	تابع حالت	عملگر
تصویر شرودینگر $H=H_s$	$i\hbar \frac{\partial}{\partial t}\|\psi,t\rangle_s = \hat{H}_s \|\psi,t\rangle_s$ $\|\psi,t\rangle_s = e^{-i\hat{H}_s t/\hbar}\|\psi,0\rangle_s$ *	$\hat{H}_s\ ,\ \hat{O}_s$ مستقل از زمان $\frac{d}{dt}\hat{O}_s = 0$
تصویر هایزنبرگ $H=H_s$	$\|\psi\rangle_H = \|\psi,0\rangle_s = e^{iH_s t/\hbar}\|\psi,t\rangle_s$ $\frac{d}{dt}\|\psi\rangle_H = 0$ ✓	$O_H(t) = e^{iH_s t/\hbar} O_s e^{-iH_s t/\hbar}$ * $\frac{d}{dt}O_H(t) = \frac{i}{\hbar}[H_s, O_H(t)] + \left(\frac{\partial O}{\partial t}\right)_H$
تصویر دیراک (برهمکنش) $H=H_0+V(t)$ غیر وابسته به زمان	✓ $\|\psi,t\rangle_I = e^{iH_0 t/\hbar}\|\psi,t\rangle_s$ $= e^{-\frac{i}{\hbar}\int V(t')dt'}\|\psi,0\rangle_s$ درصورتیکه $[H_0, V(t)]=0$	$O_I(t) = e^{iH_0 t/\hbar} O_s e^{-iH_0 t/\hbar}$ ✓ ✓ $\frac{d}{dt}O_I(t) = \frac{i}{\hbar}[H_0, O_I(t)] + \left(\frac{\partial O(t)}{\partial t}\right)_I$

$$i\hbar\frac{d}{dt}|\psi,t\rangle_I = V_I(t)\,|\psi,t\rangle_I$$

$$V_I(t) = e^{iH_0 t/\hbar}\, V(t)\, e^{-iH_0 t/\hbar}$$

$$\langle O_s\rangle_s = \langle O_I(t)\rangle_I$$

$${}_I\langle \psi,t|O_I|\psi,t\rangle_I = {}_s\langle\psi,t|\underbrace{e^{-iH_0 t/\hbar}\,O_I\,e^{iH_0 t/\hbar}}_{O_s}|\psi,t\rangle_s = {}_s\langle O_s\rangle_s$$

$$O_s = e^{-iH_0 t/\hbar}\,O_I\,e^{iH_0 t/\hbar}$$

$$e^{+iH_0 t/\hbar}\,O_s\,e^{-iH_0 t/\hbar} = O_I \quad ✓$$

$$\frac{d}{dt}O_I = \ldots$$

• اختلال وابسته به زمان (سری نویمن Neumann)

$$i\hbar\frac{\partial}{\partial t}|\psi,t\rangle_I = V_I(t)\,|\psi,t\rangle_I$$

$$i\hbar\int_{t_0}^{t}dt'\,\frac{\partial}{\partial t'}|\psi,t'\rangle_I = \int_{t_0}^{t}dt'\,V_I(t')\,|\psi,t'\rangle_I$$

$$i\hbar\left(|\psi,t\rangle_I - |\psi,t_0\rangle_I\right) = \int_{t_0}^{t}dt'\,V_I(t')\,|\psi,t'\rangle_I$$

$$|\psi,t\rangle_I = |\psi,t_0\rangle_I + \frac{1}{i\hbar}\int_{t_0}^{t}dt'\,V_I(t')\,|\psi,t'\rangle_I \qquad (*)$$

$$|\psi,t\rangle_I \overset{(*)}{=} |\psi,t_0\rangle_I + \frac{1}{i\hbar}\int_{t_0}^{t}dt'\,V_I(t')\,|\psi,t_0\rangle_I + \left(\frac{1}{i\hbar}\right)^2\int_{t_0}^{t}dt'\,V_I(t')\int_{t_0}^{t'}dt''\,V_I(t'')\,|\psi,t''\rangle_I \qquad (**)$$

$$\boxed{|\psi,t\rangle_I = |\psi,t_0\rangle_I + \frac{1}{i\hbar}\int_{t_0}^{t}dt'\,V_I(t')\,|\psi,t_0\rangle_I + \frac{1}{(i\hbar)^2}\int_{t_0}^{t}dt'\,V_I(t')\int_{t_0}^{t'}dt''\,V_I(t'')\,|\psi,t_0\rangle + \ldots}$$

$$+ \frac{1}{(i\hbar)^2} \int_{t_0}^{t} dt' \, V_I(t') \int_{t_0} dt \, V_I(t') \, |\psi, t_0\rangle + \cdots$$

$$i\hbar \frac{\partial}{\partial t} |\psi, t\rangle_I = V_I(t) \, |\psi, t\rangle_I \quad \longleftarrow \quad \boxed{|\psi, t\rangle_I = \mathcal{U}(t, t_0) \, |\psi, t_0\rangle_I}$$

بیس دو جواب

$$\mathcal{U}(t_0, t_0) = 1$$

$$i\hbar \frac{\partial}{\partial t} \left(\mathcal{U}(t, t_0) \, |\psi, t_0\rangle_I \right) = V_I(t) \left(U(t, t_0) \, |\psi, t_0\rangle_I \right)$$

$$i\hbar \frac{\partial}{\partial t} U(t, t_0) = V_I(t) \, U(t, t_0) \qquad *$$

ادعا:

$$\boxed{U(t, t_0) = T \exp\left(-\frac{i}{\hbar} \int_{t_0}^{t} V_I(t') dt' \right)}$$

عملگر ترتیب زمانی

سری دایسون - Dyson

اثبات:

$$t_0 \quad \Delta t \quad t \qquad \Delta t = \frac{t - t_0}{n}$$

$n = 1$ $\qquad t_0 \;\text{———}\; t$

$$i\hbar \frac{\partial}{\partial t} U(t, t_0) \Big|_{t_0} = V_I(t_0) \, U(t_0, t_0) = V_I(t_0)$$

$$i\hbar \; \frac{U(t_0 + \Delta t, t_0) - \underbrace{\cancel{U(t_0, t_0)}}_{=1}}{\Delta t} = V_I(t_0)$$

$$\boxed{U(t_0 + \Delta t, t_0) = 1 - \frac{i}{\hbar} \Delta t \, V_I(t_0)}$$

$n = 2$ $\qquad t_0 \xrightarrow{\Delta t} t_1 \xrightarrow{\Delta t} t_2 = t \qquad\qquad t_0 < t_1 < t_2 = t$

$$|\psi, t\rangle_I = U(t, t_0) \, |\psi, t_0\rangle_I \qquad t_0 < t$$

$$U(t, t_1) \, U(t_1, t_0) = U(t, t_0)$$

$$U(t_1 + \Delta t, t_1) \quad U(t_0 + \Delta t, t_0) = U(t, t_0)$$

$$U(t, t_0) = \left(1 - \frac{i}{\hbar} \Delta t \; V_I(t_1) \right) \left(1 - \frac{i}{\hbar} \Delta t \; V_I(t_0) \right)$$

$$= 1 - \frac{i}{\hbar} \Delta t \left(V_I(t_1) + V_I(t_0) \right) + \left(\frac{-i}{\hbar} \right)^2 \Delta t^2 \; V_I(t_1) \, V_I(t_0)$$

$$t_0 < t_1$$

$$t_0 \quad t_1 \quad t_2 \quad \cdots \quad t_N = t \qquad\qquad t_0 < t_1 < \cdots < t_N = t$$

$$U(t, t_0) = U(t_N, t_{N-1}) \cdots U(t_2, t_1) \, U(t_1, t_0)$$

$$= \left(1 - \frac{i}{\hbar} \Delta t \; V_I(t_{N-1}) \right) \cdots \cdots \left(1 - \frac{i}{\hbar} \Delta t \, V_I(t_0) \right)$$

$$= 1 - \frac{i}{\hbar} \Delta t \sum_{n=0}^{N-1} V_I(t_n)$$

$$+ \left(-\frac{i}{\hbar} \Delta t \right)^2 \sum_{\substack{m, n = 0 \\ m < n}}^{N-1} V_I(t_n) \, V_I(t_m) + \cdots$$

همزمان $\begin{cases} \Delta t \to 0 \\ N \to \infty \end{cases}$

جمع $\Big]\; N \to \infty$

$$U(t,t_0) = 1 - \frac{i}{\hbar}\int_{t_0}^{t} dt_1\, V_I(t_1) + \left(\frac{-i}{\hbar}\right)^2 \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2\; V_I(t_1)\, V_I(t_2) \qquad t_2 < t_1$$

$$+ \cdots + \left(\frac{-i}{\hbar}\right)^N \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \cdots \int_{t_0}^{t_{n-1}} dt_n\; V_I(t_1)\cdots V_I(t_n)$$

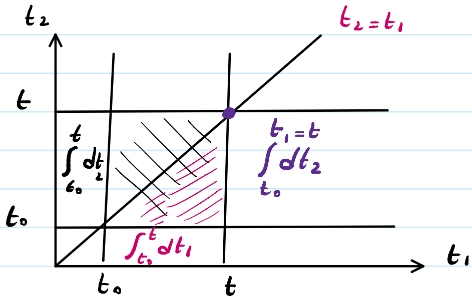
سری دایسون

$$\int_{t_0}^{t} \underset{dt_2}{dt_1} \int_{t_0}^{t_1} \underset{dt_1}{dt_2}\; \underset{t_2<t_1}{V_I(t_1)\, V_I(t_2)} \overset{!}{=} \frac{1}{2!}\int_{t_0}^{t} dt_1 \int_{t_0}^{t} dt_2\; T\Big(V_I(t_1)\, V_I(t_2)\Big)$$

$$T\Big(V_I(t_1)\, V_I(t_2)\Big) \equiv \underset{t_1>t_2}{\theta(t_1 - t_2)}\; V_I(t_1)\, V_I(t_2) + \theta(t_2 - t_1)\; V_I(t_2)\, V_I(t_1)$$

عملگر time ordering

$$U(t,t_0) = T \exp\left(\frac{-i}{\hbar}\int_{t_0}^{t} V(t')\,dt'\right)$$



$$|\psi,t\rangle_I = |\psi,t_0\rangle_I + \frac{1}{i\hbar}\int_{t_0}^{t} dt'\; V_I(t')\, |\psi,t_0\rangle_I + \cdots$$

اختلال رتبه اول : تداوم رتبه اول

$$|\psi,t\rangle_s = e^{-iH_0 t/\hbar}\, |\psi,t_0\rangle_s$$

غیروابسته $H_0 |n\rangle = E_n |n\rangle$

$$|n\rangle \rightarrow |n,t\rangle_s = e^{-iH_0 t/\hbar}\, |n\rangle$$

$$|n,t\rangle_s = e^{-iE_n t/\hbar}\, |n\rangle$$

$= t_0$: $V(t)$ $H = H_0 + V(t)$

$|\psi,t\rangle_s$

سوال: با چه احتمالی $|\psi,t\rangle_s$ برابر است با $|n,t\rangle_s$ بطوریکه

$$|n,t\rangle = e^{-iH_0 t/\hbar}\, |n\rangle \;\checkmark$$

$$\langle n,t\,|\,\psi,t\rangle_s = \langle n|\, \underbrace{e^{+iH_0 t/\hbar}\, |\psi,t\rangle_s}_{|\psi,t\rangle_I} =$$

$$\langle n|\psi,t\rangle_I = \langle n|\psi,t_0\rangle_I + \frac{1}{i\hbar}\int_{t_0}^{t} dt'\; \langle n|\, V_I(t')\, |\psi,t_0\rangle_I$$

$$|\psi,t_0\rangle_I = e^{iH_0 t_0/\hbar}\, |m,t_0\rangle_I = e^{+iH_0 t_0/\hbar}\, e^{-iH_0 t_0/\hbar}\, |m\rangle = |m\rangle$$

$$\langle n|\psi,t\rangle_I = \underbrace{\langle n|m\rangle}_{\substack{\delta_{nm} \\ 0 \;\; m\neq n}} + \frac{1}{i\hbar}\int_{t_0}^{t} dt'\; \langle n|\, V_I(t')\, |m\rangle$$

$$\langle n|\, e^{+iH_0 t/\hbar}\, V(t)\, e^{-iH_0 t/\hbar}\, |m\rangle$$

$$\langle n|\psi,t\rangle_I = \frac{1}{\cdots}\int^{t} dt'\; e^{i(E_n - E_m)t/\hbar}\; \langle n|V(t)|m\rangle$$

$$\langle n|\psi,t\rangle_I = \frac{1}{i\hbar}\int_{t_0}^{t} dt' \; e^{i(E_n-E_m)t/\hbar} \; \langle n|V(t)|m\rangle$$

where

$$H_I(t) = e^{iH_0(t-t_0)}(H_{\rm int})e^{-iH_0(t-t_0)} = \int d^3x\,\frac{\lambda}{4!}\phi_I^4 \tag{4.19}$$

is the interaction Hamiltonian written in the interaction picture. The solution of this differential equation for $U(t,t_0)$ should look something like $U \sim \exp(-iH_I t)$; this would be our desired formula for U in terms of ϕ_I. Doing it more carefully, we will show that the actual solution is the following power series in λ:

$$\begin{aligned} U(t,t_0) = 1 + (-i)\int_{t_0}^{t} dt_1\, H_I(t_1) + (-i)^2\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2\, H_I(t_1)H_I(t_2) \\ + (-i)^3\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \int_{t_0}^{t_2} dt_3\, H_I(t_1)H_I(t_2)H_I(t_3) + \cdots . \end{aligned} \tag{4.20}$$

To verify this, just differentiate: Each term gives the previous one times $-iH_I(t)$. The initial condition $U(t,t_0)=1$ for $t=t_0$ is obviously satisfied.

Note that the various factors of H_I in (4.20) stand in *time order*, later on the left. This allows us to simplify the expression considerably, using the time-ordering symbol T. The H_I^2 term, for example, can be written

$$\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2\, H_I(t_1)H_I(t_2) = \frac{1}{2}\int_{t_0}^{t} dt_1 \int_{t_0}^{t} dt_2\; T\{H_I(t_1)H_I(t_2)\}. \tag{4.21}$$

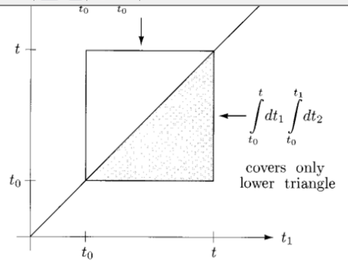


Figure 4.1. Geometric interpretation of Eq. (4.21).

The double integral on the right-hand side just counts everything twice, since in the t_1t_2-plane, the integrand $T\{H_I(t_1)H_I(t_2)\}$ is symmetric about the line $t_1 = t_2$ (see Fig. 4.1).

A similar identity holds for the higher terms:

$$\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \cdots \int_{t_0}^{t_{n-1}} dt_n\, H_I(t_1)\cdots H_I(t_n) = \frac{1}{n!}\int_{t_0}^{t} dt_1 \cdots dt_n\, T\{H_I(t_1)\cdots H_I(t_n)\}.$$

This case is a little harder to visualize, but it is not hard to convince oneself that it is true. Using this identity, we can now write $U(t,t_0)$ in an extremely compact form:

$$\begin{aligned} U(t,t_0) &= 1 + (-i)\int_{t_0}^{t} dt_1\, H_I(t_1) + \frac{(-i)^2}{2!}\int_{t_0}^{t} dt_1\, dt_2\; T\{H_I(t_1)H_I(t_2)\} + \cdots \\ &\equiv T\left\{\exp\left[-i\int_{t_0}^{t} dt'\, H_I(t')\right]\right\}, \end{aligned} \tag{4.22}$$

where the time-ordering of the exponential is just defined as the Taylor series with each term time-ordered. When we do real computations we will keep