Language Modeling

Advanced: Good Turing Smoothing

(Reading: J+M Ch3)

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[These slides were created by Dan Jurafsky and Chris Manning for an online course on NLP]

Reminder: Add-1 (Laplace) Smoothing

$$P_{Add-1}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i) + 1}{c(w_{i-1}) + V}$$

More general formulations: Add-k

$$P_{Add\text{-}k}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i) + k}{c(w_{i-1}) + kV}$$

$m = kV$

$$P_{Add\text{-}k}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i) + m(\frac{1}{V})}{c(w_{i-1}) + m}$$

Unigram prior smoothing

$$P_{Add\text{-}k}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i) + m(\frac{1}{V})}{c(w_{i-1}) + m}$$

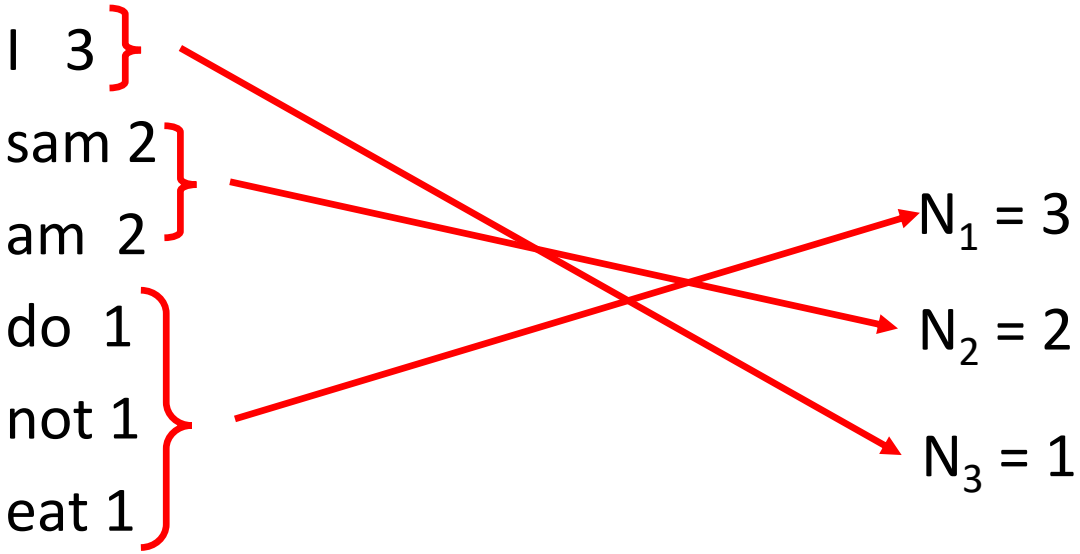
$$P_{\text{UnigramPrior}}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i) + mP(w_i)}{c(w_{i-1}) + m}$$

Advanced smoothing algorithms

- Intuition used by many smoothing algorithms
  - Good-Turing
  - Kneser-Ney
  - …
- Use the count of things we've **seen once**
  - to help estimate the count of things we've **never seen**

Notation: N_c = Frequency of frequency c

- N_c = the count of things we've seen c times
- Sam I am I am Sam I do not eat



Good-Turing smoothing intuition

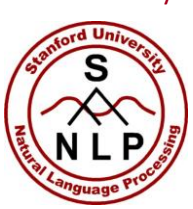
- You are fishing (a scenario from Josh Goodman), and caught:
  - 10 carp, 3 perch, 2 whitefish, 1 trout, 1 salmon, 1 eel = 18 fish
- How likely is it that next species is trout?
  - $\frac{1}{18}$
- How likely is it that next species is new (i.e. catfish or bass)
  - Let's use our estimate of things-we-saw-once to estimate the new things.
  - $\frac{3}{18}$ (because N_1=3)
- Assuming so, how likely is it that next species is trout?
  - Must be less than $\frac{1}{18}$
  - How to estimate?

Good Turing calculations

$$P^*_{GT}(\text{things with zero frequency}) = \frac{N_1}{N}$$

$$c^* = \frac{(c+1)N_{c+1}}{N_c}$$

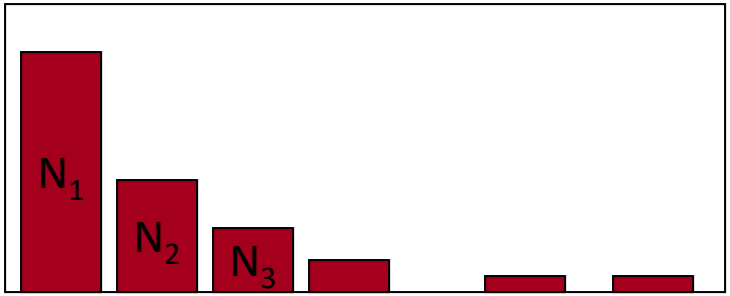
- Unseen (bass or catfish)
  - c = 0:
  - MLE p $= \frac{0}{18} = 0$
  - P^*_{GT} (unseen) $= \frac{N_1}{N} = \frac{3}{18}$
- Seen once (trout)
  - c = 1:
  - MLE p = $\frac{1}{18}$
  - C*(trout) = $\frac{(1+1) * N_2}{N_1} = \frac{2*1}{3} = \frac{2}{3}$
  - P^*_{GT}(trout) $= \frac{\frac{2}{3}}{18} = \frac{1}{27}$



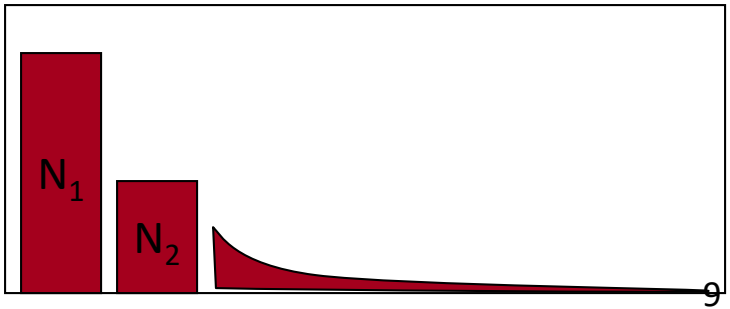
Good-Turing complications

(slide from Dan Klein)

- Problem: what about "the"? (say c=4417)
  - For small k, $N_k > N_{k+1}$
  - For large k, too jumpy, zeros wreck estimates



- Simple Good-Turing [Gale and Sampson]: replace empirical N_k with a best-fit power law once counts get unreliable



Resulting Good-Turing numbers

- Numbers from Church and Gale (1991)
- 22 million words of AP Newswire

$$c^* = \frac{(c+1)N_{c+1}}{N_c}$$

Count c	Good Turing c*
0	.0000270
1	0.446
2	1.26
3	2.24
4	3.24
5	4.22
6	5.19
7	6.21
8	7.24
9	8.25

Language Modeling

Advanced:

Kneser-Ney Smoothing

Resulting Good-Turing numbers

- Numbers from Church and Gale (1991)
- 22 million words of AP Newswire

$$c^* = \frac{(c+1)N_{c+1}}{N_c}$$

- It sure looks like c* = (c - 0.75)

Count c	Good Turing c*
0	.0000270
1	0.446
2	1.26
3	2.24
4	3.24
5	4.22
6	5.19
7	6.21
8	7.24
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Absolute Discounting Interpolation

- Save ourselves some time and just subtract 0.75 (or some d)!

discounted bigram

Interpolation weight

$$P_{\text{AbsoluteDiscounting}}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i) - d}{c(w_{i-1})} + \lambda(w_{i-1})P(w_i)$$

unigram

- (Maybe keeping a couple extra values of d for counts 1 and 2)
- But should we really just use the regular unigram $P(w_i)$?

Kneser-Ney Smoothing I

- Better estimate for probabilities of lower-order unigrams!
  - Shannon game: *I can't see without my reading*_____?
  - "Francisco" is more common than "glasses"
  - … but "Francisco" always follows "San"
- The unigram is useful exactly when we haven't seen this bigram!
- Instead of P(w): "How likely is w"
- $P_{continuation}$(w): "How likely is w to appear as a novel continuation?
  - For each word, count the number of bigram types it completes
  - Every bigram type was a novel continuation the first time it was seen

$$P_{CONTINUATION}(w) \mu \left|\{w_{i-1} : c(w_{i-1}, w) > 0\}\right|$$

Kneser-Ney Smoothing II

- How many times does w appear as a novel continuation:

$$P_{CONTINUATION}(w) \mu \left|\{w_{i-1} : c(w_{i-1}, w) > 0\}\right|$$

- Normalized by the total number of word bigram types

$$\left|\{(w_{j-1}, w_j) : c(w_{j-1}, w_j) > 0\}\right|$$

$$P_{CONTINUATION}(w) = \frac{\left|\{w_{i-1} : c(w_{i-1}, w) > 0\}\right|}{\left|\{(w_{j-1}, w_j) : c(w_{j-1}, w_j) > 0\}\right|}$$

Kneser-Ney Smoothing III

- Alternative metaphor: The number of # of word types seen to precede w

$$|\{w_{i-1} : c(w_{i-1}, w) > 0\}|$$

- normalized by the # of words preceding all words:

$$P_{CONTINUATION}(w) = \frac{|\{w_{i-1} : c(w_{i-1}, w) > 0\}|}{\sum_{w'} |\{w'_{i-1} : c(w'_{i-1}, w') > 0\}|}$$

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

i	want	to	eat	chinese	food	lunch	spend
2533	927	2417	746	158	1093	341	278

P(want)=$\frac{927}{8493} = 0.11$

P(lunch)=$\frac{341}{8493} = 0.04$

$P_{cont.}$(want)=$\frac{1}{32} = 0.031$

$P_{cont.}$(lunch)=$\frac{4}{32} = 0.125$

- A frequent word (Francisco) occurring in only one context (San) will have a low continuation probability

Kneser-Ney Smoothing IV

$$P_{KN}(w_i \mid w_{i-1}) = \frac{\max(c(w_{i-1}, w_i) - d, 0)}{c(w_{i-1})} + \lambda(w_{i-1}) P_{CONTINUATION}(w_i)$$

$$P_{KN}(food \mid chinese) = \frac{\max(82 - 0.75, 0)}{158} + 0.0142 * \frac{5}{32} = 0.52$$

$$P_{KN}(eat \mid chinese) = \frac{\max(0 - 0.75, 0)}{158} + 0.0142 * \frac{3}{32} = 0.00133$$

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

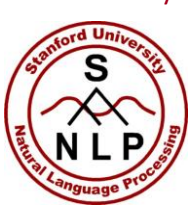
i	want	to	eat	chinese	food	lunch	spend
2533	927	2417	746	158	1093	341	278

$$\lambda(w_{i-1}) = \frac{d}{c(w_{i-1})} \left| \{w : c(w_{i-1}, w) > 0\} \right|$$

the normalized discount

The number of word types that can follow w_{i-1}
= # of word types we discounted
= # of times we applied normalized discount

$$\lambda(chinese) = \frac{0.75}{158} * 3 = 0.0142$$



Kneser-Ney Smoothing: Recursive formulation

$$P_{KN}(w_i \mid w_{i-n+1}^{i-1}) = \frac{\max(c_{KN}(w_{i-n+1}^{i}) - d, 0)}{c_{KN}(w_{i-n+1}^{i-1})} + \lambda(w_{i-n+1}^{i-1}) P_{KN}(w_i \mid w_{i-n+2}^{i-1})$$

$$c_{KN}(\bullet) = \begin{cases} count(\bullet) & \text{for the highest order} \\ continuationcount(\bullet) & \text{for lower order} \end{cases}$$

Continuation count = Number of unique single word contexts for •