Lecture 14-QM_II (15/08/00)

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اختلال غیروابسته به زمان :

$$H = H_0 + \lambda H_1$$
$$H_0 |n^{(0)}\rangle = E_n^{(0)} |n^{(0)}\rangle$$
$$H |n\rangle = E_n |n\rangle$$
$$E_n = E_n^{(0)} + \lambda E_n^{(1)} + \dots$$
$$|n\rangle = |n^{(0)}\rangle + \lambda |n^{(1)}\rangle + \dots$$

- واگنی نداریم

$$H_0 |n^{(0)}\rangle = E_n^{(0)} |n^{(0)}\rangle$$
$$n \neq k \rightarrow E_n^{(0)} \neq E_k^{(0)}$$

$E_n^{(1)}, E_n^{(2)}$ $\quad |n^{(1)}\rangle, |n^{(2)}\rangle$

$$E_n^{(1)} = \langle n^{(0)} | H_1 | n^{(0)} \rangle$$

- واگنی داریم

$$H_0 |n^{(0)}_i\rangle = E_n^{(0)} |n^{(0)}_i\rangle \qquad i = 1, \dots \ell$$

$$E_n^{(1)} \rightarrow \quad \underset{\sim}{H_1} \vec{\alpha} = E_n^{(1)} \vec{\alpha}$$

اثر اشتارک :

$$H = H_0 + H_1$$
$$H_0 = \frac{\vec{P}^2}{2m_e} - \frac{Ze^2}{r}$$
$$H_1 = e \mathcal{E} z$$

$$E^{(1)}_{n=1} = 0$$
$$n=1 \quad \ell = 0 \quad m_\ell = 0$$
$$|n \ell m_\ell\rangle = |100\rangle$$

$$\langle n \ell m_\ell | z | n' \ell' m'_\ell \rangle \neq 0$$

قواعد گزینش

$$\Delta m_\ell = 0 \qquad \Delta \ell = \pm 1$$
$$m'_\ell - m_\ell = 0 \qquad \ell' - \ell = +1$$

$$E^{(1)}_{n=2} = ?$$

$n=2$ $\quad \ell = 0, 1 \quad$ $\ell = 0 \quad m_\ell = 0 \qquad -\ell \leq m_\ell \leq \ell$

$\ell = 1 \quad m_\ell = 0, \pm 1$

$$E_n^{(0)} = -\frac{1 R_y}{n^2}$$

$$|n \ell m_\ell\rangle = \begin{cases} |2\,0\,0\rangle \\ |2\,1\,0\rangle \\ |2\,1,+1\rangle \\ |2\,1,-1\rangle \end{cases}$$

$$E^{(0)}_{n=2} = -\frac{1 R_y}{4}$$

$$\boxed{\underset{\sim}{H_1} \vec{\alpha} = E^{(1)}_{n=2} \vec{\alpha}} \qquad H_1 = e \mathcal{E} z$$

$$\sum_i \langle n_j^{(0)} | H_1 | n_i^{(0)} \rangle \alpha_i = E_n^{(1)} \alpha_j$$

$$\underset{\sim}{H_1} = e\mathcal{E} \begin{pmatrix} \cancel{\langle 200|z|200\rangle} & \langle 200|z|210\rangle & \cancel{\langle 200|z|211\rangle} & \cancel{\langle 200|z|21\text{-}1\rangle} \\ \langle 210|z|200\rangle & \cancel{\langle 210|z|210\rangle} & \cancel{\langle 210|z|211\rangle} & \cancel{\langle 210|z|21,\text{-}1\rangle} \\ \cancel{\langle 211|z|200\rangle} & \cancel{\langle 211|z|210\rangle} & \cancel{\langle 211|z|211\rangle} & \cancel{\langle 211|z|21,\text{-}1\rangle} \\ \cancel{\langle 21\text{-}1|z|200\rangle} & \cancel{\langle 21\text{-}1|z|210\rangle} & \cancel{\langle 21\text{-}1|z|211\rangle} & \cancel{\langle 21\text{-}1|z|21\text{-}1\rangle} \end{pmatrix}$$

$$\Delta m_\ell = 0 \qquad \Delta \ell = \pm 1$$

$$\langle 200 | z | 210 \rangle = -3a_0 \qquad \langle 210 | z | 200 \rangle = -3a_0$$

$$R_{n\ell}(r)\, Y_{\ell m}(\theta, \varphi) = \psi_{n\ell m}(\vec{r})$$

$$\langle 200 | z | 210 \rangle = -3a_0 \qquad \langle 210 | z | 200 \rangle = -3a_0$$

$$R_{n\ell}(r)\, Y_{\ell m}(\theta,\varphi) = \psi_{n\ell m}(\vec{r})$$

$n = 2 \qquad \ell = 1, 0 \qquad m_\ell = 0$

$$\int \psi^*_{200}(\vec{r})\, \psi_{210}(\vec{r})\, z \, d^3r = -3a_0$$

$$\underset{\sim}{H}_1 = e\mathcal{E} \begin{pmatrix} \begin{pmatrix} 0 & -3a_0 \\ -3a_0 & 0 \end{pmatrix} & \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} \\ \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} \end{pmatrix}$$

$$-3a_0 e\mathcal{E} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = \underset{\sim}{E^{(1)}_{n=2}} \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$$

$$\begin{pmatrix} -E^{(1)}_{n=2} & -3a_0 e\mathcal{E} \\ -3a_0 e\mathcal{E} & -E^{(1)}_{n=2} \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\det \begin{pmatrix} -E^{(1)}_{n=2} & -3a_0 e\mathcal{E} \\ -3a_0 e\mathcal{E} & -E^{(1)}_{n=2} \end{pmatrix} = 0$$

$$\left(E^{(1)}_{n=2}\right)^2 - \left(3a_0 e\mathcal{E}\right)^2 = 0 \longrightarrow E^{(1)}_{n=2} = \pm 3a_0 e\mathcal{E}$$

$$E^{(1)}_{n=2} = \pm 3a_0 e\mathcal{E} \qquad \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$$

$$-\cancel{3a_0 e\mathcal{E}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = \pm \cancel{3a_0 e\mathcal{E}} \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$$

$$-\begin{pmatrix} \alpha_2 \\ \alpha_1 \end{pmatrix} = \pm \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} \rightarrow \begin{matrix} \alpha_2 = -\alpha_1 \\ \alpha_2 = +\alpha_1 \end{matrix}$$

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \qquad \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$|n\rangle = N(\lambda) \left\{ \underbrace{\sum_i \alpha_i |n_i^{(0)}\rangle}_{|\tilde{n}^{(0)}\rangle} + O(\lambda) \right\}$$

$$|\tilde{n}^{(0)}\rangle = \alpha_1 |200\rangle + \alpha_2 |210\rangle + \underset{0}{\underset{\shortparallel}{\alpha_3}} |211\rangle + \underset{0}{\underset{\shortparallel}{\alpha_4}} |21,-1\rangle$$

$$E^{(1)}_{n=2} = +3a_0 e\mathcal{E} \qquad \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$$

$$|\tilde{n}^{(0)}\rangle = \frac{1}{\sqrt{2}} \left(|200\rangle - |210\rangle \right)$$

$$E^{(1)}_{n=2} = -3a_0 e\mathcal{E} \qquad \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$$

$$|\tilde{n}^{(0)}\rangle = \frac{1}{\sqrt{2}} \left(|200\rangle + |210\rangle \right)$$

$$+3a_0 e\mathcal{E} \qquad \frac{1}{\sqrt{2}} \left(|200\rangle - |210\rangle \right)$$

$-\frac{1Ry}{4}$ $\quad n=2$

$|200\rangle, |210\rangle, |211\rangle, |21,-1\rangle$

$+3a_0e\mathcal{E}$

$-3a_0e\mathcal{E}$

$\frac{1}{\sqrt{2}}\left(|200\rangle - |210\rangle\right)$ $\quad |211\rangle \quad |21-1\rangle$

$\frac{1}{\sqrt{2}}\left(|200\rangle + |210\rangle\right)$

m_ℓ

Stark

اتم هیدروژن واقعی

- اثر تصحیح انرژی جنبشی نسبیتی :

غیرنسبیتی $\quad E = \frac{p^2}{2m} = \frac{1}{2}mv^2 \qquad \frac{(mv)^2}{2m} = \frac{1}{2}mv^2$

$$E = \sqrt{\vec{p}^2c^2 + m_e^2c^4}$$

$$E_{kin} = E - m_ec^2 = \sqrt{\vec{p}^2c^2 + m_e^2c^4} - m_ec^2$$

$$= \sqrt{m_e^2c^4\left(\frac{\vec{p}^2}{m_e^2c^2} + 1\right)} - m_ec^2$$

$\vec{p}^2 \ll m_e^2c^2$

$$\sim m_ec^2\left(1 + \frac{1}{2}\frac{\vec{p}^2}{m_e^2c^2} - \frac{1}{8}\left(\frac{\vec{p}^2}{m_e^2c^2}\right)^2 + \cdots\right) - m_ec^2$$

$$= \cancel{m_ec^2} + \frac{\vec{p}^2}{2m_e} - \frac{1}{8}\frac{(\vec{p}^2)^2}{m_e^3c^2} + \cdots - \cancel{m_ec^2}$$

انرژی جنبشی غیرنسبیتی $\qquad$ اولین تصحیح به انرژی جنبشی غیرنسبیتی

$m_e^2\vec{v}^2 \ll m_e^2c^2 \qquad v^2 \lesssim c^2$

$$H = H_0 + H_1$$

$$H_0 = \frac{\vec{p}^2}{2m_e} - \frac{Ze^2}{r}$$

$$H_1 = -\frac{(\vec{p}^2)^2}{8m_e^3c^2} \qquad E_n^{(1)}$$

$$H_1 = -\frac{1}{2m_ec^2}\left(\frac{\vec{p}^2}{2m_e}\right)^2$$

$n \quad \ell \quad m_\ell$

$\underline{2n^2} = 2\sum_{\ell=0}^{n-1}(2\ell+1)$ $\quad$ درجه واگنی

$$H_1 = -\frac{1}{2m_ec^2}\left(H_0 + \frac{Ze^2}{r}\right)^2$$

$$E_n^{(1)} = {}^{(0)}\langle n\ell m_\ell | H_1 | n\ell m_\ell\rangle^{(0)} =$$

$$= -\frac{1}{2m_ec^2}\,{}^{(0)}\langle n\ell m_\ell | \left(H_0 + \frac{Ze^2}{r}\right)^2 | n\ell m_\ell\rangle^{(0)}$$

$$= -\frac{1}{2m_ec^2}\,{}^{(0)}\langle n\ell m_\ell | H_0^2 + \left(\frac{Ze^2}{r}\right)^2 + 2H_0\frac{Ze^2}{r} | n\ell m_\ell\rangle^{(0)}$$

$$H_0|n\ell m_\ell\rangle^{(0)} = E_n^{(0)}|n\ell m_\ell\rangle^{(0)}$$

$$E_n^{(1)} = -\frac{1}{2m_ec^2}\left\{(E_n^{(0)})^2 + (Ze^2)^2\,{}^{(0)}\langle n\ell m_\ell|\frac{1}{r^2}|n\ell m_\ell\rangle^{(0)} + E_n^{(0)}Ze^2\,{}^{(0)}\langle n\ell m_\ell|\frac{1}{r}|n\ell m_\ell\rangle^{(0)}\right\}$$

$\langle \frac{1}{r} \rangle = \frac{Z}{} \qquad \langle \frac{1}{r} \rangle = \frac{Z^2}{}$

$$+ \quad E_n' \quad Ze \qquad \langle n l m_l \,|\, \tfrac{1}{r} \,|\, n l m_l \rangle' \int$$

$$\left\langle \frac{1}{r} \right\rangle_{(0)} = \frac{Z}{a_0 n^2} \qquad\qquad \left\langle \frac{1}{r^2} \right\rangle = \frac{Z^2}{a_0^2 n^2 (l+\frac{1}{2})}$$

$$E_n^{(1)} = - \frac{m_e c^2 (Z\alpha)^4}{2n^4} \left(\frac{n}{l+\frac{1}{2}} - \frac{3}{4} \right) \qquad 10^{-8}$$

$$E_n^{(0)} = - \frac{1 Ry}{n^2} = - \frac{1}{2} m_e c^2 (Z\alpha)^2 \frac{1}{n^2} \qquad \alpha = \frac{1}{137}$$

$\alpha = \dfrac{e^2}{\hbar c}$ بدون بُعد