Lecture 12-QM_II (08/08/00)

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اختلال غیروابسته به زمان (واگنی نداریم)

$$H = H_0 + \lambda H_1$$

$$H_0 |n^{(0)}\rangle = E_n^{(0)} |n^{(0)}\rangle$$

$$H |n\rangle = E_n |n\rangle$$

$$E_n - E_n^{(0)} \sim O(\lambda)$$

$$|n\rangle - |n^{(0)}\rangle \sim O(\lambda)$$

$$\star \quad E_n = E_n^{(0)} + \sum_{i=1}^{\infty} \lambda^i E_n^{(i)}$$

$$|n\rangle = |n^{(0)}\rangle + \sum_{i=1}^{\infty} \lambda^i |n^{(i)}\rangle$$

Ansatz:
$$|n\rangle = \left\{ N(\lambda) \left(|n^{(0)}\rangle + \sum_{\substack{k\neq n \\ k}} C_{nk}(\lambda) \, |k^{(0)}\rangle \right) \right\}$$

$$C_{nk}(\lambda) = \sum_{i=1}^{\infty} \lambda^i C_{nk}^{(i)}$$

$$\langle n | n \rangle = 1$$

$$H |n\rangle = E_n |n\rangle$$

$$(H_0 + \lambda H_1) \{ \quad \} = (\star) \{ \quad \}$$

$$\lambda^0 : \quad H_0 |n^{(0)}\rangle = E_n^{(0)} |n^{(0)}\rangle$$

$$\lambda^1 : \quad H_1 |n^{(0)}\rangle + \sum_{k\neq n} C_{nk}^{(1)} H_0 |k^{(0)}\rangle = E_n^{(1)} |n^{(0)}\rangle + \sum_{k\neq n} C_{nk}^{(1)} E_n^{(0)} |k^{(0)}\rangle$$

$$\boxed{\begin{aligned} \lambda^2 : \quad & \sum_{k\neq n} C_{nk}^{(1)} H_1 |k^{(0)}\rangle + \sum_{k\neq n} C_{nk}^{(2)} H_0 |k^{(0)}\rangle = \\ & = \sum_{k\neq n} C_{nk}^{(2)} E_n^{(0)} |k^{(0)}\rangle + \sum_{k\neq n} C_{nk}^{(1)} E_n^{(1)} |k^{(0)}\rangle \\ & \quad + E_n^{(2)} |n^{(0)}\rangle \end{aligned}}$$

λ^3:

⋮

$$\boxed{E_n^{(1)} = \langle n^{(0)} | H_1 | n^{(0)} \rangle} \qquad E_{n=0}^{(1)}$$

$$|n^{(1)}\rangle = N(\lambda) \sum_{k\neq n} C_{nk}^{(1)} |k^{(0)}\rangle$$

$$\langle n | n \rangle = 1 \rightarrow (1 + O(\lambda^2))$$

$$C_{nk}^{(1)} = \frac{\langle k^{(0)} | H_1 | n^{(0)} \rangle}{E_n^{(0)} - E_k^{(0)}}$$

$n=0$

$$\boxed{|n^{(1)}\rangle = \sum_{\substack{k\neq n \\ k}} \frac{|k^{(0)}\rangle \langle k^{(0)} | H_1 | n^{(0)} \rangle}{E_n^{(0)} - E_k^{(0)}}}$$

$$\sum_{k\neq n} C_{nk}^{(2)} E_k^{(0)} \underbrace{\cancel{\langle n^{(0)} | k^{(0)} \rangle}}_{0} + \sum_{k\neq n} C_{nk}^{(1)} \langle n^{(0)} | H_1 | k^{(0)} \rangle$$

$$= \sum_{\substack{n\neq k \\ k}} C_{nk}^{(2)} E_n^{(0)} \underbrace{\cancel{\langle n^{(0)} | k^{(0)} \rangle}}_{0} + \sum_{k\neq n} C_{nk}^{(1)} E_n^{(1)} \underbrace{\cancel{\langle n^{(0)} | k^{(0)} \rangle}}_{0}$$

$$+ E_n^{(2)} \underbrace{\langle n^{(0)} | n^{(0)} \rangle}_{=1}$$

$+ \; C_n \quad \underbrace{\langle n \;|\; n \rangle}_{=1}$

$\langle n^{(0)} | k^{(0)} \rangle = \delta_{nk} = 0 \qquad n \neq k$ اگر

$\langle n^{(0)} | n^{(0)} \rangle = \delta_{nn} = 1$

$$E_n^{(2)} = \sum_{k \neq n} C_{nk}^{(1)} \langle n^{(0)} | H_1 | k^{(0)} \rangle$$

$$E_n^{(2)} = \sum_{\substack{k \neq n \\ k}} \frac{\langle n^{(0)} | H_1 | k^{(0)} \rangle \langle k^{(0)} | H_1 | n^{(0)} \rangle}{E_n^{(0)} - E_k^{(0)}}$$

$$= \sum_{\substack{k \\ k \neq n}} \frac{| \langle n^{(0)} | H_1 | k^{(0)} \rangle |^2}{E_n^{(0)} - E_k^{(0)}}$$

$H = H_0 + \lambda H_1$

$\vec{\mathcal{E}} = {}^{\circ}\mathcal{E}\, \hat{e}_z$

$H_1 = e\, \vec{\mathcal{E}} \cdot \vec{x}$

$\underline{n = 0} \qquad k \neq 0 \qquad E_{n=0}^{(2)} < 0$

$$E_n = E_n^{(0)} + \lambda E_n^{(1)} + \lambda^2 E_n^{(2)} \,/\, + \cdots$$

truncation

$$C_{nm}^{(2)} = \sum_{k \neq n} \frac{\langle m^{(0)} | H_1 | k^{(0)} \rangle \langle k^{(0)} | H_1 | n^{(0)} \rangle}{(E_n^{(0)} - E_m^{(0)})\,(E_n^{(0)} - E_k^{(0)})} + \frac{\langle m^{(0)} | H_1 | n^{(0)} \rangle \langle n^{(0)} | H_1 | n^{(0)} \rangle}{(E_n^{(0)} - E_m^{(0)})}$$

$$| n^{(2)} \rangle = \underbrace{N(\lambda)} \left\{ \sum_{\substack{k \neq n \\ k}} C_{nk}^{(2)} | k^{(0)} \rangle \right\}$$

$$1 - \frac{1}{2} \sum_{k \neq n} |C_{nk}^{(1)}|^2 \lambda^2$$

نظریه اختلال دائمی (غیروابسته به زمان)

$n \neq k \iff E_n^{(0)} \neq E_k^{(0)}$ تبهگنی نداریم

$n \neq k \not\Rightarrow E_n^{(0)} \neq E_k^{(0)}$ تبهگنی داریم

$|n^{(0)}\rangle \neq |k^{(0)}\rangle$

$$H^{(0)} | n_i^{(0)} \rangle = \epsilon_n^{(0)} | n_i^{(0)} \rangle \qquad i = 1, \ldots \ell$$

$$\langle n_i^{(0)} | m_j^{(0)} \rangle = \delta_{ij}\, \delta_{mn} \qquad \langle n^{(0)} | m^{(0)} \rangle = \delta_{nm}$$

Ansatz:

$$| n \rangle = N(\lambda) \left\{ | n^{(0)} \rangle + \sum_{\substack{n \neq k \\ k}} C_{nk}(\lambda) \; | k^{(0)} \rangle \right\}$$ تبهگنی نداریم

تبهگنی داریم $$| n \rangle = N(\lambda) \left\{ \sum_{i=1}^{\ell} \alpha_i | n_i^{(0)} \rangle + \sum_{k \neq n} C_{nk}(\lambda) \sum_{i=1}^{\ell} \beta_i | k_i^{(0)} \rangle \right\}$$

$$C_{nk}(\lambda) = \sum_{j=1}^{\infty} \lambda^j C_{nk}^{(j)}$$

$$H_0 | n \rangle = E_n | n \rangle$$

$\ell = 1$

$$H\,|n\rangle = E_n\,|n\rangle$$

$$(H_0 + \lambda H_1)\quad \left\{ \sum_{i=1}^{\ell} \alpha_i\, |n_i^{(0)}\rangle + \sum_{k\neq n} \lambda\, C_{nk}^{(1)} \sum_{i=1}^{\ell} \beta_i\, |k_i^{(0)}\rangle + \cdots \right\}$$

$$= \left(E_n^{(0)} + \lambda E_n^{(1)} + \cdots \right) \left\{ \sum_{i=1}^{\ell} \alpha_i\, |n_i^{(0)}\rangle + \sum_{k\neq n} \lambda C_{nk}^{(1)} \sum_{i=1}^{\ell} \beta_i\, |k_i^{(0)}\rangle + \cdots \right\}$$

λ^0 :
$$H_0 \sum_{i=1}^{\ell} \alpha_i\, |n_i^{(0)}\rangle = \underline{E_n^{(0)}} \sum_{i=1}^{\ell} \alpha_i\, |n_i^{(0)}\rangle$$

$$E_n^{(0)} \sum_{i=1}^{\ell} \beta_i\, |k_i^{(0)}\rangle$$

$\ast$ λ^1 :
$$H_1 \sum_{i=1}^{\ell} \alpha_i\, |n_i^{(0)}\rangle + \sum_{k\neq n} C_{nk}^{(1)}\; \underline{H_0 \sum_{i=1}^{\ell} \beta_i\, |k_i^{(0)}\rangle}$$
$$= E_n^{(0)} \sum_{k\neq n} C_{nk}^{(1)} \sum_{i=1}^{\ell} \beta_i\, |k_i^{(0)}\rangle + \underline{E_n^{(1)}} \sum_{i=1}^{\ell} \alpha_i\, |n_i^{(0)}\rangle$$

λ^2 : $\cdots$

$E_n^{(1)} = ?$

$$\sum_{i=1}^{\ell} \alpha_i\, \langle n_j^{(0)} | H_1 | n_i^{(0)} \rangle + \sum_{k\neq n} C_{nk}^{(1)}\, E_n^{(0)} \sum_{i=1}^{\ell} \beta_i\, \cancel{\underbrace{\langle n_j^{(0)} | k_i^{(0)} \rangle}_{\delta_{nk}\,\delta_{ij}}}$$
$$= E_n^{(0)} \sum_{\substack{k\neq n\\ k}} C_{nk}^{(1)} \sum_{i=1}^{\ell} \beta_i\, \cancel{\underline{\langle n_j^{(0)} | k_i^{(0)} \rangle}} + E_n^{(1)} \sum_{i=1}^{\ell} \alpha_i\, \underbrace{\langle n_j^{(0)} | n_i^{(0)} \rangle}_{\delta_{ij}}$$

$$\langle n_j^{(0)} | k_i^{(0)} \rangle = \delta_{ij}\, \delta_{nk} = 0 \qquad k \neq n$$

$$\sum_{i=1}^{\ell} \langle n_j^{(0)} | H_1 | n_i^{(0)} \rangle\, \alpha_i = E_n^{(1)}\, \alpha_j$$

$$(H_1)_{ji}\, \alpha_i = E_n^{(1)}\, \alpha_j$$

$$\boxed{\underset{\sim}{H_1}\, \vec{\alpha} = E_n^{(1)}\, \vec{\alpha}}$$