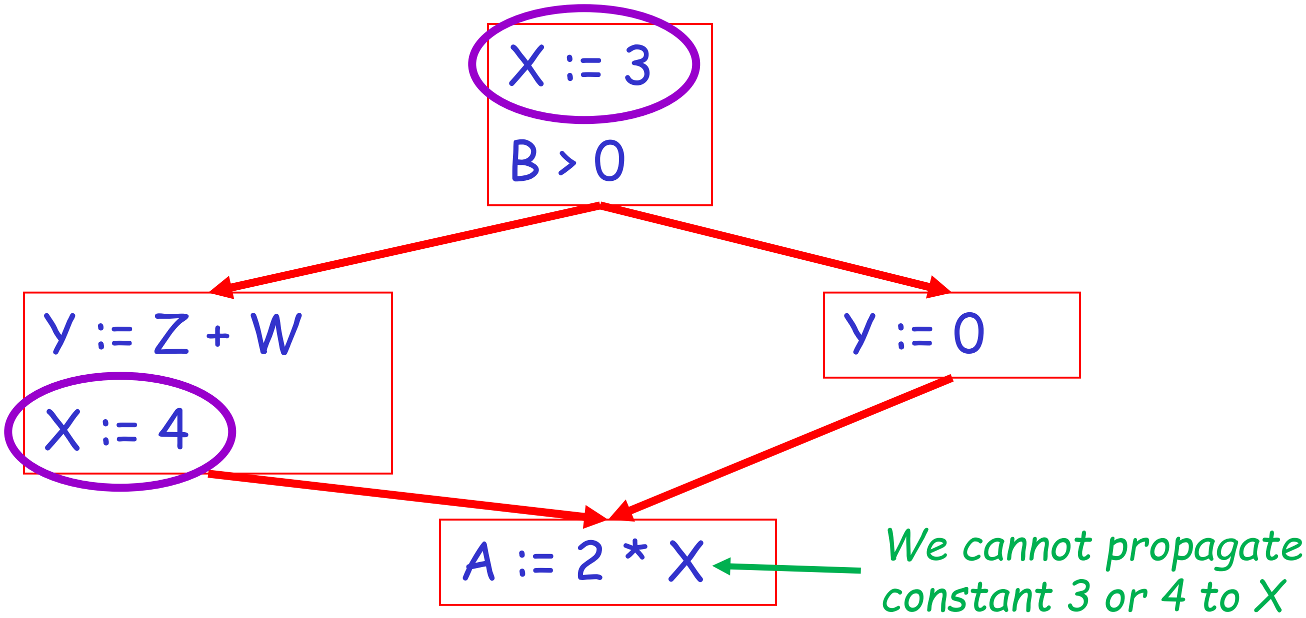


Correctness

- How do we know it is OK to globally propagate constants?
- There are situations where it is incorrect:

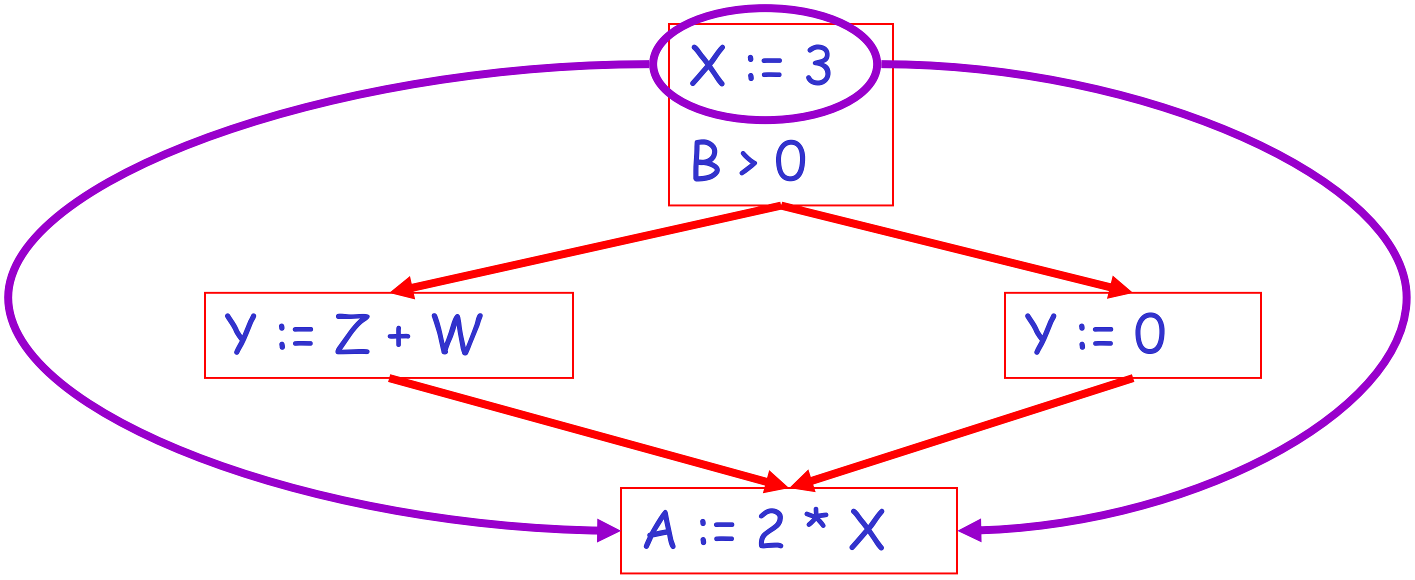


Correctness (Cont.)

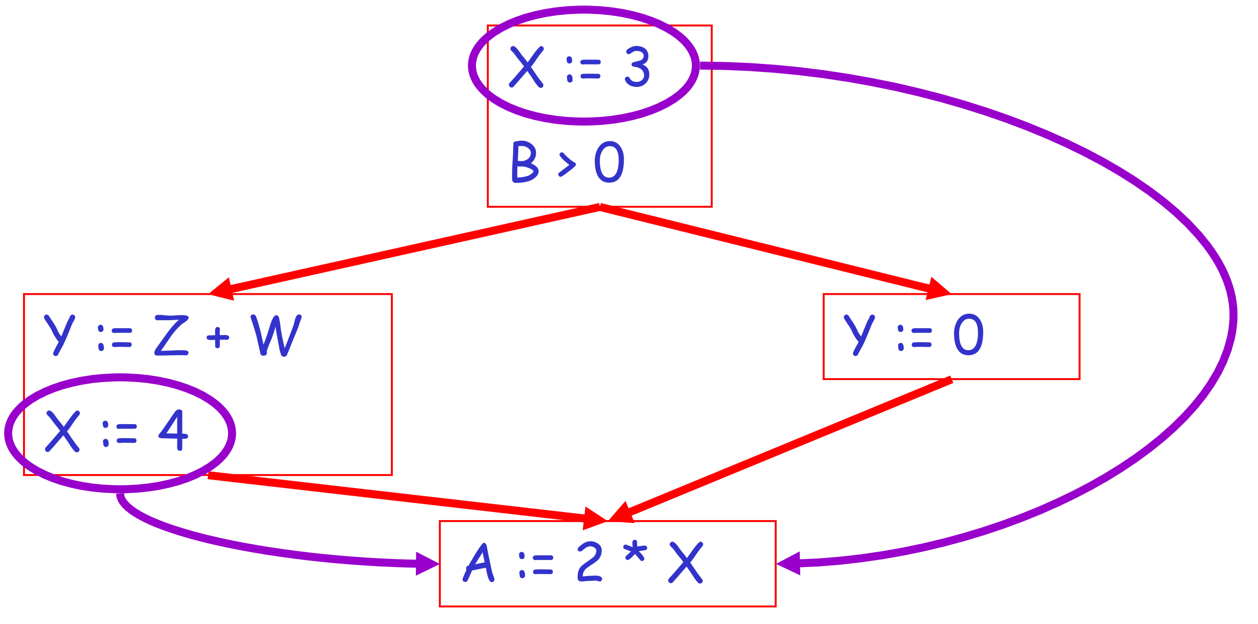
To replace a use of x by a constant k we must know that:

On every path to the use of x, the last assignment to x is x := k **

Example 1 Revisited



Example 2 Revisited



Discussion

- The correctness condition is not trivial to check

- "All paths" includes paths around loops and through branches of conditionals

- Checking the condition requires global dataflow analysis
  - An analysis of the entire control-flow graph

Global Analysis

Global optimization tasks share several traits:

- The optimization depends on knowing a property X at a particular point in program execution
- Proving X at any point requires knowledge of the entire program
- It is OK to be conservative. If the optimization requires X to be true, then want to know either
  - X is definitely true
  - Don't know if X is true (we don't do the optimization)

- It is always safe to say "don't know"

Global Analysis (Cont.)

- *Global dataflow analysis* is a standard technique for solving problems with these characteristics

- Global constant propagation is one example of an optimization that requires global dataflow analysis

Global Constant Propagation

- Global constant propagation can be performed at any point where ** holds

On every path to the use of x, the last assignment to x is x := k (**)

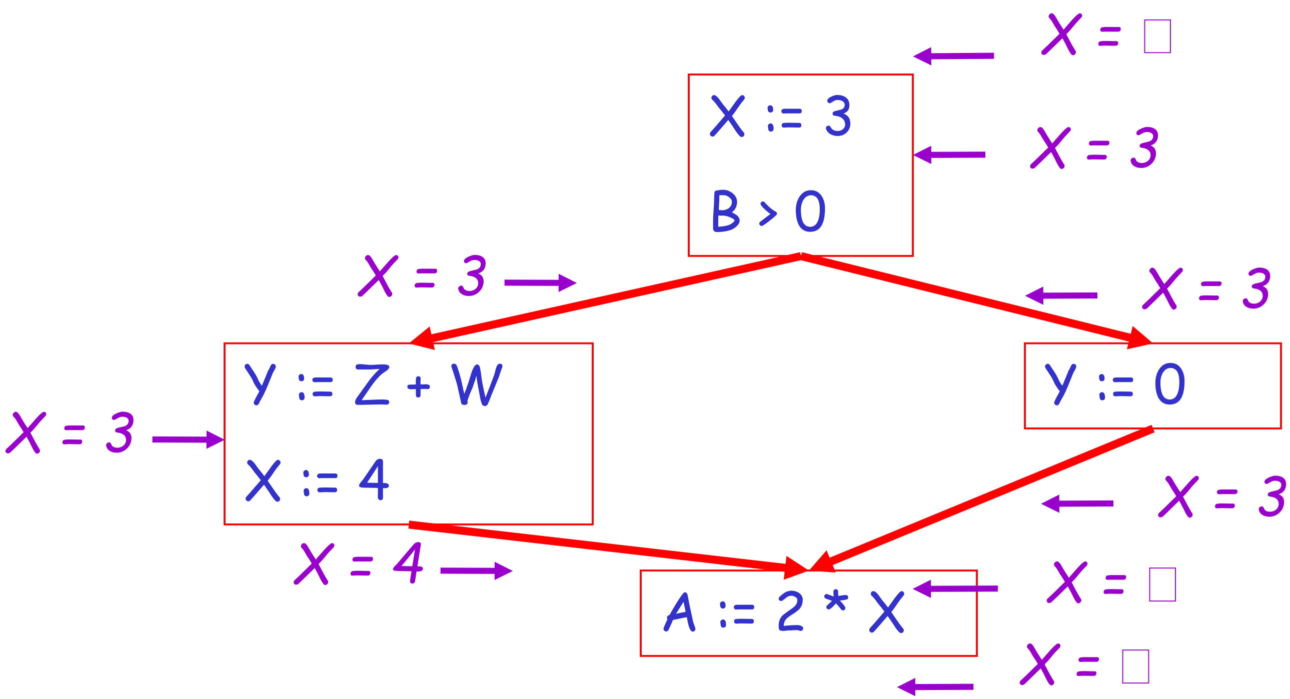
- Consider the case of computing ** for a single variable X at all program points

Global Constant Propagation (Cont.)

- To make the problem precise, we associate one of the following values with X at every program point

value	*interpretation*
⊗	This statement never executes
c	X = constant c
□	X is not a constant

Example



Using the Information

- Given global constant information, it is easy to perform the optimization
  - Simply inspect the $x = ?$ associated with a statement using x
  - If x is constant at that point replace that use of x by the constant

- But how do we compute the properties $x = ?$

The Idea

The analysis of a complicated program can be expressed as a combination of simple rules relating the change in information between adjacent statements

Explanation

- The idea is to "push" or "transfer" information from one statement to the next

- For each statement s, we compute information about the value of x immediately before and after s

  C(s,x,in) = value of x before s is executed

  C(s,x,out) = value of x after s is executed

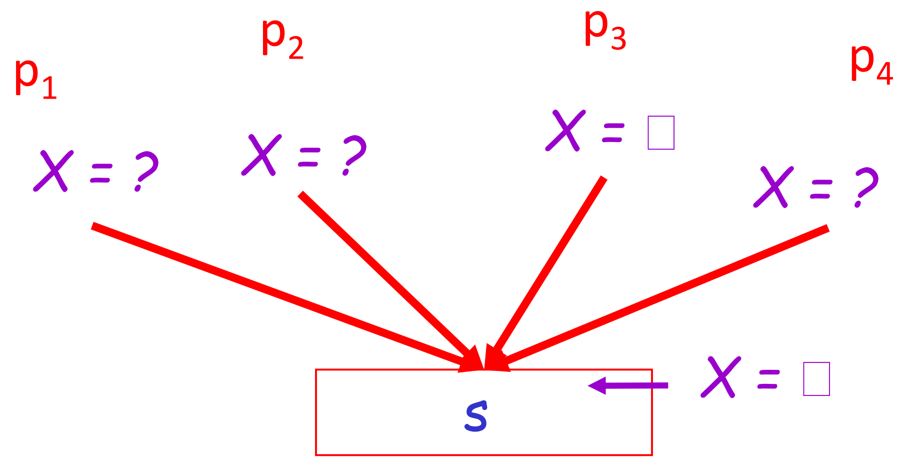
Stands for 'constant' information

Transfer Functions

- Define a *transfer* function that transfers information one statement to another

- In the following rules, let statement s have immediate predecessor statements $p_1,\dots,p_n$

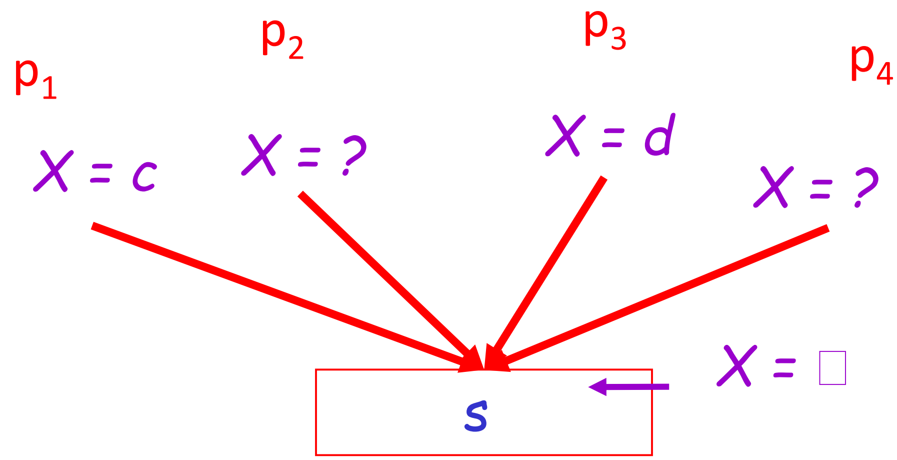
Rule 1



if $\exists_i$ ($C(p_i, x, out) = \square$)

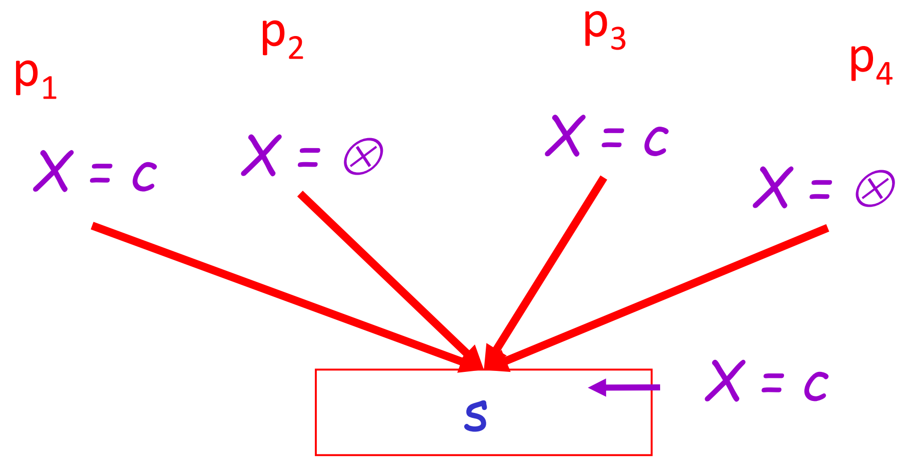
then $C(s, x, in) = \square$

Rule 2



$$\text{if } \exists_{i,j}\ (C(p_i, x, out) = c \ \& \ C(p_j, x, out) = d \ \& \ d <> c)$$
$$\text{then } C(s, x, in) = \square$$

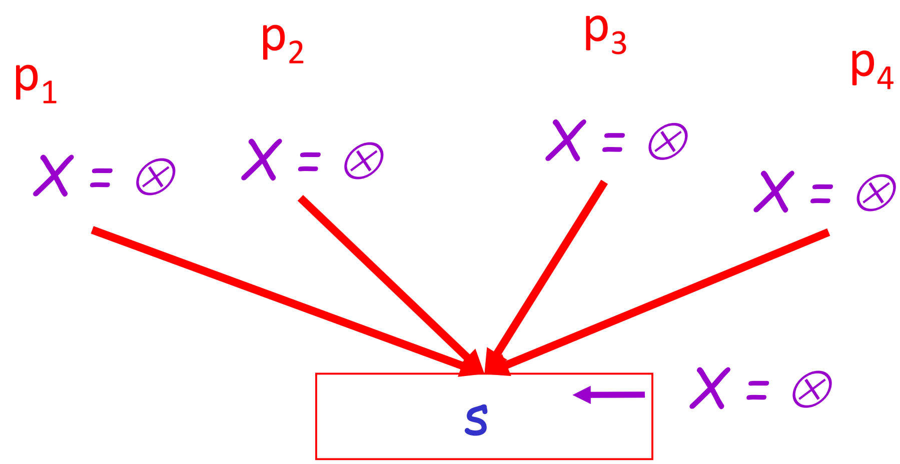
Rule 3



if $\forall_i$ ($C(p_i, x, out) = c$ or $\otimes$)

then $C(s, x, in) = c$

Rule 4



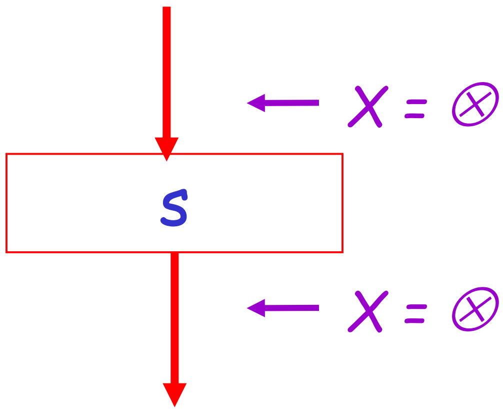
if $\forall_i$ ($C(p_i, x, out) = \otimes$)

then $C(s, x, in) = \otimes$

The Other Half

- Rules 1-4 relate the *out* of one statement to the *in* of the next statement
- Now we need rules relating the *in* of a statement to the *out* of the same statement

Rule 5



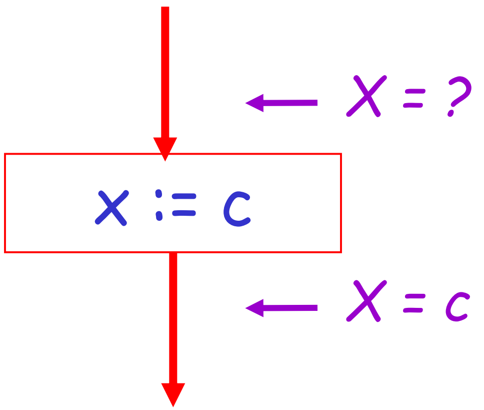
if $C(s, x, in) = \otimes$

then $C(s, x, out) = \otimes$

Rule 6

Rule 6 has a lower priority than Rule 5

R6 is checked only when R5 cannot be applied



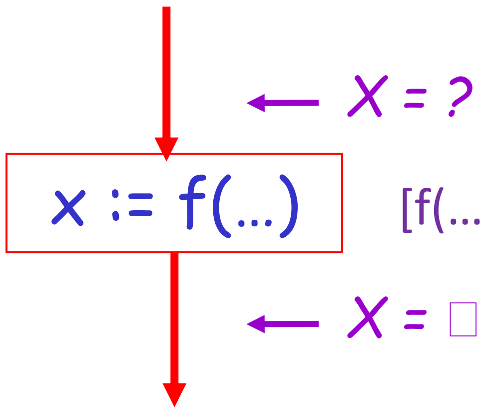
if c is a constant

then C(x := c, x, out) = c

Rule 7

Rule 7 has a lower priority than Rule 5

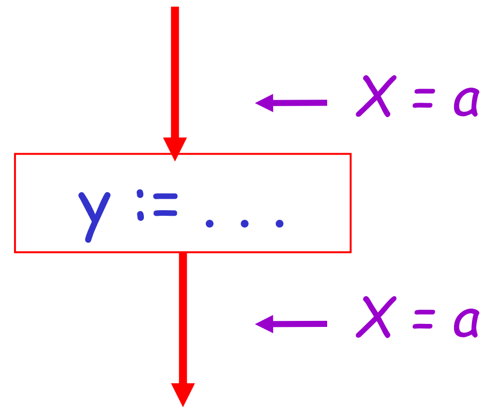
R7 is checked only when R5 cannot be applied



[f(...) is anything except a constant]

C(x := f(...), x, out) = □

Rule 8



if x <> y

then C(y := …, x, out) = C(y := …, x, in)

An Algorithm

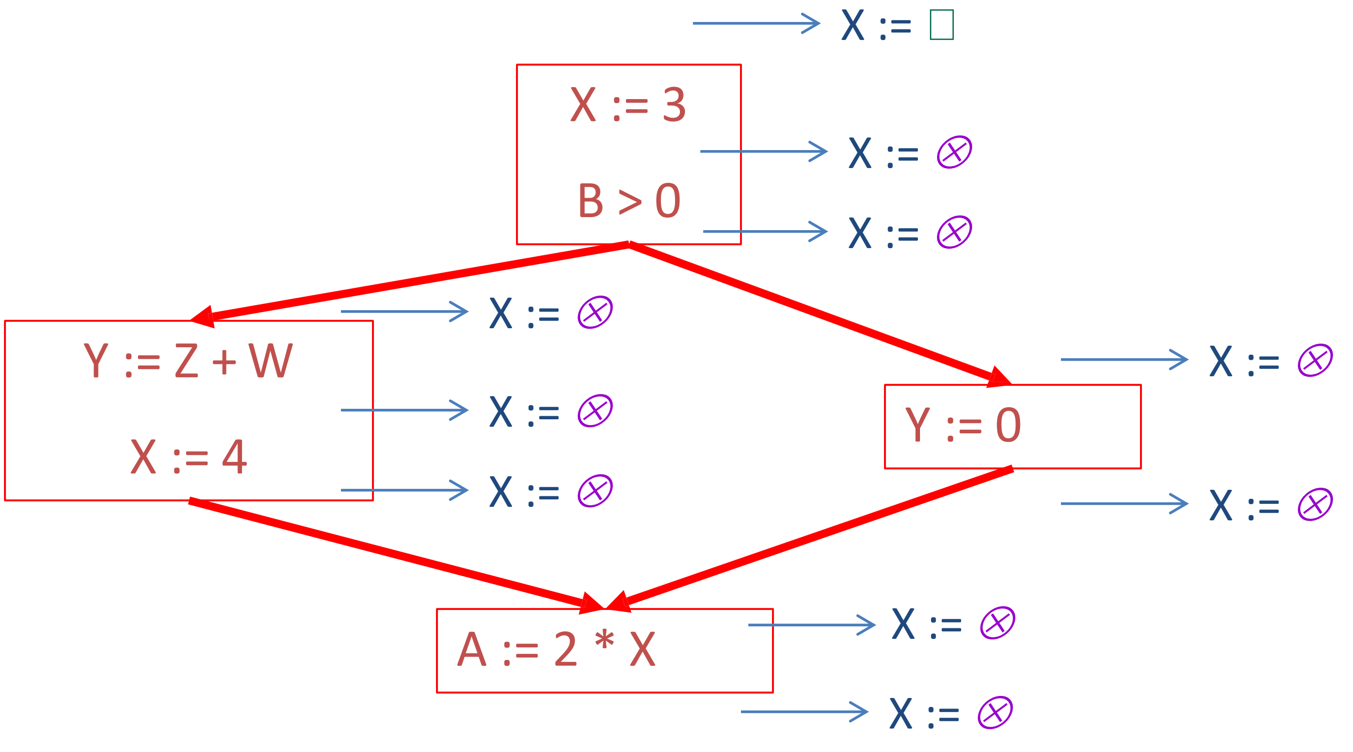
1. For every entry *s* to the program, set $C(s, x, in) = \square$

2. Set $C(s, x, in) = C(s, x, out) = \otimes$ everywhere else

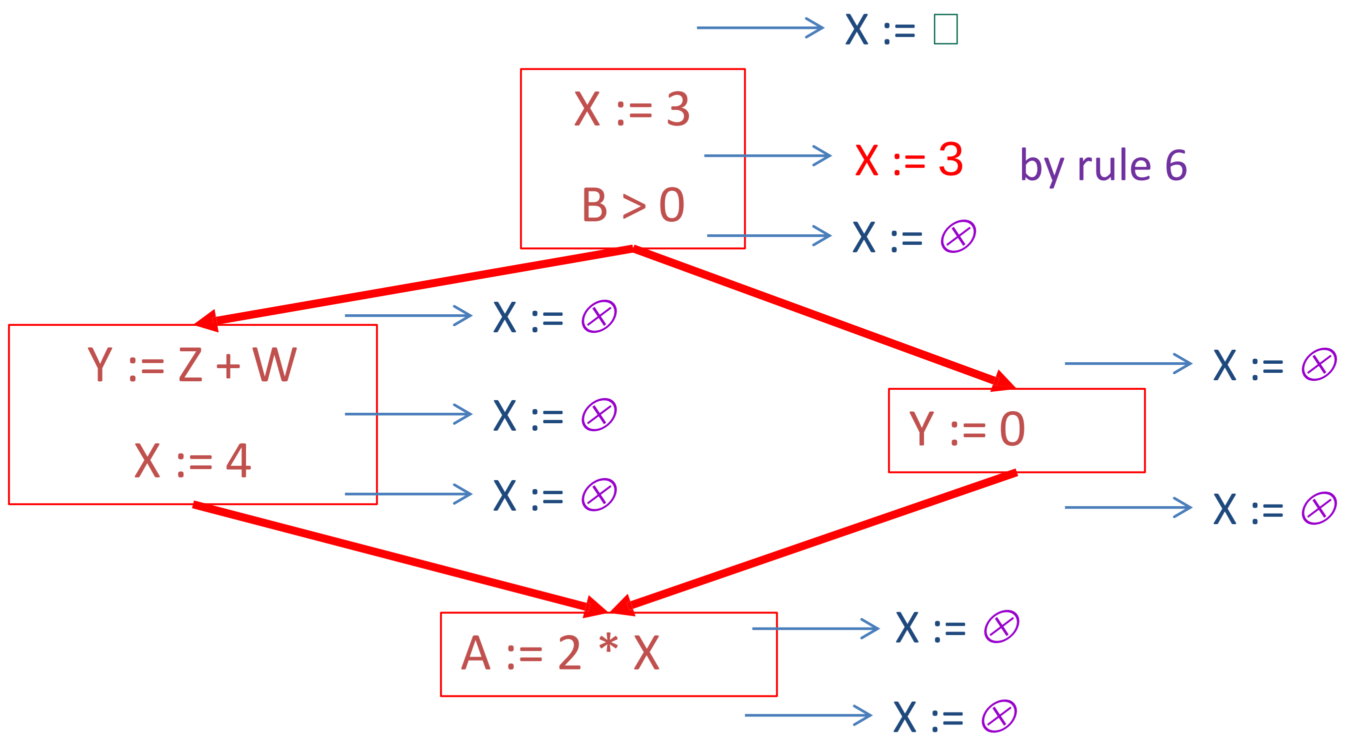
3. Repeat until all points satisfy 1-8:

   Pick *s* not satisfying 1-8 and update using the appropriate rule

Constant Propagation

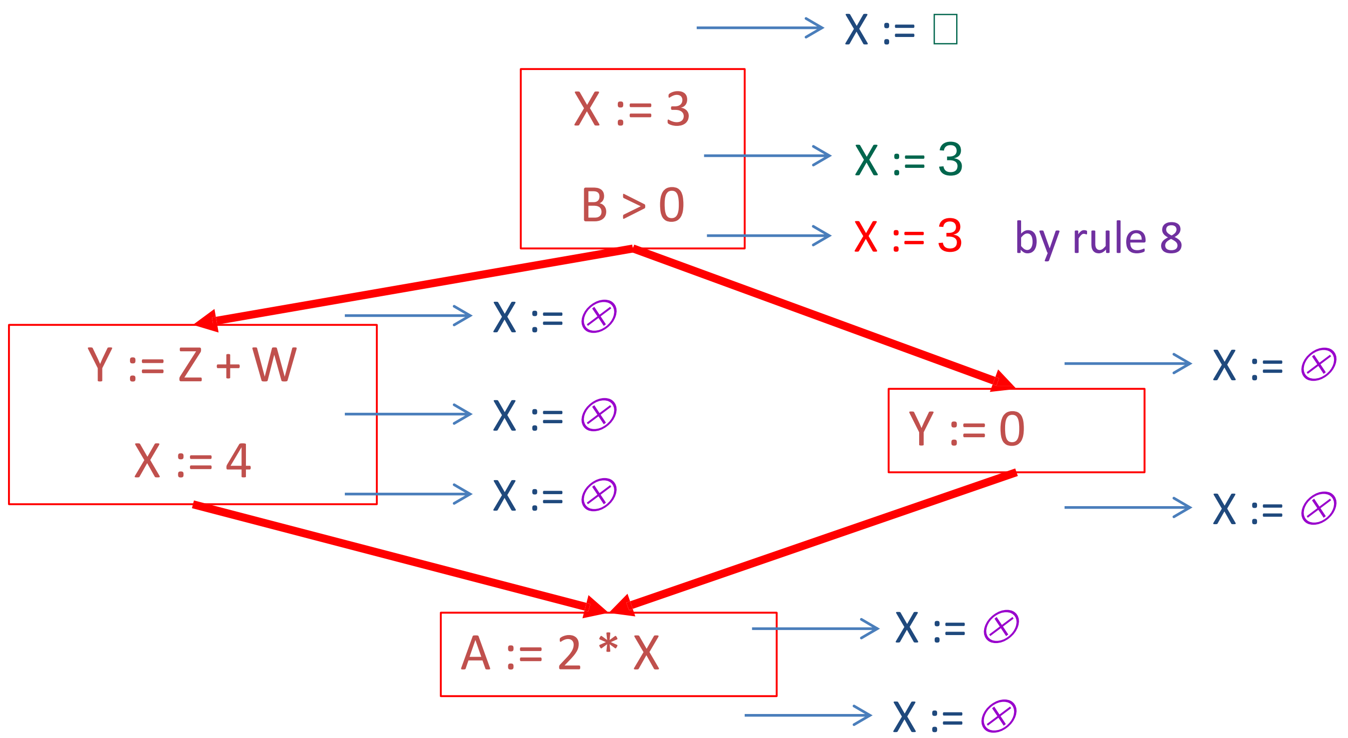


Constant Propagation



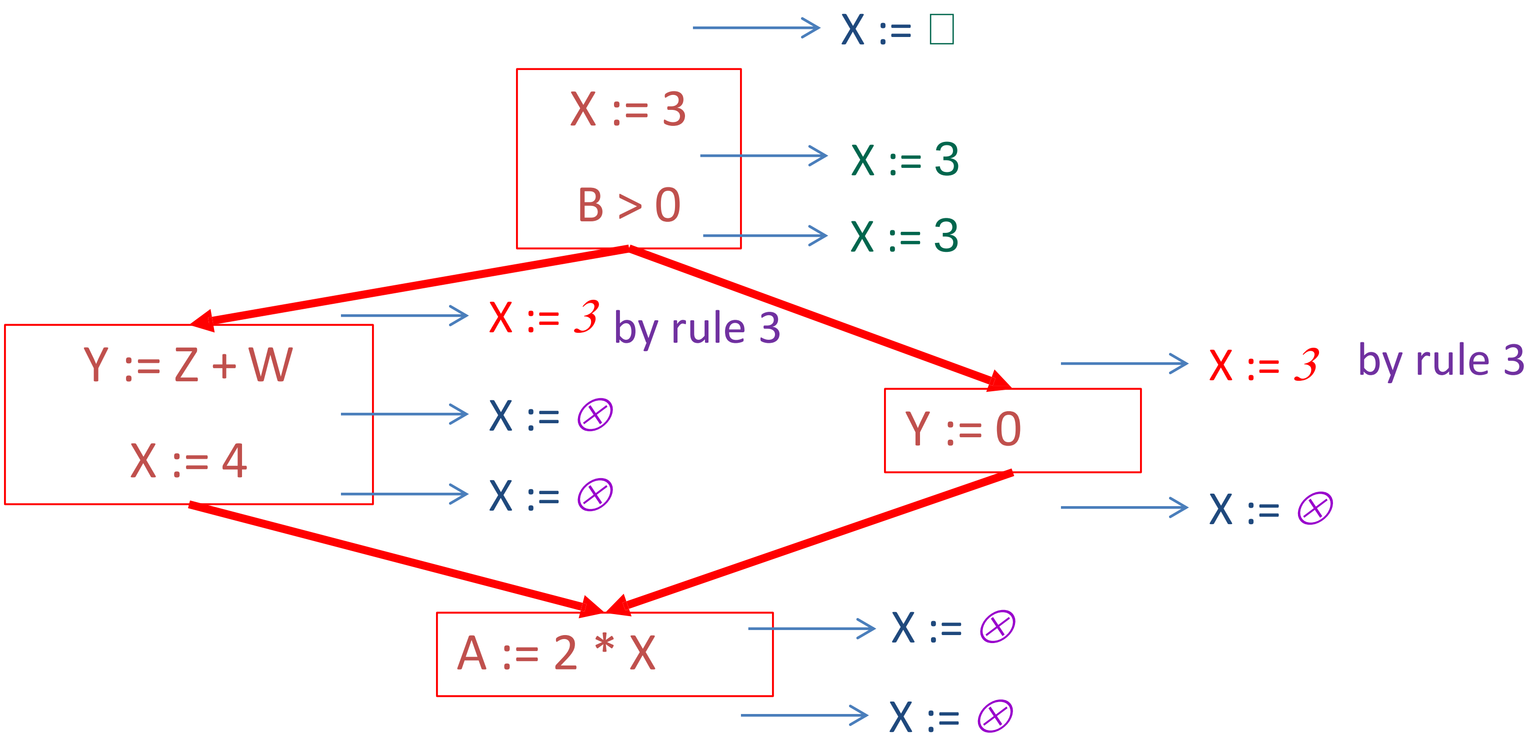
Rule 6: if c is a constant then C(x := c, x, out) = c

Constant Propagation



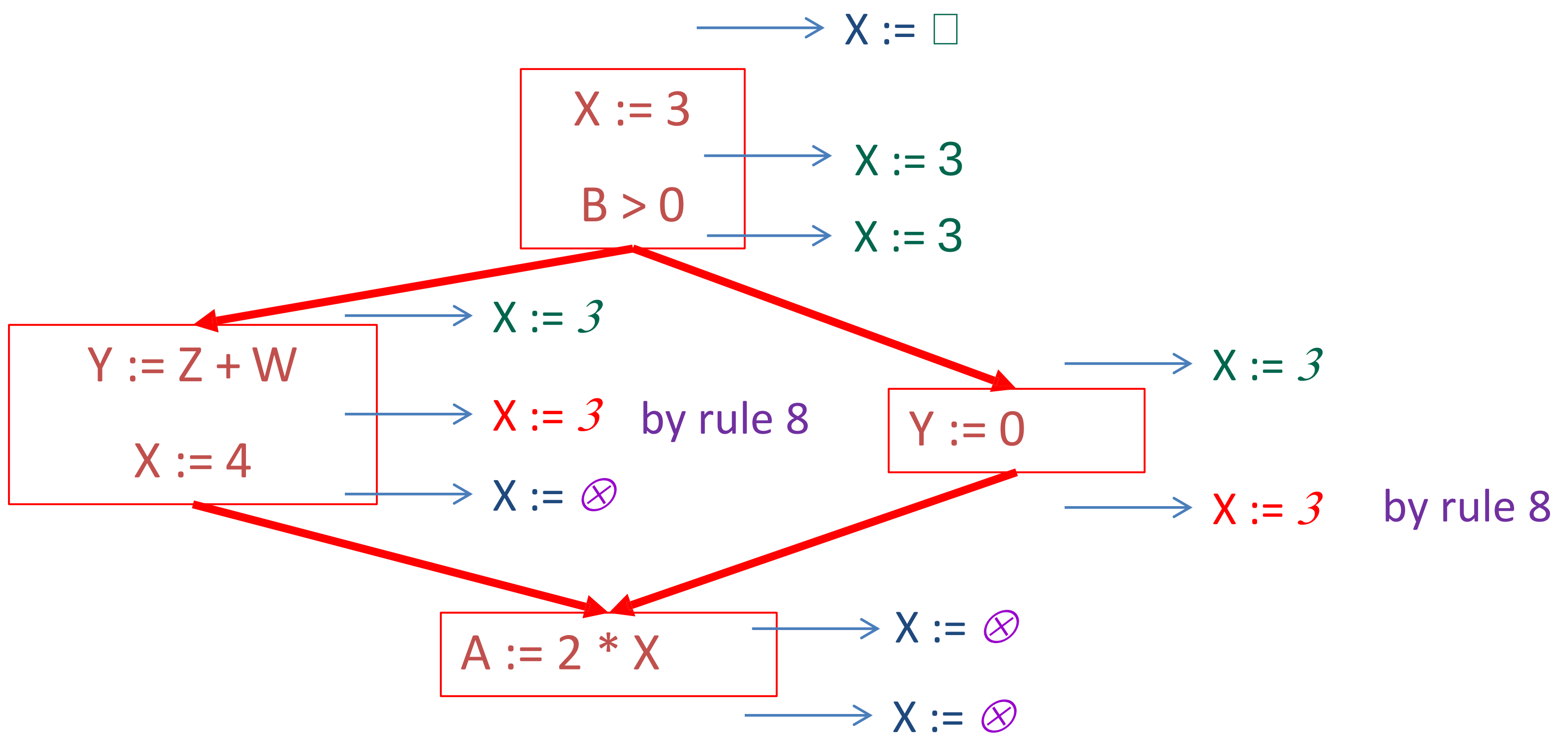
Rule 8: if x <> y then C(y := …, x, out) = C(y := …, x, in)

Constant Propagation



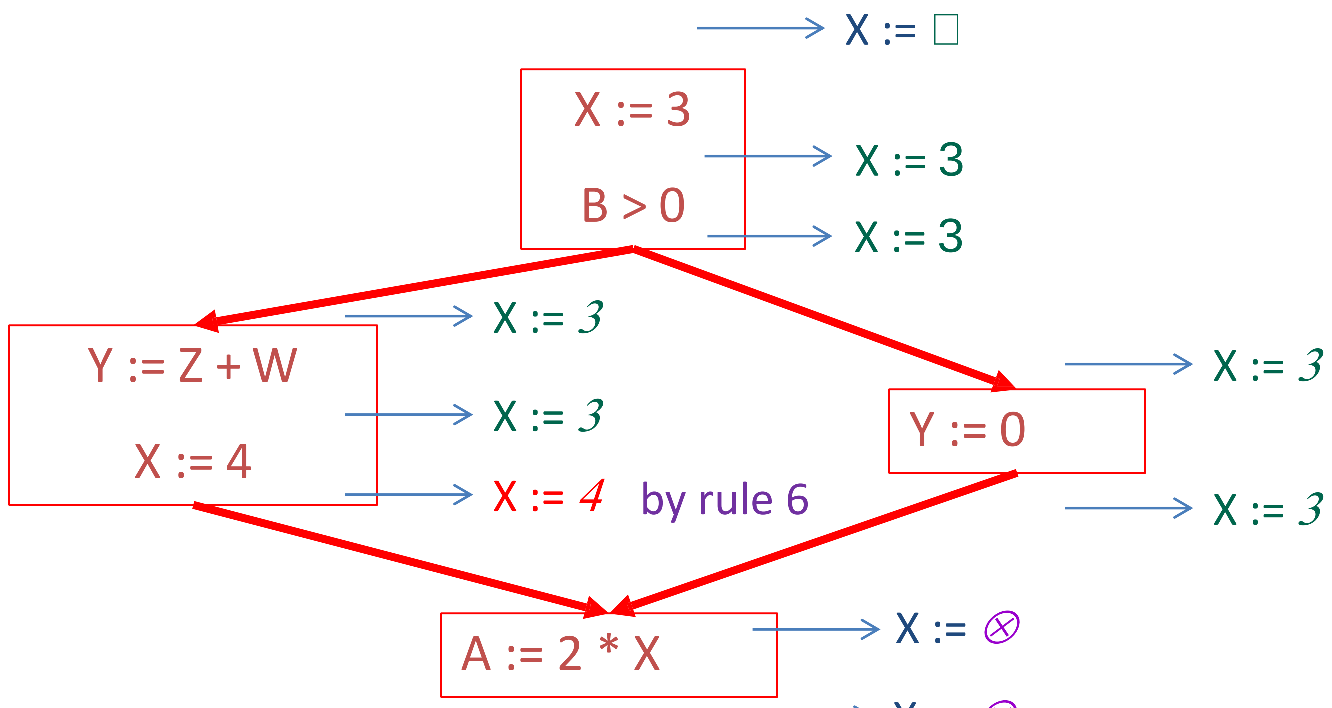
Rule 3: *if* $\forall_i$ ($C(p_i, x, out) = c$ *or* ⊗) *then* $C(s, x, in) = c$

Constant Propagation



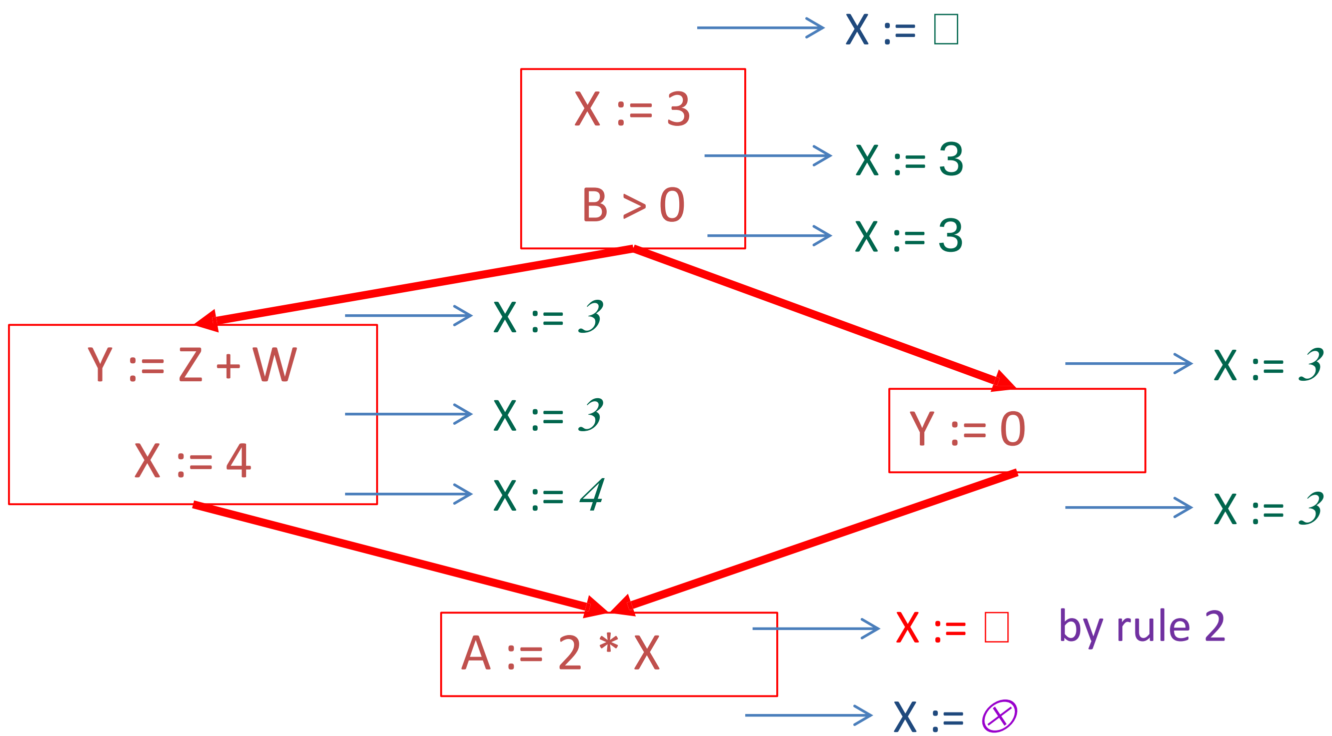
Rule 8: if x <> y then C(y := …, x, out) = C(y := …, x, in)

Constant Propagation



Rule 6: if c is a constant then C(x := c, x, out) = c

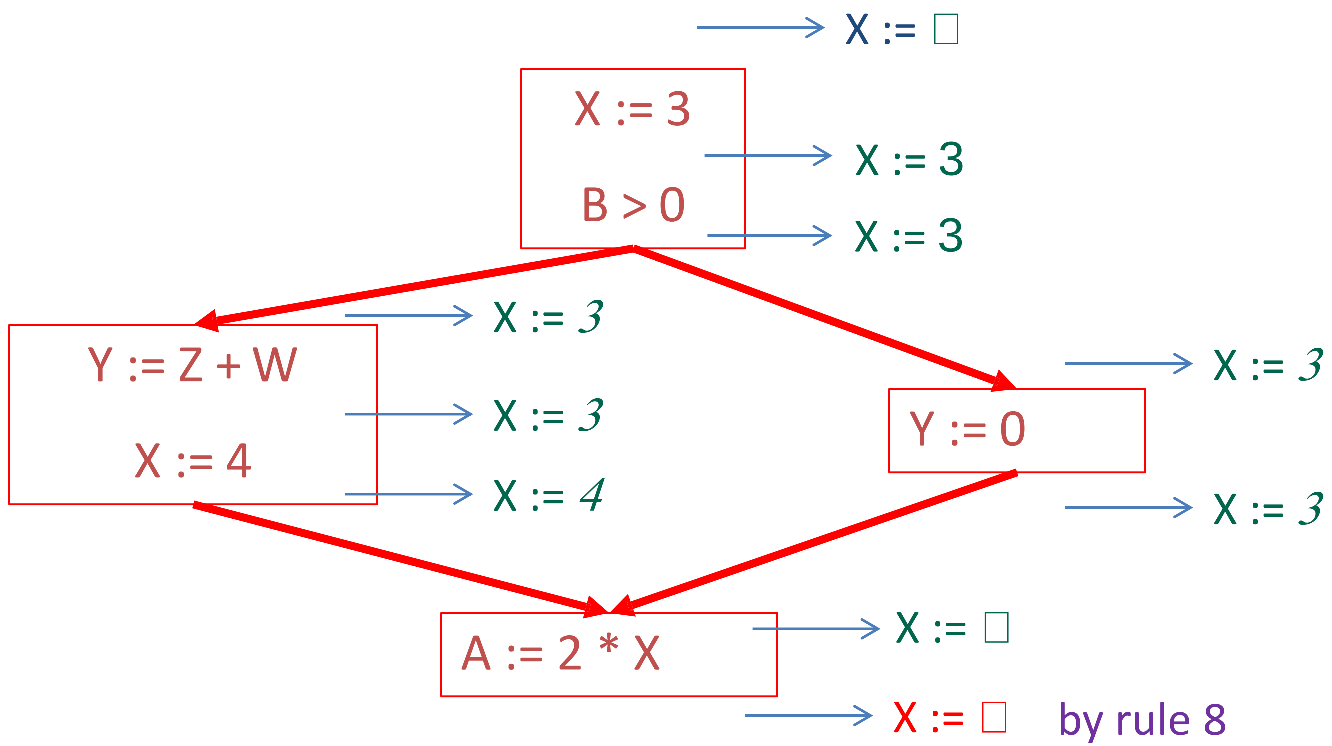
Constant Propagation



Rule 2: if $\exists_{i,j}$ ($C(p_i$, x, out) = c & $C(p_j$, x, out) = d & d <> c)

then C(s, x, in) = □

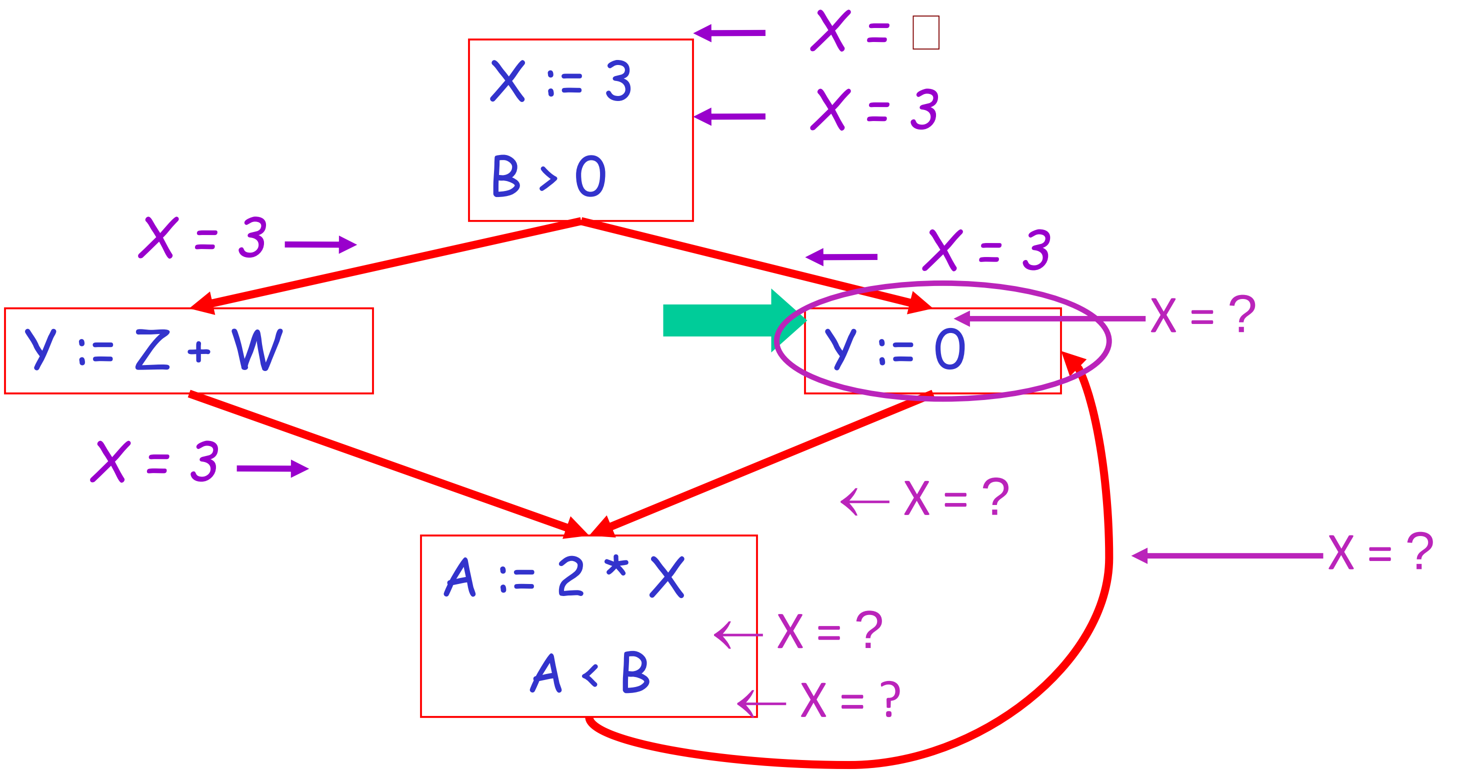
Constant Propagation



Rule 8: if x <> y then *C*(y := …, x, out) = *C*(y := …, x, in)

The Value ⊗

- To understand why we need ⊗, look at a loop



Discussion

- Consider the statement Y := 0
- To compute whether X is constant at this point, we need to know whether X is constant at the two predecessors
  - X := 3
  - A := 2 * X
- But info for A := 2 * X depends on its predecessors, including Y := 0!

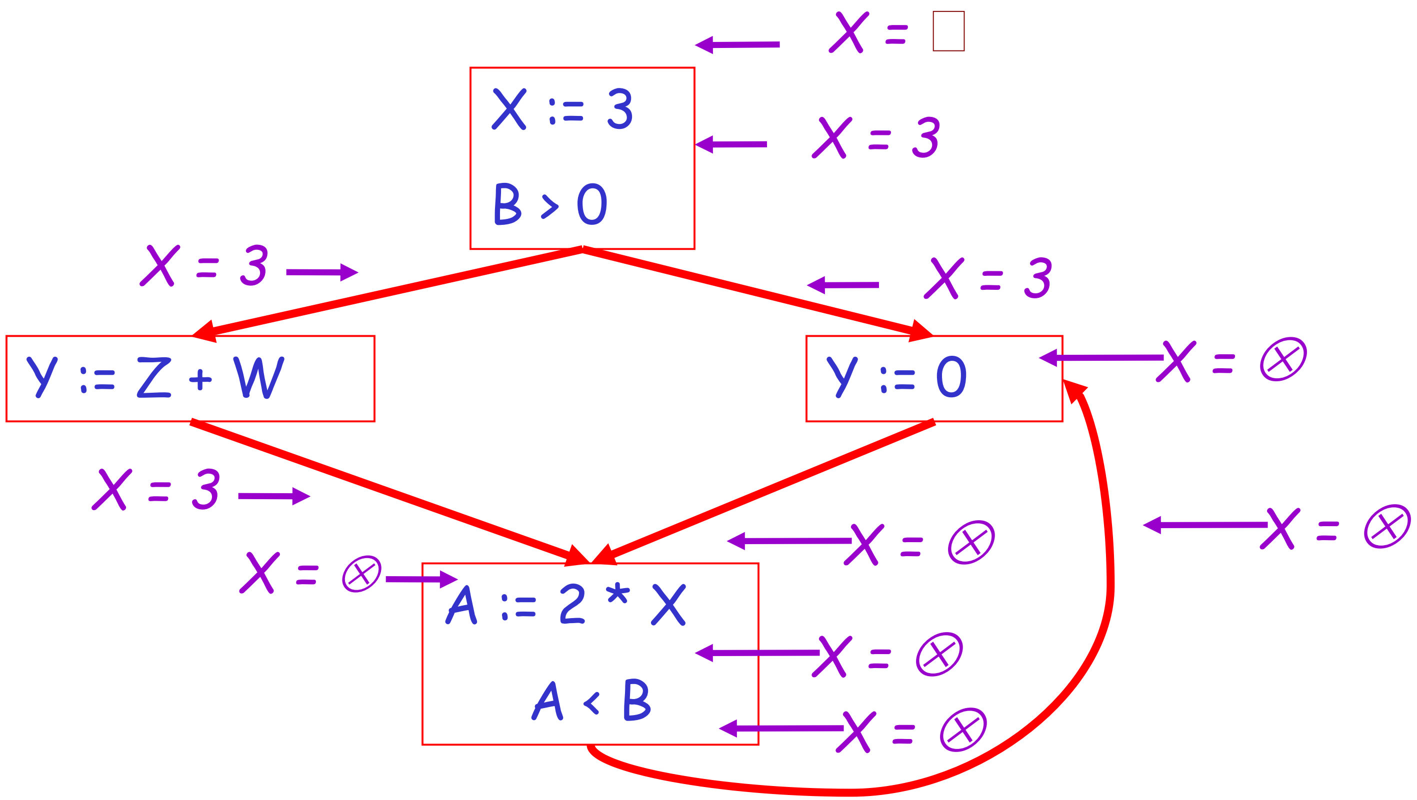
The Value ⊗ (Cont.)

- Because of cycles, all points must have values at all times

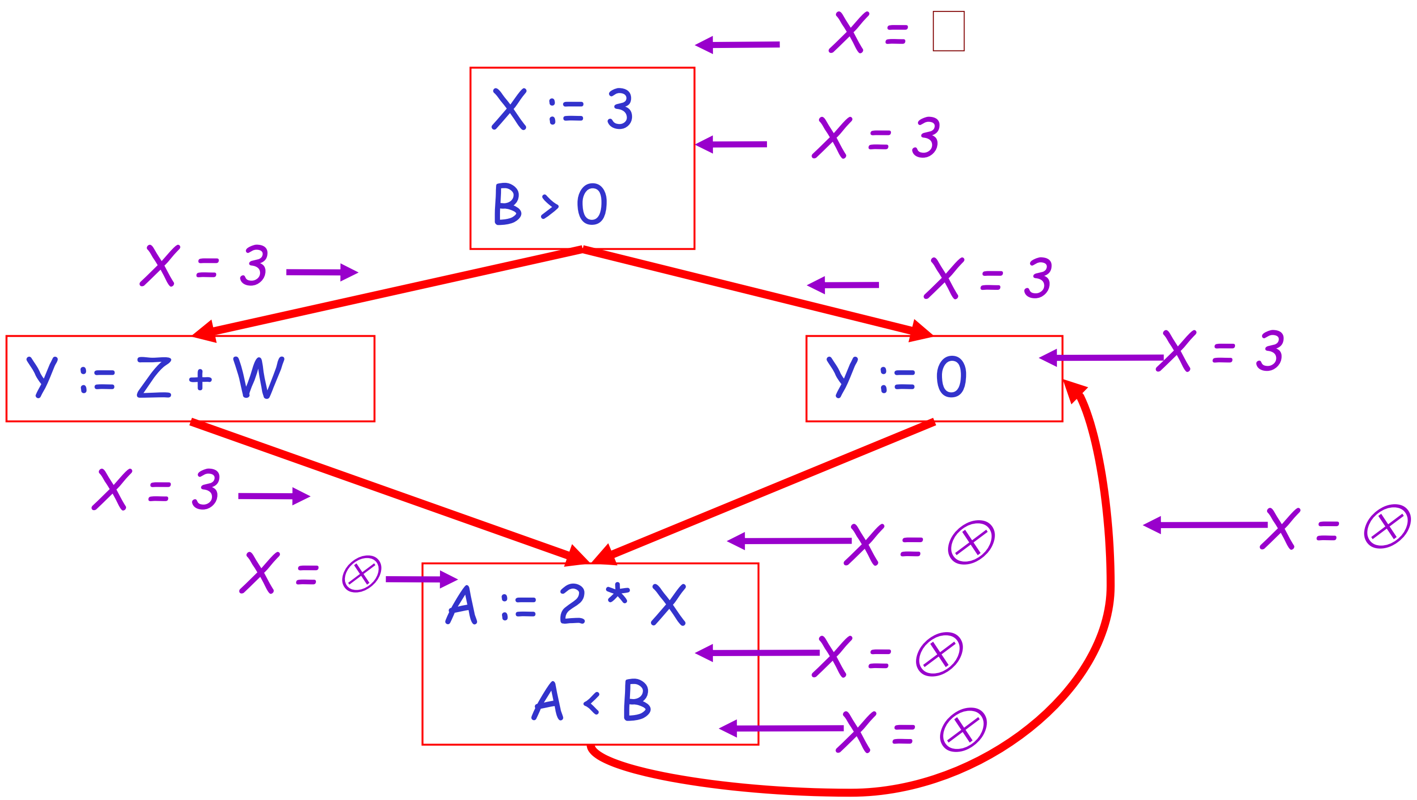
- Intuitively, assigning some initial value allows the analysis to break cycles

- The initial value ⊗ means "So far as we know, control never reaches this point"

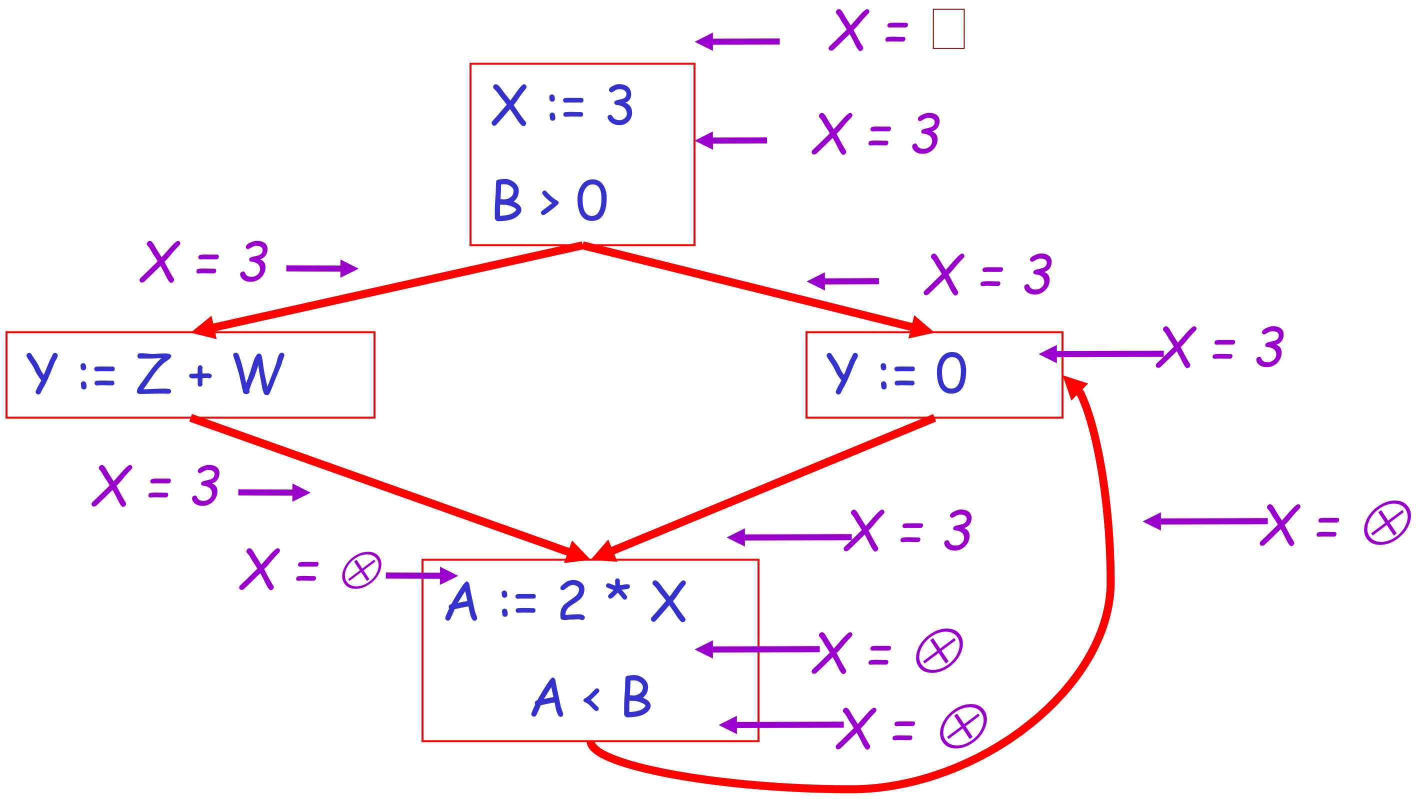
Example



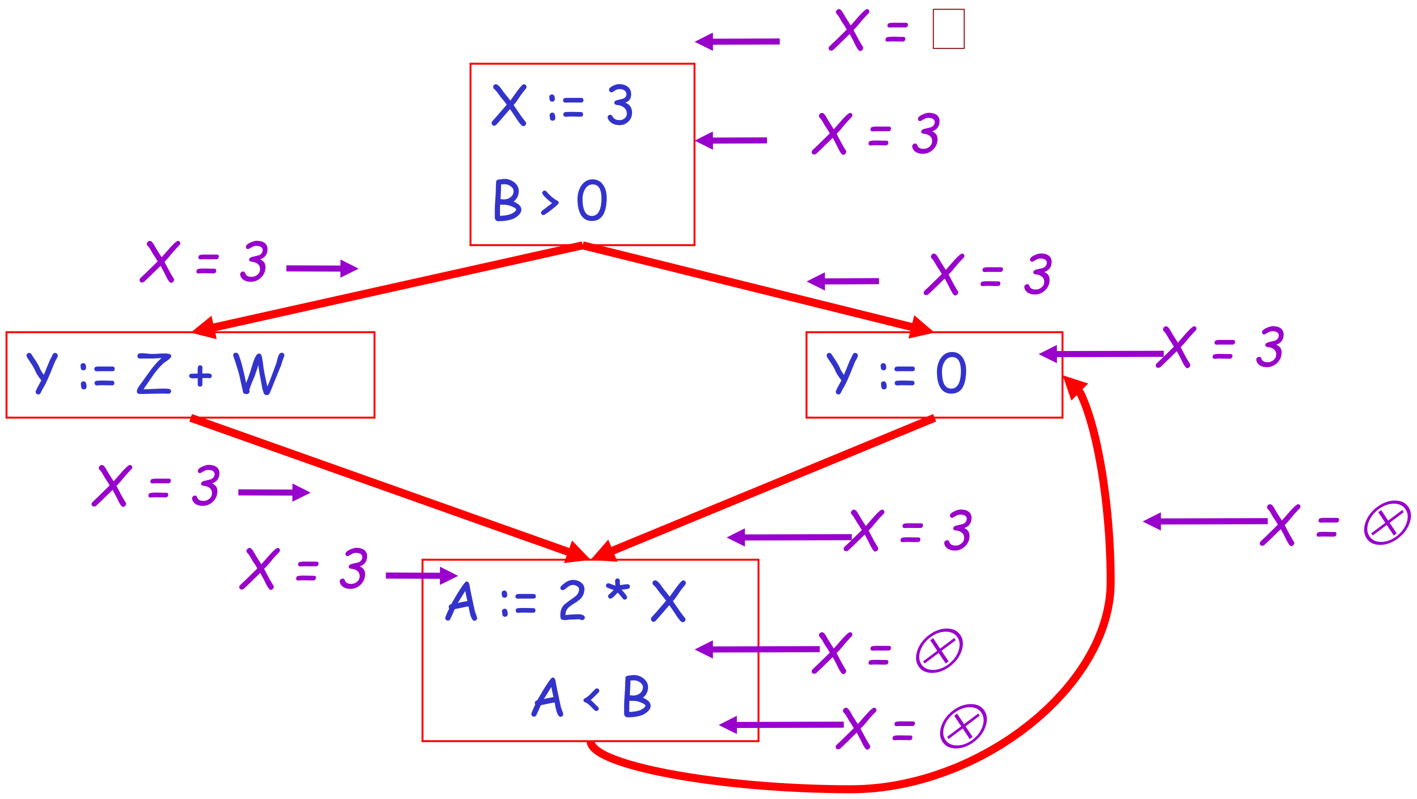
Example



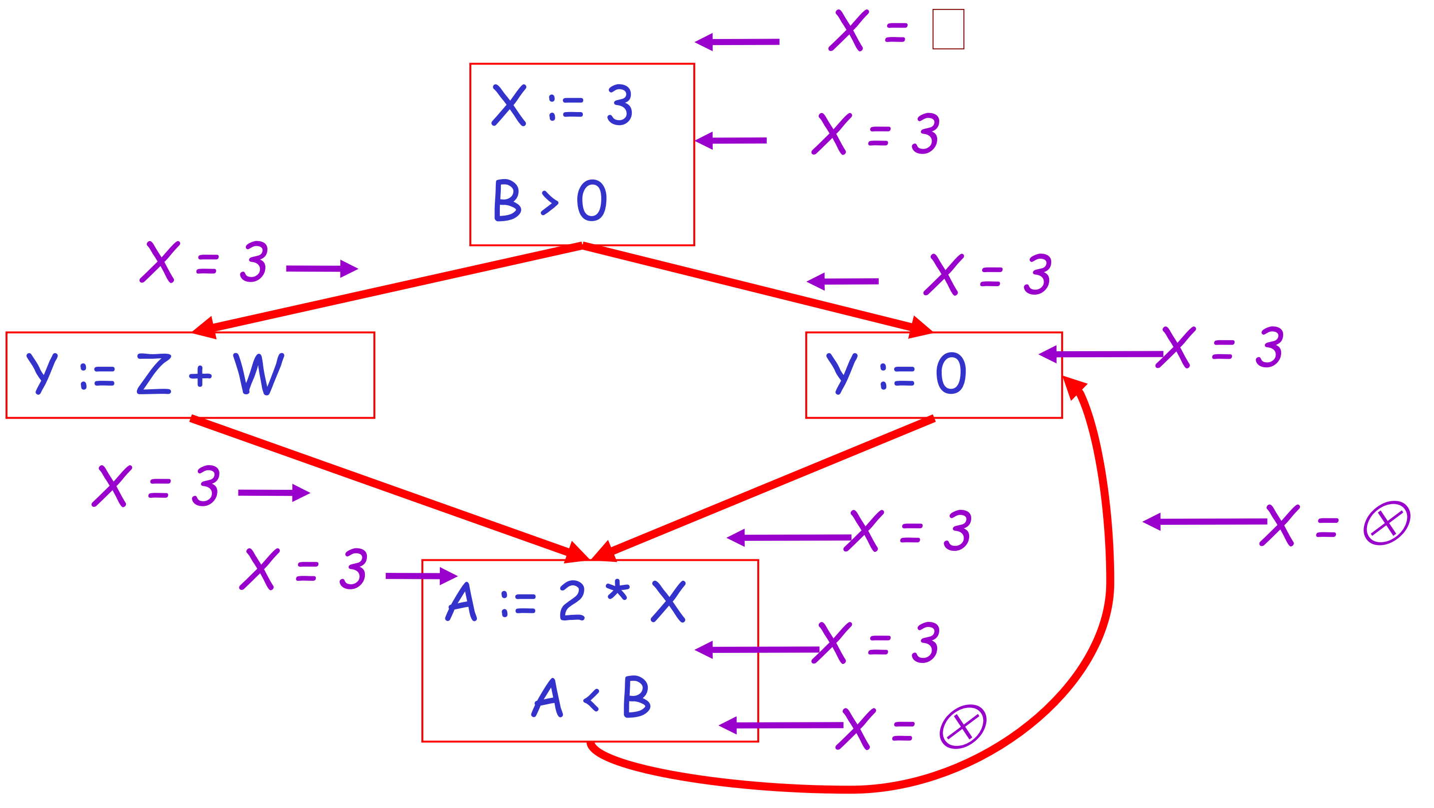
Example



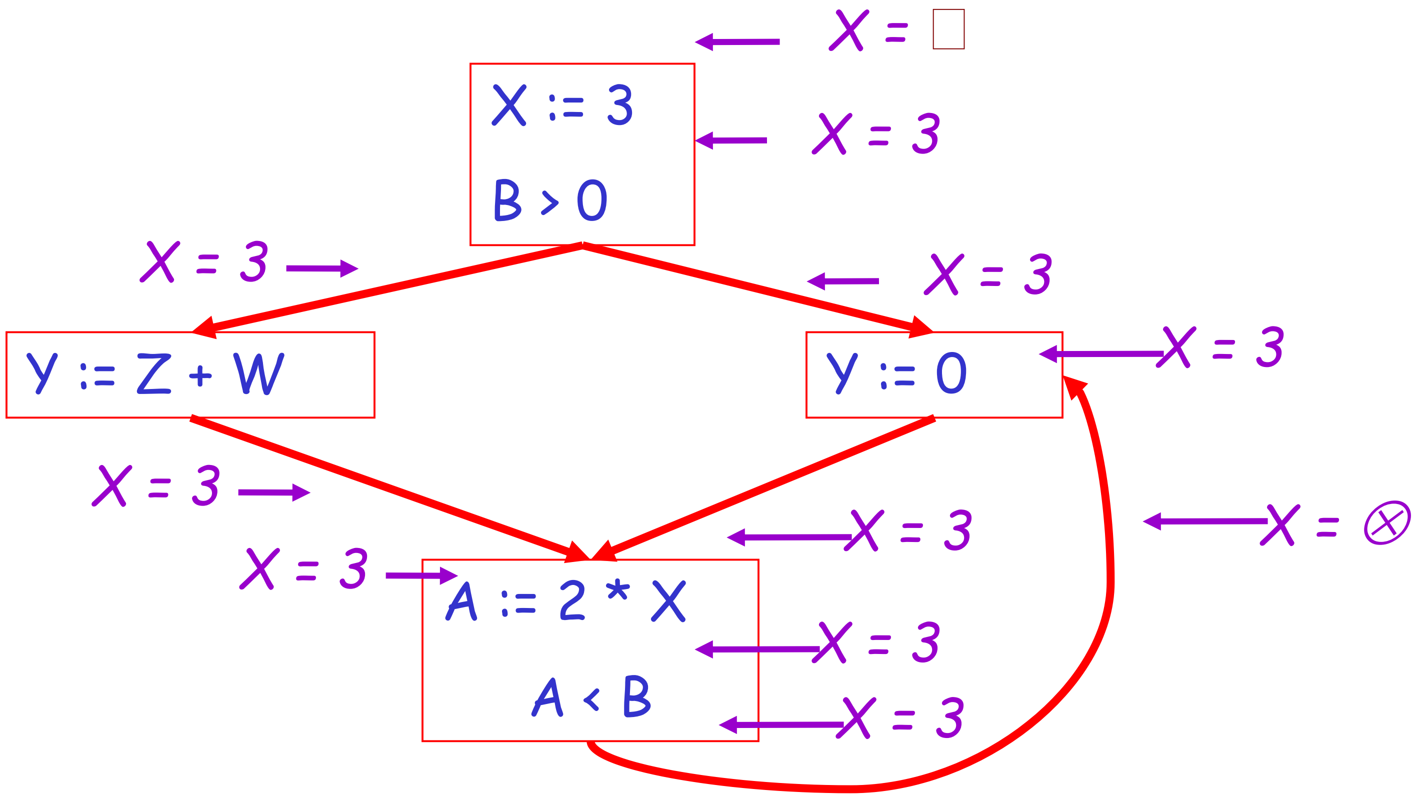
Example



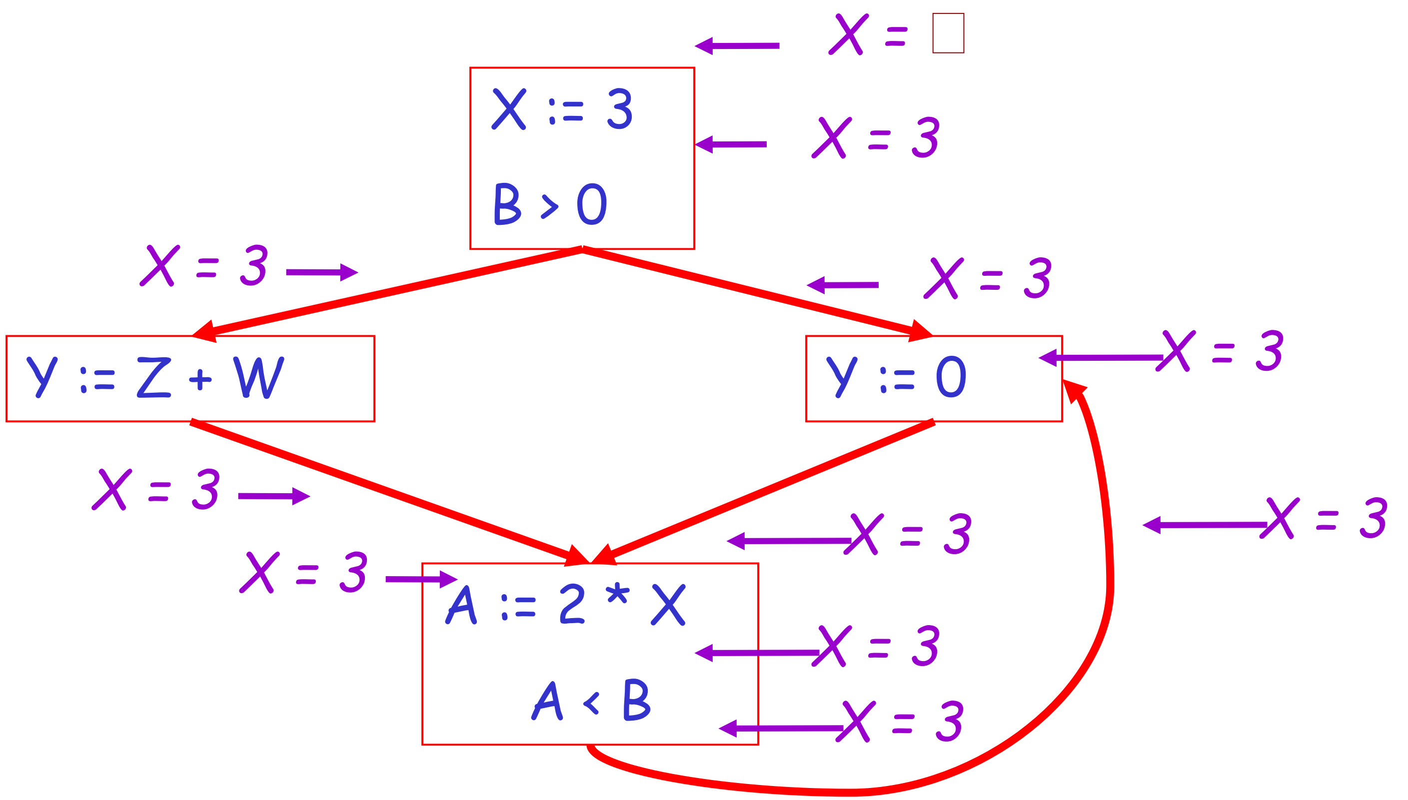
Example



Example



Example



Orderings

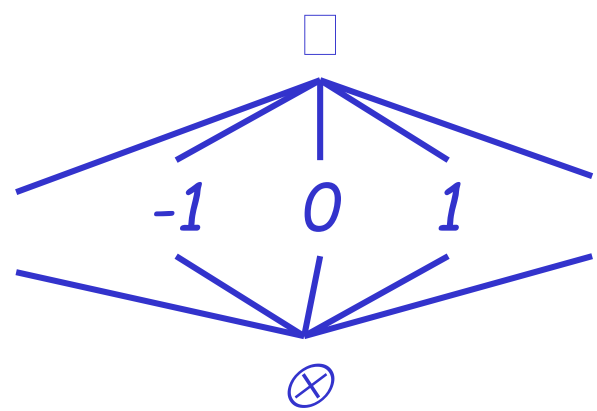
- We can simplify the presentation of the analysis by ordering the values

these are abstract values

⊗ < c < □

e.g., □ stands for any possible run time value

- Drawing a picture with “lower” values drawn lower, we get



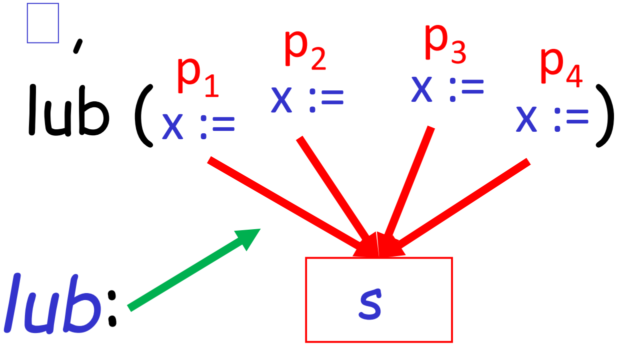
constants are not comparable

e.g, 0 is not less than 1

Orderings (Cont.)

- □ is the greatest value, ⊗ is the least
  - All constants are in between and incomparable

- Let *lub* be the least-upper bound in this ordering
  - Examples: lub(1, 2) = □, lub(□, ⊗) = □, lub(1, ⊗) = 1, …



- Rules 1-4 are in fact computing *lub*:

C(s, x, in) = lub { C(p, x, out) | p is a predecessor of s }

Termination

- Simply saying "repeat until nothing changes" doesn't guarantee that eventually nothing changes

- The use of lub explains why the algorithm terminates
  - Values start as $\otimes$ and only *increase*

    $\otimes$ can change to a constant, and a constant to □
  - Thus, $C(s, x, \_)$ can change at most twice

Termination (Cont.)

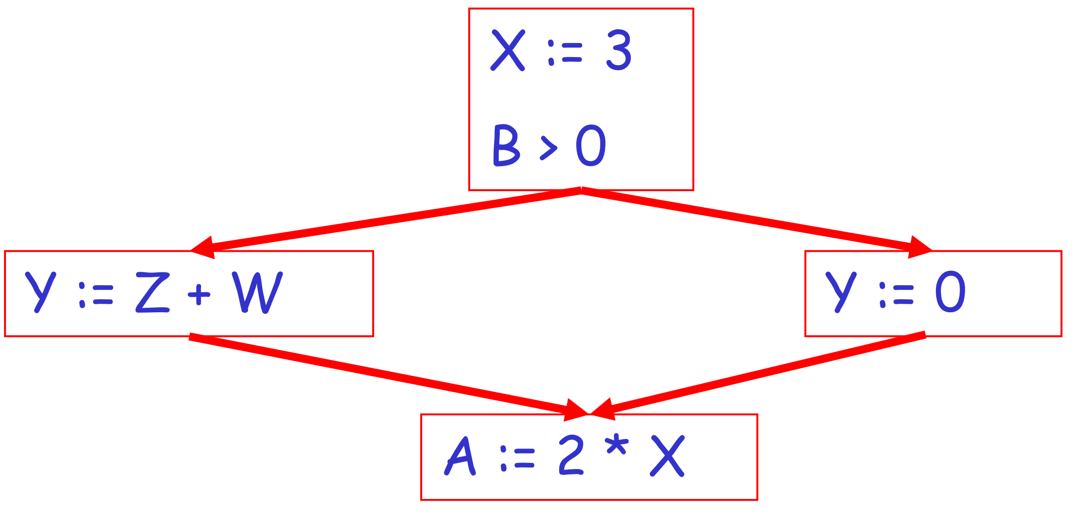
Thus the algorithm is linear in program size

Number of steps =
Number of $C(....)$ value computed * 2 =
Number of program statements * 4

for each statement s, we have
one c(s, x, in) and one c(s, x, out),
each can change at most twice

Liveness Analysis

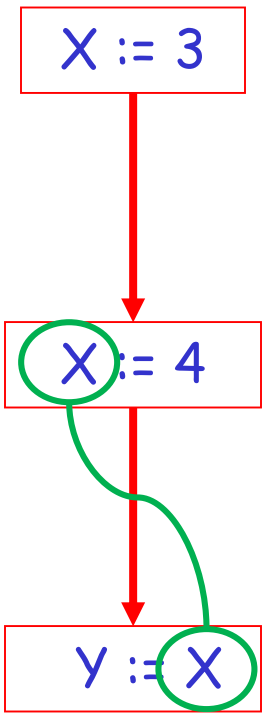
Once constants have been globally propagated, we would like to eliminate dead code



After constant propagation, X := 3 is dead (assuming X not used elsewhere)

Live and Dead

- The first value of x is *dead* (never used)
- The second value of x is *live* (may be used in the future)
- Liveness is an important concept



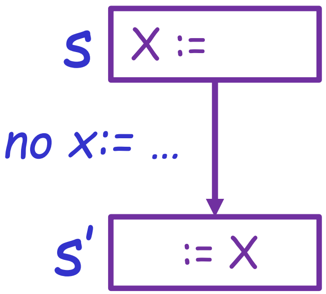
this value of x is never used

in this eg., x is guaranteed to be used

Liveness

A variable x is live at statement s if

- There exists a statement s′ that uses x
- There is a path from s to s′
- That path has no intervening assignment to x



Global Dead Code Elimination

- A statement x := … is dead code if x is dead after the assignment

- Dead statements can be deleted from the program

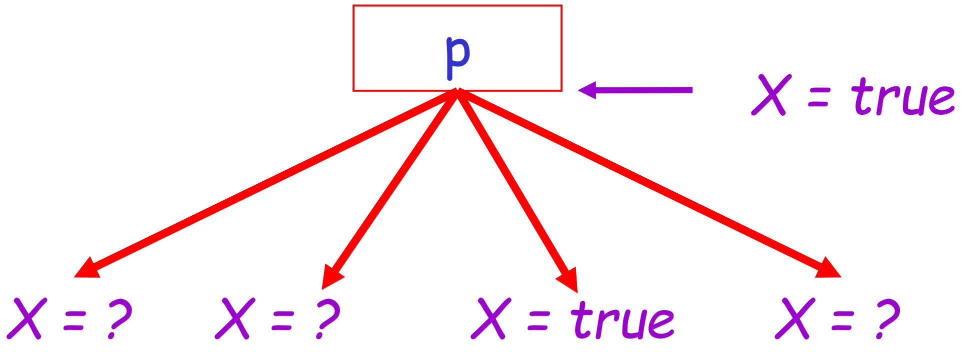
- But we need liveness information first . . .

Computing Liveness

- We can express liveness in terms of information transferred between adjacent statements, just as in copy propagation

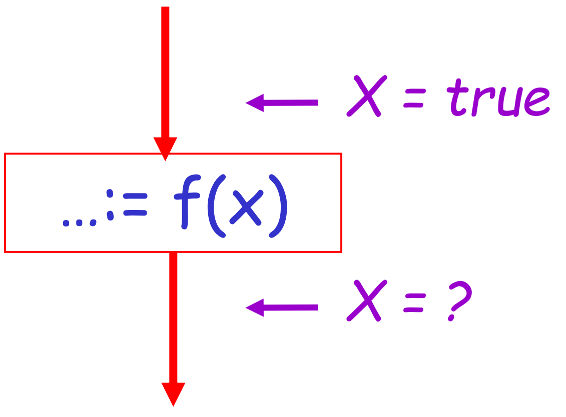
- Liveness is simpler than constant propagation, since it is a boolean property (true or false)

Liveness Rule 1



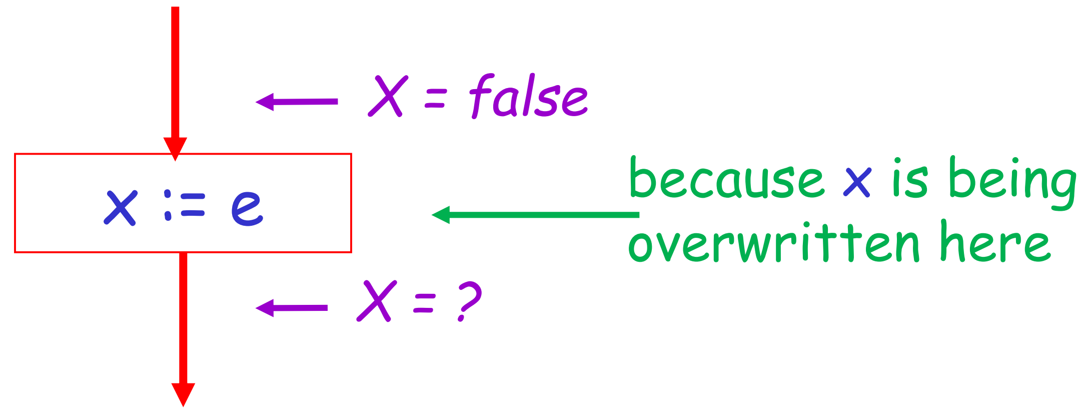
$L(p, x, out) = \vee \{ L(s, x, in) \mid s$ a successor of $p \}$

Liveness Rule 2



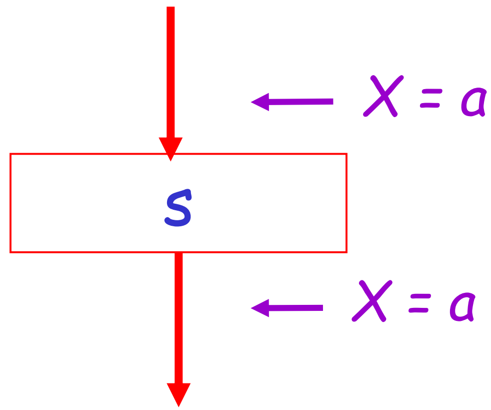
$L(s, x, in) = true$ if s refers to x on the rhs

Liveness Rule 3



$L(x := e, x, in) = false$ if e does not refer to x

Liveness Rule 4



L(s, x, in) = L(s, x, out) if s does not refer to x

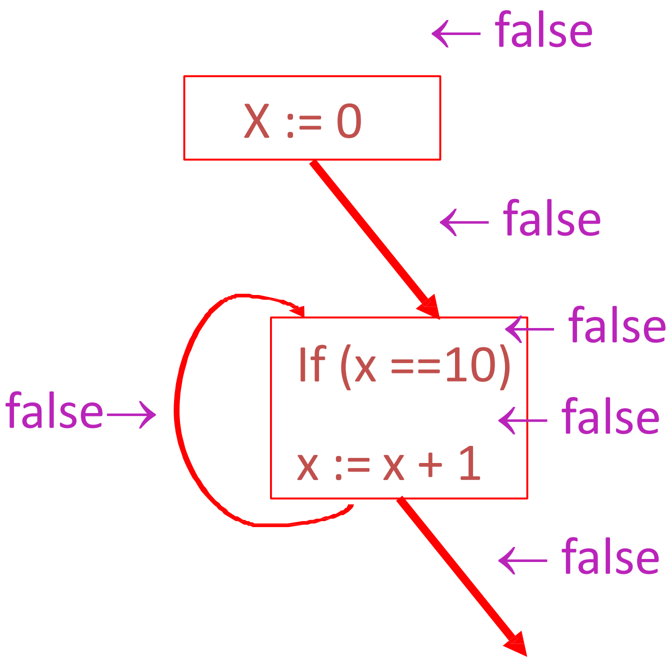
Algorithm

1. Let all L(…) = false initially

2. Repeat until all statements s satisfy rules 1-4

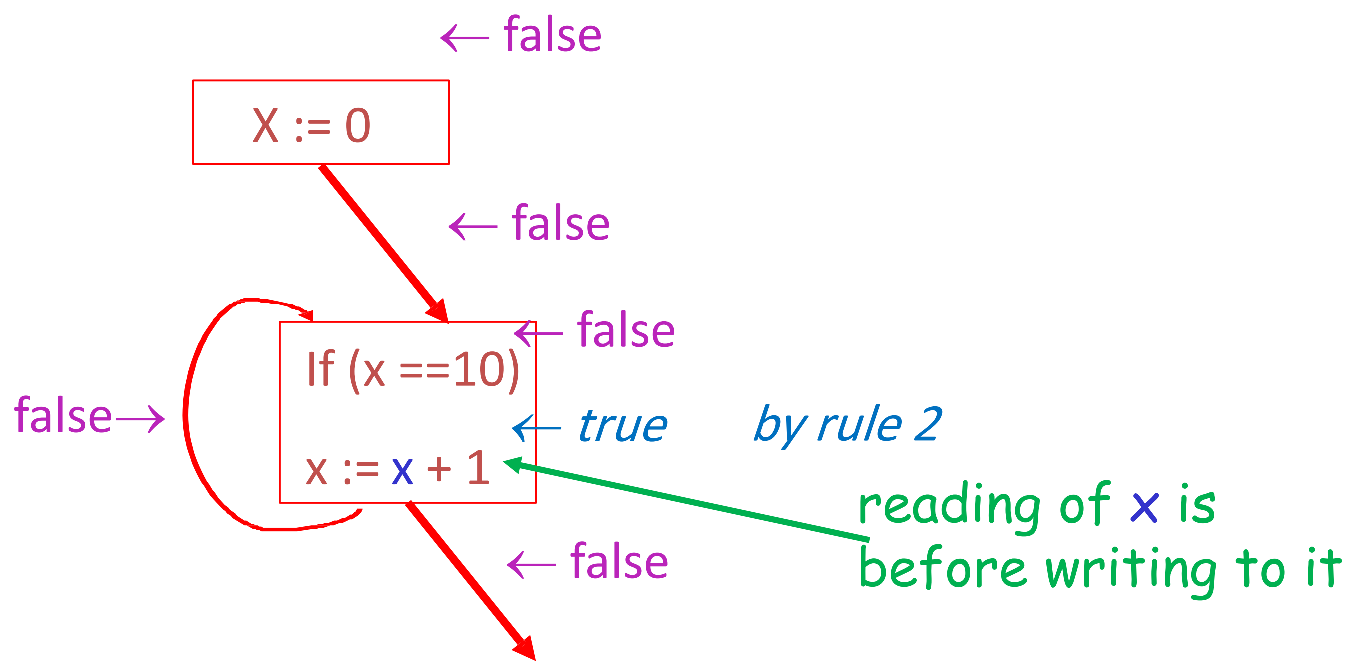
   Pick s where one of 1-4 does not hold and update using the appropriate rule

Example



where x is live?

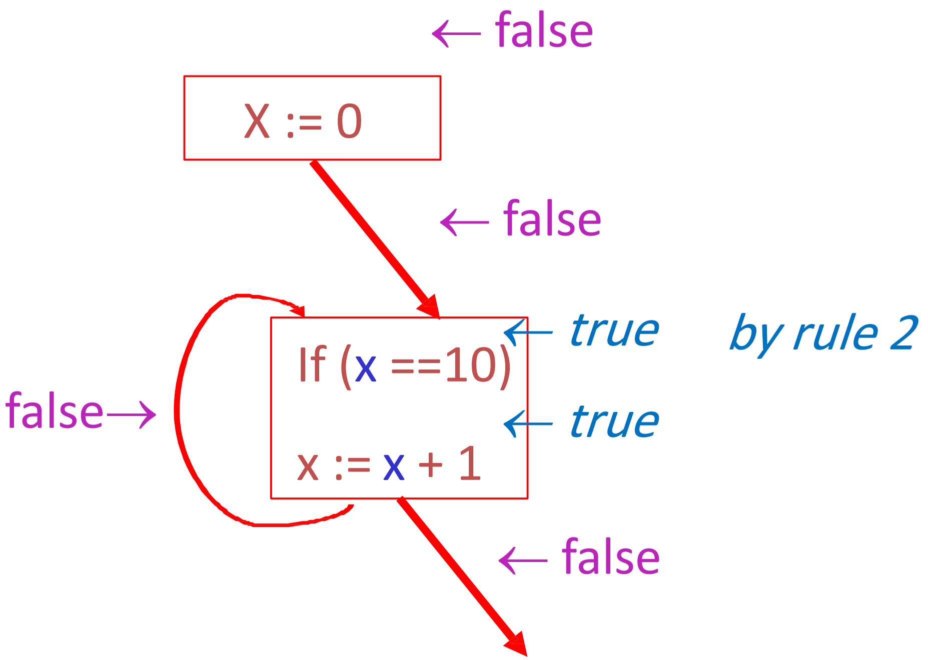
Example



where x is live?

Rule 2: L(s, x, in) = true if s refers to x on the rhs

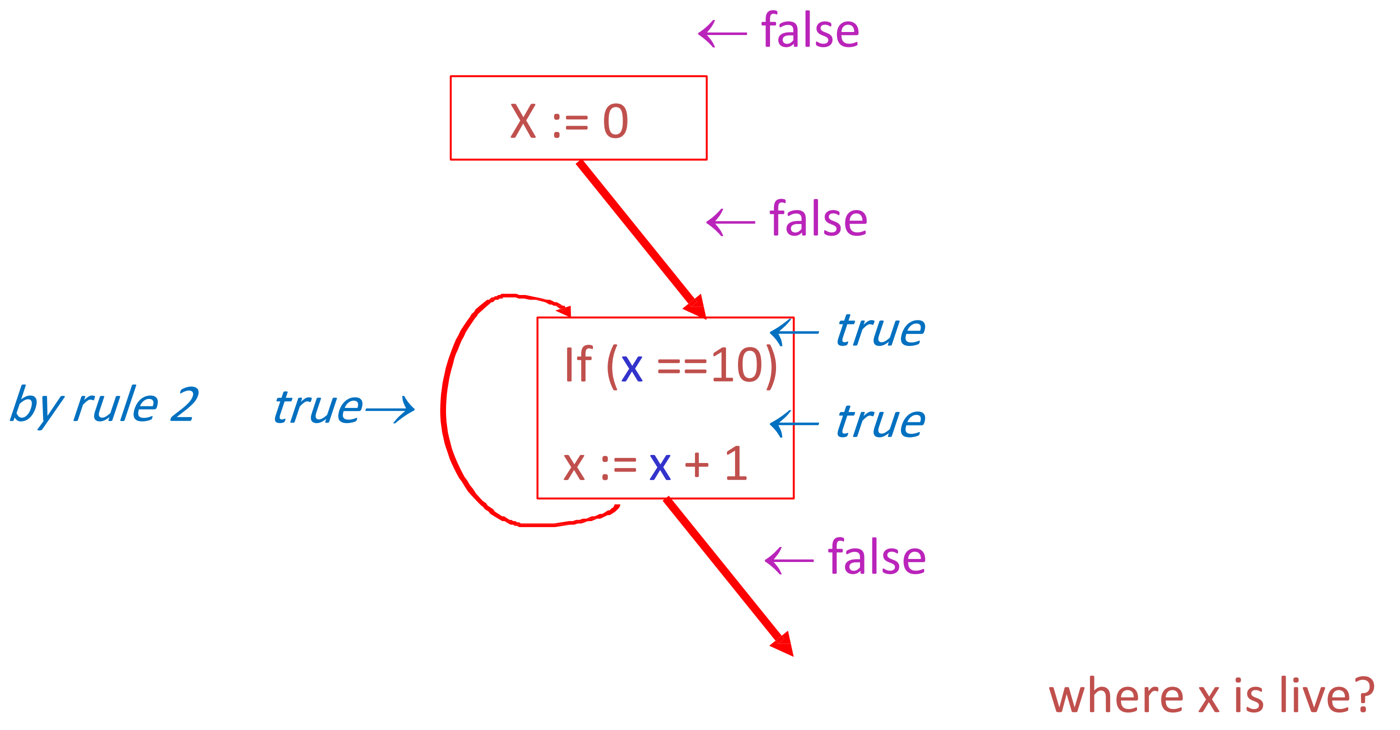
Example



where x is live?

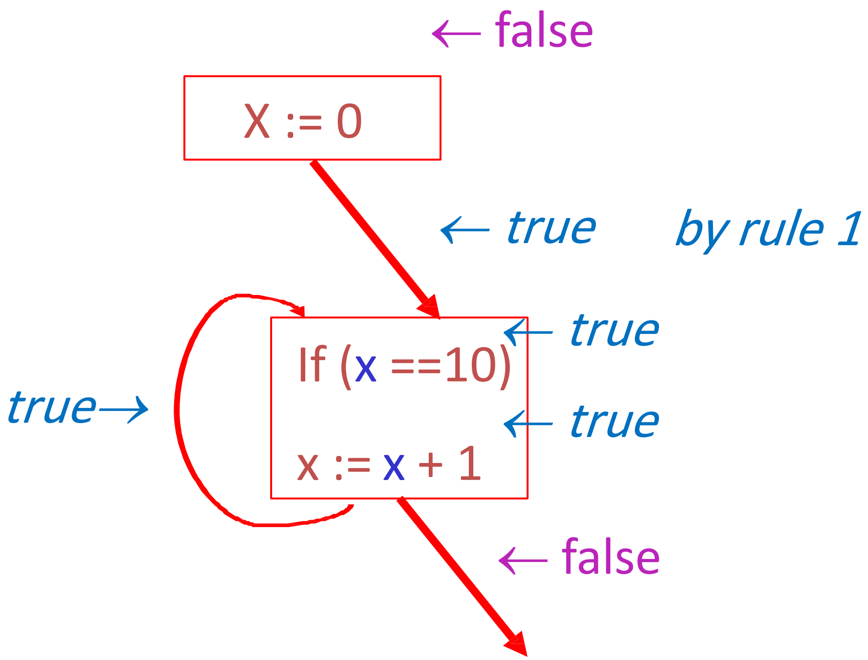
Rule 2: L(s, x, in) = true if s refers to x on the rhs

Example



Rule 2: L(s, x, in) = true if s refers to x on the rhs

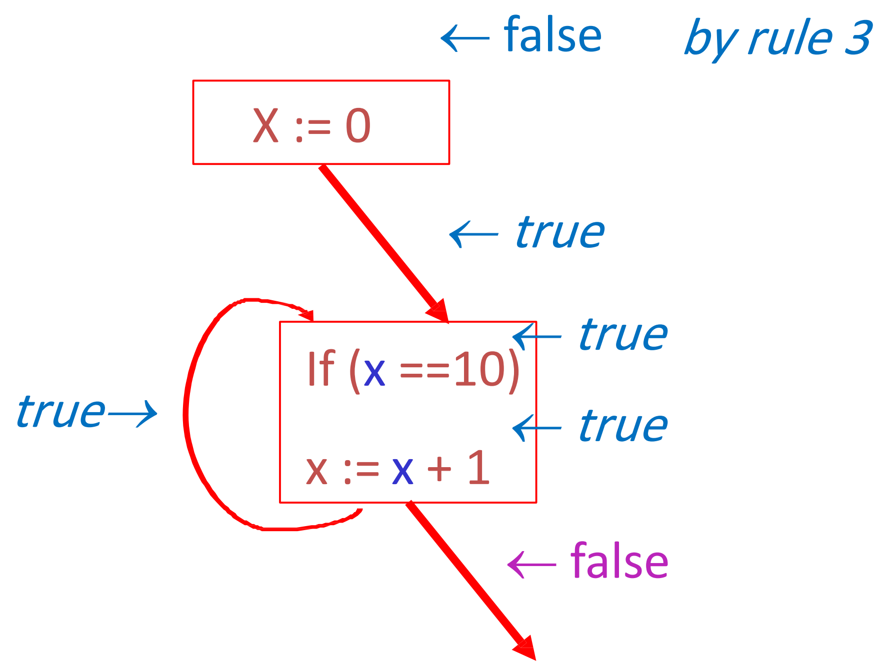
Example



where x is live?

Rule 1: L(p, x, out) = ∨ { L(s, x, in) | s a successor of p }

Example



where x is live?

Rule 3: L(x := e, x, in) = false if e does not refer to x

Termination

- A value can change from *false* to *true*, but not the other way around

false < *true*

- Each value can change only once, so termination is guaranteed

- Once the analysis is computed, it is simple to eliminate dead code

Forward vs. Backward Analysis

We've seen two kinds of analysis:

Constant propagation is a *forwards* analysis: information is pushed from inputs to outputs

Liveness is a *backwards* analysis: information is pushed from outputs back towards inputs

Analysis

- There are many other global flow analyses

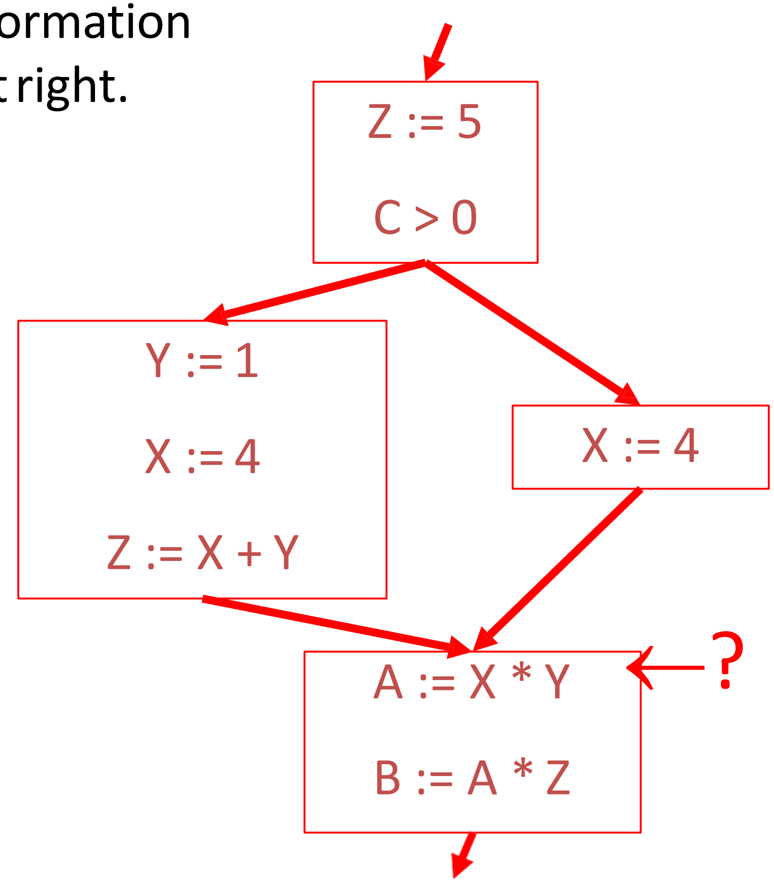
- Most can be classified as either forward or backward

- Most also follow the methodology of local rules relating information between adjacent program points

Question?

After running the constant propagation algorithm to completion, choose the correct dataflow information for X, Y, and Z at the program point labeled at right.

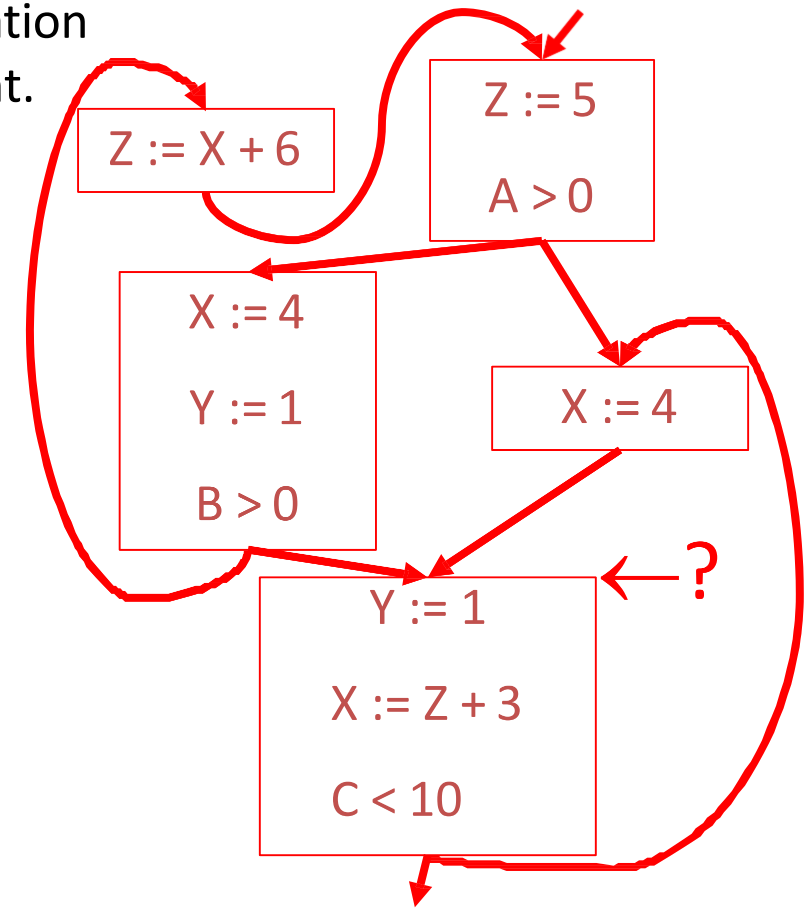
	X	Y	Z
○	4	□	□
○	4	□	5
○	4	1	5
○	□	□	□



Question?

After running the constant propagation algorithm to completion, choose the correct dataflow information for X, Y, and Z at the program point labeled at right.

	X	Y	Z
○	□	1	□
○	4	□	5
○	4	1	5
○	4	□	□



Question?

After running the liveness analysis algorithm to completion, which of W, X, Y, and Z are live at the program point labeled at right? Assume all variables are dead on exit.

- ☐ W
- ☐ X
- ☐ Y
- ☐ Z