Lecture 28-QM_II (11/10/00)

Saturday, January 1, 2022 2:07 PM

1) حالت مقید

$$V(x) = -V_0\,\theta(a-|x|)$$
$$E<0$$

$$\kappa_B^2 = \frac{2m|E|}{\hbar^2}$$

$$q^2 = \frac{2m(V_0-|E|)}{\hbar^2}$$

$V_0 \to \infty$ در حد

1) $\boxed{\frac{\kappa_B}{q} = \operatorname{tg} qa}$ $\quad qa \simeq \frac{\pi}{2a}(2n+1)$ $\quad n=0,1,\dots$

2) $\boxed{\frac{\kappa_B}{q} = -\operatorname{cotg} qa}$ $\quad q_n a \simeq n\pi$ $\quad$ "

1) $E_n \simeq -V_0 + \frac{(2n+1)^2\pi^2\hbar^2}{8ma^2}$ جواب با پاریته زوج

2) $E_n \simeq -V_0 + \frac{n^2\pi^2\hbar^2}{2ma^2}$ جواب با پاریته فرد

حالت کلی برای l های دلخواه

مقید $\boxed{-|E_l| \simeq -V_0 + \frac{l^2\pi^2\hbar^2}{8ma^2}}$

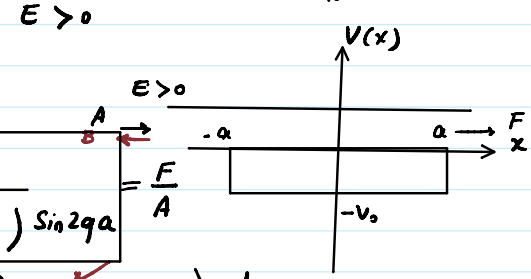
$l=0,1,\dots$

2) حالت پراکندگی

$$V(x) = -V_0\,\theta(a-|x|)$$
$$E>0$$

$$k^2 = \frac{2mE}{\hbar^2}$$

$$q^2 = \frac{2m(V_0+E)}{\hbar^2}$$



ضریب عبور → $\boxed{S(E) = \frac{e^{-2ika}}{\cos 2qa - \frac{i}{2}\left(\frac{q}{k}+\frac{k}{q}\right)\sin 2qa}}$ $\quad = \frac{F}{A}$

$$|S(E)|^2 = \left(1 + \frac{\sin^2 2qa}{4\left(\frac{E}{V_0}\right)\left(1+\frac{E}{V_0}\right)}\right)^{-1}$$

$$R(E) = \frac{B}{A}$$

$$\underbrace{|R(E)|^2}_{=0} + \underbrace{|S(E)|^2}_{=1} = 1$$

تشدید $\sin^2 2qa = 1$

$$\boxed{E_R = -V_0 + \frac{n^2\pi^2\hbar^2}{8ma^2}} > 0$$

قطب های $S(E)$

$$\cos 2qa = \frac{i}{2}\left(\frac{q}{k}+\frac{k}{q}\right)\sin 2qa$$

$$\operatorname{cotg} 2qa = \frac{1}{2}(\operatorname{cotg} qa - \operatorname{tg} qa) = \frac{i}{2}\left(\frac{q}{k}+\frac{k}{q}\right) \quad (E)$$

a) $\operatorname{cotg} qa = i\frac{q}{k} \rightarrow \operatorname{tg} qa = -i\frac{k}{q}$

b) $\operatorname{cotg} qa = i\frac{k}{q} \rightarrow \operatorname{tg} qa = -i\frac{q}{k}$

$q^* = \sqrt{\frac{2m(V_0+E)}{\hbar^2}}$

$k^2 = \frac{2mE}{\hbar^2}$

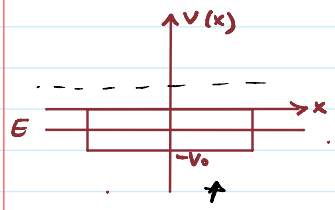
پراکندگی $\begin{cases} V_0+E>0 \\ E>-V_0 \end{cases}$ $\quad E>0$

(b) اگر k, q هردو حقیقی باشند

(a) اگر q, k هر دو حقیقی باشند $\Rightarrow$ $E<0$ $\quad V_0+E<0$ $\quad E<-V_0$

(b.1 q مختلط و k حقیقی $\Rightarrow$ $E>0$ $\quad V_0+E<0$ $\quad E<-V_0$

(b.2 q حقیقی و k مختلط



$V_0+E>0$ $\quad E>-V_0$ $\quad E<0$ } مقید

$k^2=-\kappa_B^2 \rightarrow k=+i\kappa_B$

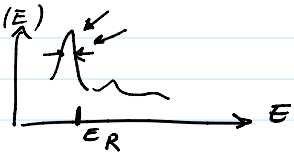
b) $\cot g\, qa = \dfrac{i(+i\kappa_B)}{q}$ حالت مقید با پاریته فرد $\cdot$ $\cot g\, qa = -\dfrac{\kappa_B}{q}$ ✓

a) $\cot g\, qa = i\dfrac{q}{k} =$

$\cot g\, qa = \dfrac{iq}{i\kappa_B} = \dfrac{q}{\kappa_B}$

حالت مقید با پاریته زوج $\quad tg\, qq = \dfrac{\kappa_B}{q}$ ✓

$|S(E)|^2 \rightsquigarrow$ (6) (E) سطح مقطع پراکندگی



$$e^{2ika}S(E) = \frac{1}{\cos 2qa\left(1 - \frac{i}{2}\left(\frac{q}{k}+\frac{k}{q}\right)tg\,2qa\right)}$$

$*$ شرط تشدید $2qa = n\pi$ $\quad E_R$

$\frac{1}{2}\left(\frac{q}{k}+\frac{k}{q}\right)tg\,2qa = f(E)$

$f(E) = \underbrace{f(E_R)}_{=0} + (E-E_R)f'(E_R) + \cdots$

$\cos 2qa\,|_{2qa=n\pi} = (-1)^n$

$f(E_R) = \frac{1}{2}\left(\frac{2aq}{2ak}+\frac{2ak}{2aq}\right)tg\,2qa\,|_{2qa=n\pi} = 0$

$f'(E_R) = \left.\dfrac{df(E)}{dE}\right|_{E_R} = \dfrac{df}{\partial(2qa)}\dfrac{d(2qa)}{dE}\Big|_{\substack{2qa=n\pi \\ E=E_R}}$

$= \frac{1}{2}\left(\frac{q}{k}+\frac{k}{q}\right)\frac{1}{\cos^2 2qa}\Big|_{2qa=n\pi}\left.\frac{d(2qa)}{dE}\right|_{E=E_R}$

$f'(E_R) = \frac{1}{2}\left(\frac{q}{k}+\frac{k}{q}\right)\left(\frac{d(2qa)}{dE}\right)_{E=E_R}$

$$e^{2ika}S(E) = \left.\frac{1}{\cos 2qa\,(1-if(E))}\right|_{E_R}$$

$$= (-1)^n \frac{1}{1 - i(E-E_R)\underbrace{f'(E_R)}_{\frac{2}{\Gamma}} + \cdots}$$

$2ika$ $\quad (-1)^n\, i\Gamma/2$

$$e^{2ika}\, S(E) \simeq (-1)^n \frac{1}{1-\frac{2i}{\Gamma}(E-E_R)} = \frac{(-1)^n\, i\Gamma/2}{E-E_R+\frac{i\Gamma}{2}}$$

$$\frac{2}{\Gamma} = f'(E_R) = \frac{1}{2}\frac{\sqrt{2m}}{\hbar}\, a \left(\frac{2E_R+V_0}{\sqrt{E_R}\,(E_R+V_0)}\right)$$

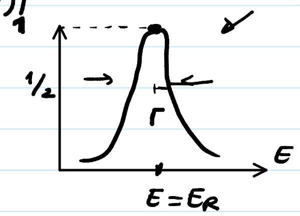
1) $\boxed{E = E_R - \frac{i\Gamma}{2}}$ ✓ $\longrightarrow$ $S(E)$ قطب دارد $\to \infty$

2) $$|S(E)|^2 = \frac{\Gamma^2/4}{(E-E_R)^2+\frac{\Gamma^2}{4}} = \frac{1}{2}$$

$\sigma \sim |S(E)|^2$

Γ Spectral width

Breit-Wigner function



پراکندگی $\sqrt{E}$ branch cut.

$E>0 \qquad \sqrt{E} = \pm |E|^{1/2}$

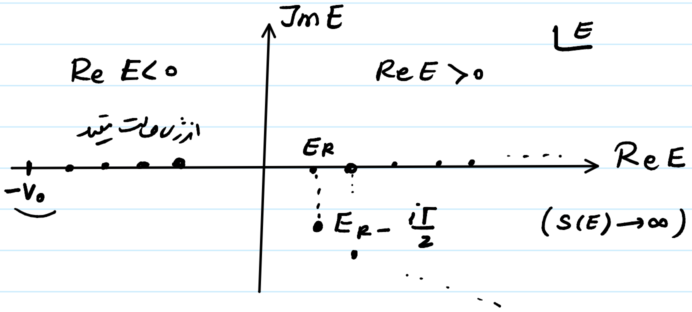
$E>0 \qquad E = |E|$ ✓ ✓ $\longrightarrow |E|^{1/2}$

$E = |E|\, e^{2\pi i} \longrightarrow |E|^{1/2}\, e^{\pi i} = -|E|^{1/2}$

$\cos 2\pi + i \sin 2\pi \qquad\qquad \cos\pi + i\sin\pi = -1$

Riemann ادامه تحلیلی در صفحه دوم

$\mathcal{L}E$



$$-\frac{n^2\pi^2\hbar^2}{8ma^2} - V_0 = -|E_n|$$

حالت مقید

$|S(E)|^2 = 1$ حالت پراکندگی (تشدید)

$$E_R = -V_0 + \frac{n^2\pi^2\hbar^2}{8ma^2}$$