Lecture 5-QM_II (12/07/00)

Monday, October 4, 2021 2:57 PM

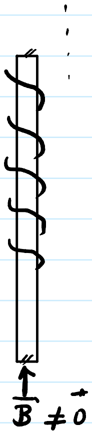
Aharonov- Bohm اثر

$$\vec{\nabla} \times \vec{A} = \vec{B}$$
$$\vec{A} \to \vec{A}' = \vec{A} + \vec{\nabla}\lambda$$
$$\varphi \to \varphi' = \varphi - \frac{1}{c}\frac{\partial}{\partial t}\lambda$$

$$i\hbar \frac{\partial}{\partial t} \psi'(\vec{x},t) = \left(\frac{1}{2m}\left(\vec{p} - \frac{q}{c}\vec{A}'\right)^2 + \cancel{e\varphi'} + V(\vec{x})\right)\psi'(\vec{x},t)$$

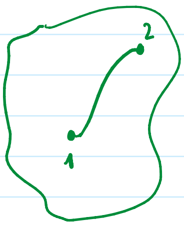
$$\psi(\vec{x},t) \to \psi'(\vec{x},t) = e^{i\alpha(\vec{x},t)}\,\psi(\vec{x},t)$$
$$\alpha(\vec{x},t) = \frac{q}{\hbar c}\lambda(\vec{x},t)$$

$$|\psi'(\vec{x},t)|^2 = \cancel{e^{-i\alpha}}\,\psi^*\,\cancel{e^{i\alpha}}\,\psi = |\psi(\vec{x},t)|^2$$



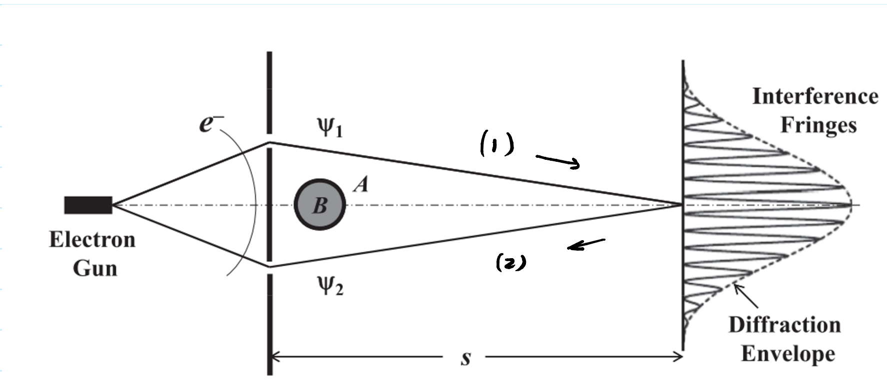
$\vec{B} \neq \vec{0}$

$$\underset{\sim\sim\sim}{\vec{B} = \vec{0}} = \vec{\nabla}\times\vec{A} \Rightarrow \vec{A} = -\vec{\nabla}\lambda \qquad \text{Pure gauge}$$
$$\vec{A}' = \vec{A} + \vec{\nabla}\lambda = -\vec{\nabla}\lambda + \vec{\nabla}\lambda = \vec{0}$$



$$i\hbar \frac{\partial}{\partial t}\psi' = \left(\frac{1}{2m}\vec{p}^{\,2} + V(\vec{x})\right)\psi'$$
$$\psi' = e^{i\alpha}\,\psi \qquad e^{-i\alpha}\psi' = \psi$$

$$\underset{\sim\sim\sim}{\vec{A} = -\vec{\nabla}\lambda} \qquad \lambda = -\int_1^2 \vec{A}\cdot d\vec{s}$$



$$\psi_1 = \psi_0\, e^{i\alpha_1}$$
$$\psi_2 = \psi_0\, e^{i\alpha_2}$$
$$\psi_1 + \psi_2 = \psi = \psi_0 e^{i\alpha_1} + \psi_0 e^{i\alpha_2}$$
$$|\psi_1+\psi_2|^2 = |\psi_0|^2\left(e^{i\alpha_1}+e^{i\alpha_2}\right)\left(e^{-i\alpha_1}+e^{-i\alpha_2}\right)$$
$$= |\psi_0|^2\left(1 + 1 + e^{i(\alpha_1-\alpha_2)} + e^{-i(\alpha_1-\alpha_2)}\right)$$
$$= |\psi_0|^2\, 2\left(1 + \cos(\alpha_1-\alpha_2)\right)$$

$$\alpha = \frac{q}{\hbar c}\lambda = \frac{q}{\hbar c}\int \vec{A}\cdot d\vec{s}$$

$$\alpha_1 - \alpha_2 = \frac{q}{\hbar c}\left(\int_1 \vec{A}\cdot d\vec{s} - \int_2 \vec{A}\cdot d\vec{s}\right)$$

$$= \frac{q}{\hbar c}\oint_{\text{Curve}} \vec{A}\cdot d\vec{s} = \frac{q}{\hbar c}\int_{S(C)} (\vec{\nabla}\times\vec{A})\cdot d\vec{F}$$

$$= \frac{q}{\hbar c}\int_{S(C)} \vec{B}\cdot d\vec{F} = \frac{q}{\hbar c}\,\Phi_B$$

شار مغناطیسی

$$H = H_0 + H_{magn}$$

$$H_0 = -\frac{\hbar^2}{2m_e}\vec{\nabla}^2 - \frac{e^2}{r} \qquad E_n = -R_y\frac{1}{n^2}$$

$$H_{magn} = \mu_B\frac{\vec{B}\cdot\vec{L}}{\hbar} \qquad \mu_B = \frac{e\hbar}{2m_e c} \quad \text{Bohr Magneton}$$

$$\vec{B} = B\hat{e}_z \qquad \vec{B}\cdot\vec{L} = BL_z$$

$$L_z\,|nlm_l\rangle = \hbar m_l\,|nlm_l\rangle$$

$$E = E_n + \Delta E_{m_l} \qquad \Delta E_{m_l} = \mu_B B\,(m_l + 2m_s)$$

$\uparrow \downarrow$ $\qquad m_s = \pm 1/2$

اثر زیمن

$l=1$ ———— $m_l = 1$, $m_l = 0$, $m_l = -1$

$$\Delta E = \mu_B B\,(m_l + 2m_s) = \mu_B B\,(m_l \pm 1)$$

g

$l=1$

$m_l = 0$: $\quad \begin{cases} +\mu_B B = \mu_B B(+1) \\ -\mu_B B = \mu_B B(-1) \end{cases}$

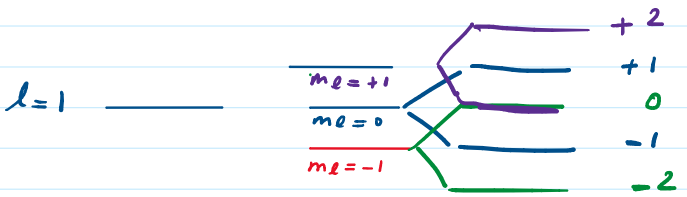
$m_l = 1$: $\quad \begin{cases} \mu_B B\,(1+1) = \mu_B B\,(2) \\ \mu_B B\,(1-1) = \mu_B B\,(0) \end{cases}$

$m_l = -1$: $\quad \begin{cases} \mu_B B\,(-1+1) = \mu_B B\,(0) \\ \mu_B B\,(-1-1) = \mu_B B\,(-2) \end{cases}$

$$H_{magn} = \frac{\mu_B}{\hbar}\,\vec{B}\cdot(\vec{L} + 2\vec{S})$$

l, m_l تکانه زاویه ای مداری

s, m_s $\quad S = \frac{1}{2} \quad m_s = \pm\frac{1}{2}$ up ↑ / down ↓



Paschen - Back

$\mu_B B$

$\vec{L}\cdot\vec{B}$

$\vec{A}\cdot\vec{\nabla}\psi$

$-\frac{1}{2}\vec{x}\times\vec{B}$

$\left(\vec{P} + \frac{e}{c}\vec{A}\right)^2$

$$\left(\vec{P}+\frac{e}{c}\vec{A}\right)^2$$

گشتاور مغناطیسی $\qquad \vec{\mu}_J = g_J \frac{\mu_B}{\hbar} \vec{J}$

$\vec{J}=\vec{L} \qquad \vec{\mu}_L = g_\ell \frac{\mu_B}{\hbar}\vec{L}$ ✓ $\qquad g_\ell = 1$ $\qquad$ ضریب ژیرومغناطیسی

$\vec{J}=\vec{S} \qquad \vec{\mu}_S = g_s \frac{\mu_B}{\hbar}\vec{S}$ ✓ $\qquad g_s = 2$

$$\vec{L} = (L_x, L_y, L_z) \qquad [L_i, L_j] = i\epsilon_{ijk} L_k$$

$$L^2 \,|\ell, m_\ell\rangle = \hbar^2 \ell(\ell+1)\,|\ell, m_\ell\rangle \qquad \ell = 0,1,2,\dots$$

$$L_z \,|\ell, m_\ell\rangle = \hbar m_\ell \,|\ell, m_\ell\rangle \qquad -\ell \leq m_\ell \leq \ell$$

$$\vec{S} = (S_x, S_y, S_z) \qquad [S_i, S_j] = i\epsilon_{ijk} S_k$$

$$\vec{S}^2 \,|s, m_s\rangle = \hbar^2 s(s+1)\,|s, m_s\rangle$$

$$S_z \,|s, m_s\rangle = \hbar m_s \,|s, m_s\rangle$$

الکترون:

$$s = \frac{1}{2}$$
$$m_s = \pm\frac{1}{2}$$

$$|\uparrow\rangle \equiv |s=\tfrac{1}{2}, m_s = +\tfrac{1}{2}\rangle \qquad \text{up}$$
$$|\downarrow\rangle \equiv |s=\tfrac{1}{2}, m_s = -\tfrac{1}{2}\rangle \qquad \text{down}$$

$$S^2 |\uparrow\rangle = S^2 |s=\tfrac{1}{2}, m_s = +\tfrac{1}{2}\rangle = \hbar^2 \frac{1}{2}\left(\frac{1}{2}+1\right)|\uparrow\rangle = \hbar^2 \frac{3}{4}|\uparrow\rangle$$

$$\begin{cases} S^2 |\uparrow\rangle = \frac{3}{4}\hbar^2 |\uparrow\rangle \\ S^2 |\downarrow\rangle = \frac{3}{4}\hbar^2 |\downarrow\rangle \end{cases}$$

$$S_z |s=\tfrac{1}{2}, m_s = +\tfrac{1}{2}\rangle = +\frac{\hbar}{2} |s=\tfrac{1}{2}, m_s = +\tfrac{1}{2}\rangle$$

$$S_z |\uparrow\rangle = +\frac{\hbar}{2}|\uparrow\rangle$$
$$S_z |\downarrow\rangle = -\frac{\hbar}{2}|\downarrow\rangle$$

$$L_\pm |\ell, m_\ell\rangle = \hbar\sqrt{\ell(\ell+1) - m_\ell(m_\ell \pm 1)}\;|\ell, m_\ell \pm 1\rangle$$

$$S_\pm = S_x \pm i S_y \qquad [S_i, S_j] = i\epsilon_{ijk} S_k$$

$$S_\pm |s, m_s\rangle = \hbar\sqrt{s(s+1) - m_s(m_s\pm 1)}\;|s, m_s \pm 1\rangle$$

a) $$S_+ \underbrace{|\uparrow\rangle}_{|s=\frac{1}{2}, m_s = +\frac{1}{2}\rangle} = \hbar\sqrt{\cancel{\tfrac{1}{2}(\tfrac{1}{2}+1) - \tfrac{1}{2}}\,(\tfrac{1}{2}+1)}\;|\dots\rangle = 0$$

b) $$S_+ \underbrace{|\downarrow\rangle}_{|s=\frac{1}{2}, m_s = -\frac{1}{2}\rangle} = \hbar\sqrt{\tfrac{1}{2}(\tfrac{1}{2}+1) + \tfrac{1}{2}(-\tfrac{1}{2}+1)}\;|\underset{s}{\tfrac{1}{2}}, \underbrace{-\tfrac{1}{2}+1}_{m_s = \frac{1}{2}}\rangle$$

$$S_+ |\downarrow\rangle = \hbar\,|\uparrow\rangle$$

c) $S_- \,|\uparrow\rangle = \hbar \,|\downarrow\rangle$

d) $S_- \,|\downarrow\rangle = 0$

$$\left.\begin{aligned} S_+ &= S_x + i S_y \\ S_- &= S_x - i S_y \end{aligned}\right\} \qquad \begin{aligned} S_x &= \frac{1}{2}\left(S_+ + S_-\right) \\ S_y &= \frac{-i}{2}\left(S_+ - S_-\right) \end{aligned}$$

$$S_x\,|\uparrow\rangle = \frac{1}{2}\left(S_+ + S_-\right)|\uparrow\rangle = 0 + \frac{\hbar}{2}\,|\downarrow\rangle$$

$$S_x\,|\downarrow\rangle = \frac{1}{2}\left(S_+ + S_-\right)|\downarrow\rangle = \frac{\hbar}{2}\,|\uparrow\rangle$$

$$S_y\,|\uparrow\rangle = \frac{-i}{2}\left(S_+ - S_-\right)|\uparrow\rangle = +\frac{i}{2}\hbar\,|\downarrow\rangle$$

$$S_y\,|\downarrow\rangle = -\frac{i}{2}\left(S_+ - S_-\right)|\downarrow\rangle = -\frac{i}{2}\hbar\,|\uparrow\rangle$$

$$S_z\,|\uparrow\rangle = \frac{\hbar}{2}\,|\uparrow\rangle$$

$$S_z\,|\downarrow\rangle = -\frac{\hbar}{2}\,|\downarrow\rangle$$

$$S_i = \begin{pmatrix} \langle\uparrow|S_i|\uparrow\rangle & \langle\uparrow|S_i|\downarrow\rangle \\ \langle\downarrow|S_i|\uparrow\rangle & \langle\downarrow|S_i|\downarrow\rangle \end{pmatrix}_{2\times 2}$$

$$S_x = \begin{pmatrix} {}^*\langle\uparrow|S_x|\uparrow\rangle & \langle\uparrow|S_x|\downarrow\rangle \\ \langle\downarrow|S_x|\uparrow\rangle & \langle\downarrow|S_x|\downarrow\rangle \end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$\langle\uparrow|\uparrow\rangle = 1$ فرض

$\langle\uparrow|\downarrow\rangle = \langle\downarrow|\uparrow\rangle = 0$

$\langle\downarrow|\downarrow\rangle = 1$

$\langle\uparrow|S_x|\uparrow\rangle = \frac{\hbar}{2} \quad \langle\uparrow|\downarrow\rangle = 0$

$\langle\uparrow|S_x|\downarrow\rangle = \frac{\hbar}{2} \quad \langle\uparrow|\uparrow\rangle = \frac{\hbar}{2}$

$\langle\downarrow|S_x|\uparrow\rangle = \frac{\hbar}{2}$

$\langle\downarrow|S_x|\downarrow\rangle = 0$

$$S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad S_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\vec{S} = \frac{\hbar}{2}\,\vec{\sigma} \longrightarrow (\sigma_x, \sigma_y, \sigma_z)$$

Pauli ماتریس

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$