Lecture 2-QM_II (29/06/00)

Monday, September 20, 2021 2:44 PM

۱) مروری بر الکترودینامیک کلاسیک

Maxwell
معادلات M تحت تبدیلات
پیمانه‌ای ناوردا هستند.

$$\vec\nabla\cdot\vec B = 0$$
$$\vec\nabla\times\vec E + \frac{1}{c}\frac{\partial}{\partial t}\vec B = 0$$
$$\vec\nabla\cdot\vec E = 4\pi\rho$$
$$\vec\nabla\times\vec B - \frac{1}{c}\frac{\partial}{\partial t}\vec E = \frac{4\pi}{c}\vec J$$

$\vec B(\vec x,t)$, $\vec E(\vec x,t)$, $\rho(\vec x,t)$, $\vec J(\vec x,t)$

معادله پیوستگی

$$\frac{d}{dt}\int d^3x\,\rho = \dot Q = 0 \quad\leftrightarrow\quad \frac{\partial\rho}{\partial t} + \vec\nabla\cdot\vec J = 0$$

$$\int d^3x\,\frac{\partial\rho}{\partial t} + \underbrace{\int d^3x\,\vec\nabla\cdot\vec J}_{=0} = 0$$

$$\frac{\partial}{\partial t}\underbrace{\int d^3x\,\rho}_{Q} = 0$$

$$m\ddot{\vec x} = q\left(\vec E + \frac{1}{c}\vec v\times\vec B\right) = \vec F_L \qquad q=-e$$

نیروی لورنتس

$$\vec A(\vec x,t) \qquad \varphi(\vec x,t)$$

$$\leftrightarrow\ \vec\nabla\cdot\vec B = 0 \longrightarrow \vec B = \vec\nabla\times\vec A$$

$$\vec\nabla\times\vec E + \frac{1}{c}\frac{\partial}{\partial t}\vec B = 0 \longrightarrow \vec\nabla\times\left(\vec E + \frac{1}{c}\frac{\partial}{\partial t}\vec A\right) = 0 \Longrightarrow$$

$$\vec\nabla\times\vec A = \vec B$$

$$\vec E = -\frac{1}{c}\frac{\partial}{\partial t}\vec A - \vec\nabla\varphi$$

اگر

$\underbrace{f(\vec x,t)}_{\text{دلخواه}}$ تبدیلات پیمانه‌ای

$$\begin{cases}\vec A'(\vec x,t) = \vec A(\vec x,t) + \vec\nabla f\\ \varphi'(\vec x,t) = \varphi(\vec x,t) - \frac{1}{c}\frac{\partial}{\partial t}f\end{cases}$$

$$\vec B' = \vec\nabla\times\vec A' \overset{!}{=} \vec\nabla\times\vec A = \vec B$$

میدان مغناطیسی تحت تبدیل
پیمانه‌ای فوق ناورداست

$$\vec B' = \vec\nabla\times\vec A' = \vec\nabla\times(\vec A + \vec\nabla f) = \vec\nabla\times\vec A + \underbrace{\vec\nabla\times\vec\nabla f}_{=0} = \vec B$$

میدان الکتریکی تحت تبدیل
پیمانه‌ای فوق ناورداست.

$$\vec E' = -\vec\nabla\varphi' - \frac{1}{c}\frac{\partial}{\partial t}\vec A' \overset{!}{=} -\vec\nabla\varphi - \frac{1}{c}\frac{\partial}{\partial t}\vec A = \vec E$$

در پیمانه کولنی

$$\underbrace{\vec\nabla\cdot\vec A = 0}_{\star}, \qquad \frac{\partial\varphi}{\partial t} = 0$$

1) $\vec\nabla\cdot\vec E = 4\pi\rho \Longrightarrow +\Delta\varphi = -4\pi\rho$

2) $\vec\nabla\times\vec B - \frac{1}{c}\frac{\partial}{\partial t}\vec E = \frac{4\pi}{c}\vec J \Longrightarrow \Delta\vec A - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec A = \frac{4\pi}{c}\vec J$

1) اثبات

$$\vec\nabla\cdot\left(-\vec\nabla\varphi - \frac{1}{c}\frac{\partial}{\partial t}\vec A\right) = 4\pi\rho$$

$$-\Delta\varphi - \frac{1}{c}\frac{\partial}{\partial t}\underbrace{\vec\nabla\cdot\vec A}_{\star=0} = 4\pi\rho \Rightarrow \boxed{\Delta\varphi = -4\pi\rho}$$

در پیمانه لورنتس

$$\partial_\mu A^\mu = 0 \qquad \vec\nabla\cdot\vec A + \frac{1}{c}\frac{\partial}{\partial t}\varphi = 0$$

$$\Rightarrow \vec\nabla\cdot\vec A = -\frac{1}{c}\frac{\partial}{\partial t}\varphi$$

1) اثبات

$$\vec\nabla\cdot\vec E = -\Delta\varphi - \frac{1}{c}\frac{\partial}{\partial t}\vec\nabla\cdot\vec A = 4\pi\rho$$

$$\Rightarrow \quad \nabla \cdot A = -\frac{1}{c}\frac{\partial}{\partial t}\varphi$$

۱) اثبات

$$\vec{\nabla}\cdot\vec{E} = -\Delta\varphi - \frac{1}{c}\frac{\partial}{\partial t}\vec{\nabla}\cdot\vec{A} = 4\pi\rho$$

$$\boxed{\Delta\varphi - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\varphi = -4\pi\rho}$$

اثر آهارونوف-بوهم

Aharonov-Bohm Effect.

• معادلات حرکت یک ذره باردار (الکتریکی) در حضور میدان الکترومغناطیسی در فرمالیسم لاگرانژ

$$L = L(\vec{x}, \dot{\vec{x}}; t) = T - U$$

$$= \frac{1}{2}m\dot{\vec{x}}^2 - q\,\varphi(\vec{x},t) + \frac{q}{c}\underbrace{\vec{A}(\vec{x},t)\cdot\dot{\vec{x}}}_{A_j\dot{x}_j}$$

معادلات لاگرانژ

$$\boxed{\frac{\partial L}{\partial x_i} - \frac{d}{dt}\frac{\partial L}{\partial \dot{x}_i} = 0}$$

$$\frac{\partial L}{\partial x_i} = -q\,\partial_i\varphi + \frac{q}{c}\,\partial_i A_j\,\dot{x}_j \quad v_j$$

$$\frac{\partial L}{\partial \dot{x}_i} = m\dot{x}_i + \frac{q}{c}A_i \xrightarrow{\frac{d}{dt}} m\ddot{x}_i + \frac{q}{c}\frac{d}{dt}A_i = \frac{d}{dt}\frac{\partial L}{\partial \dot{x}_i}$$

$$\frac{d}{dt}A_i = \frac{\partial}{\partial t}A_i + \dot{x}_j\,\partial_j A_i \qquad \vec{A}(\vec{x},t)$$

$$-q\,\partial_i\varphi + \frac{q}{c}\,\partial_i A_j\,v_j = m\ddot{x}_i + \frac{q}{c}\left(\frac{\partial}{\partial t}A_i + v_j\,\partial_j A_i\right)$$

$$m\ddot{x}_i = +q\underbrace{\left(-\partial_i\varphi - \frac{1}{c}\frac{\partial}{\partial t}A_i\right)}_{+E_i} + \frac{q}{c}v_j\underbrace{\left(\partial_i A_j - \partial_j A_i\right)}_{F_{ij}}$$

$$F_{ij} = \partial_i A_j - \partial_j A_i = \epsilon_{ijk}B_k$$

$$\begin{pmatrix} 0 & B_x & -B_y \\ -B_x & 0 & B_z \\ +B_y & -B_z & 0 \end{pmatrix} = F$$

$$\frac{q}{c}v_j\,\epsilon_{ijk}B_k = \frac{q}{c}(\vec{v}\times\vec{B})_i$$

$$\boxed{m\ddot{\vec{x}} = q\vec{E} + \frac{q}{c}(\vec{v}\times\vec{B})}$$

$$\underline{\underline{H}}\psi = E\psi$$

• در فرمالیسم هامیلتون

$$[\hat{x}_i, \hat{p}_j] = i\hbar\,\delta_{ij}$$

$$L = L(\vec{x}, \dot{\vec{x}}; t)$$
$$H = H(\vec{x}, \vec{p}; t)$$

$$\frac{\partial L}{\partial \dot{x}_i} = p_i$$

$$\frac{\partial H}{\partial x_i} = -\dot{p}_i$$

$$\frac{\partial H}{\partial p_i} = \dot{x}_i$$

$$H = \frac{\vec{p}^2}{2m} + V(\vec{x})$$

$$\frac{\partial H}{\partial x_i} = -\dot{p}_i = -\partial_i V$$

$2m$

$$\left.\begin{aligned}\frac{\partial H}{\partial x_i} &= -\dot{p}_i = -\partial_i V \\ \frac{\partial H}{\partial p_i} &= \dot{x}_i = \frac{p_i}{m}\end{aligned}\right\} \longrightarrow \ddot{x}_i = \frac{\dot{p}_i}{m} \Rightarrow \ddot{x}_i = -\frac{\partial_i V}{m}$$

$$m\ddot{x}_i = -\partial_i V = F_i$$
$$m\ddot{\vec{x}} = -\vec{\nabla} V = \vec{F}$$

اّرش

$$H = \frac{1}{2m}\left(\vec{P} - \frac{e}{c}\vec{A}\right)^2 + e\varphi$$

1) $\frac{\partial H}{\partial x_i} = -\dot{P}_i$

2) $\frac{\partial H}{\partial p_i} = \dot{x}_i$

$\dot{x}_j = v_j$

1) $\frac{\partial H}{\partial x_i} = -\frac{e}{c}\frac{1}{m}\left(\vec{P} - \frac{e}{c}\vec{A}\right)_j \partial_i A_j + e\,\partial_i\varphi = -\dot{P}_i$

2) $\frac{\partial H}{\partial p_i} = \frac{1}{m}\left(\vec{P} - \frac{e}{c}\vec{A}\right)_i = \dot{x}_i$

1) $\frac{\partial H}{\partial x_i} = -\dot{p}_i = -\frac{e}{c} v_j \partial_i A_j + e\,\partial_i\varphi$

$$\dot{x}_i = \frac{1}{m}\left(p_i - \frac{e}{c}A_i\right) \longrightarrow m\ddot{x}_i = \dot{p}_i - \frac{e}{c}\frac{d}{dt}A_i$$
$$= \dot{p}_i - \frac{e}{c}\frac{\partial}{\partial t}A_i - \frac{e}{c}v_j\partial_j A_i$$

$$m\ddot{x}_i = \frac{e}{c}v_j\,\partial_i A_j - e\,\partial_i\varphi - \frac{e}{c}\frac{\partial}{\partial t}A_i - \frac{e}{c}v_j\,\partial_j A_i$$
$$= e\left(-\partial_i\varphi - \frac{1}{c}\frac{\partial}{\partial t}A_i\right) + \frac{e}{c}v_j\underbrace{\left(\partial_i A_j - \partial_j A_i\right)}_{F_{ij} = \epsilon_{ijk}B_k}$$
$$= eE_i + \frac{e}{c}v_j\,\epsilon_{ijk}B_k$$
$$m\ddot{\vec{x}} = e\vec{E} + \frac{e}{c}\vec{v}\times\vec{B} = \vec{F}_L$$

در کوانتوم مکانیک

$$\hat{H} = \frac{1}{2m}\left(\hat{\vec{P}} - \frac{e}{c}\hat{\vec{A}}(\vec{x})\right)^2 + e\,\hat{\varphi}(\vec{x})$$

$\psi(\vec{x},t) \rightsquigarrow \hat{H}\psi(\vec{x}) = E\psi(\vec{x})$

$e^{\frac{i}{\hbar}Et}$

$\hat{\vec{P}} = \frac{\hbar}{i}\vec{\nabla}_x$

$\hat{\vec{x}} = \vec{x}$

$$\left.\begin{aligned}\hat{\vec{A}}(\vec{x}) &= \vec{A}(\vec{x}) \\ \hat{\varphi}(\vec{x}) &= \varphi(\vec{x})\end{aligned}\right\}$$

$$\boxed{\left(\frac{1}{2m}\left(\frac{\hbar}{i}\vec{\nabla} - \frac{e}{c}\vec{A}(\vec{x})\right)^2 + e\,\varphi(\vec{x})\right)\psi(\vec{x}) = E\,\psi(\vec{x})}$$