Sharif University of Technology - Department of Physics

Quantum Mechanics II - Fall 2021

Problem Set 7

Due Saturday 1400/10/11

Problem 1 (10 pts)

An atomic electron of mass m, charge $e = -|e|$, and spin $\boldsymbol{S}$ interacts with a monochromatic radiation field of angular frequency $\omega = ck$ with $k = |\boldsymbol{k}|$. The Hamiltonian of the system is

$$H = H_0 + H(t) = H_0 - \frac{e}{mc}\boldsymbol{A}\cdot\boldsymbol{p} - \frac{e}{mc}\left(\boldsymbol{\nabla}\times\boldsymbol{A}\right)\cdot\boldsymbol{S}$$

where $H(t)$ is a small perturbation (the low-intensity limit). The vector potential $\boldsymbol{A}(\boldsymbol{x},t)$ is given by the following plane-wave

$$\begin{aligned}\boldsymbol{A}(\boldsymbol{x},t) &= 2|A_0|\boldsymbol{\epsilon}\cos\left(\boldsymbol{k}\cdot\boldsymbol{x} - \omega t + \theta\right) \\ &= A_0\boldsymbol{\epsilon}e^{i(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t)} + A_0^{\star}\boldsymbol{\epsilon}e^{-i(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t)},\end{aligned}$$

where $A_0 = |A_0|e^{i\theta}$ is a complex number, $\boldsymbol{\epsilon}$ is a unit vector in the direction of polarization, $\boldsymbol{k}$ is the wave vector, and $\boldsymbol{\epsilon}\cdot\boldsymbol{k} = 0$ (transversal gauge). Let $|i\rangle$ and $|f\rangle$ be two eigenstates of the unperturbed Hamiltonian H_0 , which correspond to the energy levels E_i and E_f , respectively ($E_i \neq E_f$). Assuming that the perturbation $H(t)$ is turned on at $t = 0$, calculate the probability of the transition $|i\rangle \to |f\rangle$.