Final Exam: Monetary Economics

Spring 2021, Mohammad H. Rahmati *

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This is an open book exam. You have 24 hours to send me a "clean" response by email Your answers to question 1 shouldn't go beyond 2 lines per part.

1. (10 points) At date $t = 0$, informed agents understand that 10 quarters from now, at $t = 10$ the productivity growth will increase permanently by 5%. So, prior to $t = 10$ productivity growth was 0% and its growth raises to 5% afterwards. (For example, they believe in a new international agreement due to the election of a new president) Assume there is no labor in this economy and the production function is a simple AK model. For simplicity, assume $\beta = 1/1.05, \delta = 0.05$ (capital depreciation), $\sigma = 1$ (log-utility)

    1.1. If monetary and fiscal policy are passive, and information is sticky such that in each period with probability half of agents understand the news, draw inflation, real and nominal interest rate, investment, and growth rate. If you like to have a framework for thinking consult Mankiw, Riess, QJE, 2002

    1.2. Now assume the monetary authority pegs the interest rate to 10%. Assume the monetary authority has the power and commitment to do so. Draw inflation, real interest rate, investment, and growth rate.

    1.3. Now assume the monetary policy is passive, but the fiscal authority borrow from international market in the first ten quarters to fully smooth consumption. It will pay back the international loan by levying lump-sum tax from $t = 11$. Draw inflation, real interest rate, investment, and growth rate.

    1.4. Now assume only the central bank observes the change in growth in advance. Therefore, all households and firms learn about the growth surge at $t = 10$. What monetary policy do you propose? Should the central bank inform agents about the incoming growth? If so, what monetary policy would you propose? Draw inflation, real interest rate, investment, and growth rate if the monetary authority would do and say nothing.

2. (No point, I just put it here as a fun question!) The Central Bank of Iran (CBI) proposes to issue "Digital Currency" instead of bills and coins. The new money is distributed among banks and like hard currency stored in wallets and used for shopping. It is exactly as Rial, however the new "Digital Currency" will be traced by CBI. Discuss what is the monetary implication of this new currency?

3. (60 points) This is a long question and I want to play around with New-Keynesian framework to see how the basic equations change. So, feel free to make any additional and necessary assumptions, but you should be clear why are you making these assumptions and what are they in a explicit language.

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3.1. (10 points) Lets start with "IS" equation. Assume the same model as N-K we discussed in the class (I follow the notation by McKay, Nakamura, and Steinsson,AER, 2016). However, the households consume both home ($c_{h,t}$) and alien goods ($c_{f,t}$). There are a "wedge" between home and alien goods ζ_t in respect to their expenses that is stochastic and follow AR(1) process as $\zeta_t = \rho_\zeta \zeta_{t-1} + \varepsilon_{\zeta,t}$. I assume the utility function is as follows:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_{h,t}^{1-\gamma}}{1-\gamma} + \frac{c_{f,t}^{1-\delta}}{1-\delta} - \frac{l_t^{1+\psi}}{1+\psi} \right]$$

The household budget constraints are:

$$c_{h,t} + \frac{b_{t+1}}{1+r_t} + m_t = b_t + W_t l_t - \tau_t \bar{\tau}$$

$$c_{f,t} = e_t + \zeta_t m_t$$

Where b_t is claims in domestic consumption. e_t is endowment in alien goods and is exogenous with AR(1) process as $r_t = \rho_e e_{t-1} + \varepsilon_{e,t}$. This equation shows that ζ_t is relative exchange value of foreign to domestic goods in real term. It represents "Pricing-to-Market" shock introduced by Betts, Devereux (2000). It also represents shock to "purchasing power parity" relations like what discussed in Obsfeld and Rogoff (1995, JPE). To me as a monetary economics, I see it as an "international mark-up shock". So, here two alien shocks (i.e. endowment e_t and PTM ζ_t) directly affect the alien marginal consumption. Find the "IS" equation. Notice, $c_{h,t}$ is not total production as we had in closed economy N-K models. In other words, you can see m_t as a kind of export of home goods. Be careful on how you define output gap as well as "IS" curve. Recall we called "IS" as "demand equation".

3.2. (10 points) Now assume there are competitive final producers that use intermediate goods to make domestic goods (c_h) such that $Y_t = \left(\int_0^1 y_{j,t}^{1/\mu} dj \right)^{\mu}$. She will minimize her costs ($\sum_j p_{j,t} y_{j,t}$) and demand for intermediate goods. Also consider a standard Calvo monopolistic firms that update their prices with probability θ in each period. Firms are risk neutral that discount future with rate β. Intermediate firms demand labor and pay laborers (real) wage W_s to produce $y_{j,s} = n_{j,t}$. If firms choose the optimal price $P_t^\star$, using law of aggregate pricing $P_t = \left[\theta (P_t^\star)^{1/(1-\mu)} + (1-\theta)(P_{t-1})^{1/(1-\mu)} \right]^{1-\mu}$ and the labor first order condition of households find the Phillips curve.

3.2.1. If we set monetary instruments as short term nominal interest rate, does monetary policy can stabilize both production and inflation in response to ζ_t?

3.2.2. Does monetary policy can stabilize both production and inflation in response to e_t?

3.2.3. Can you think of another monetary instrument in a way that stabilize the economy in response to alien shocks?

3.3. (5 points) Use Taylor approximation of the utility function, as is done by Rotemberg and Woodford [1998] (or an appendix in chapter 8 of Walsh's monetary textbook) to calculate the welfare equation. If you can't don't stop on this part, just assume a reasonable welfare function!

3.4. (7 points) Using the welfare function you find in 3.3, demand and supply equations in 3.1. and 3.2, find the monetary optimal policy in response to ζ_t? Also, solve for optimal monetary policy in response to e_t?

3.5. (8 points) Does innovation in ζ_t lead to Zero-Lower-Bound limit on nominal interest rates? If so, solve the optimal monetary policy with the ZLB restriction? How would you recommend an optimal monetary policy in presence of this limit?

3.6. (20 points) Economy 3.1. is an Autarky economy in terms of an alien goods! We change the economy 3.1 such that household can transfer their alien goods between periods. They can borrow and lend in alien claims d_t.(This debt could be international sovereign bonds or assets hold by household in alien goods, like gold!)

$$c_{f,t} + \frac{d_{t+1}}{1 + r_t^f(d_{t+1})} = d_t + e_t + \zeta_t m_t$$

$r_t^f(d_{t+1})$ is interest rate on alien debts that $r_t^{f'} > 0$ and $r_t^{f''} > 0$. So, interest rate is determined by the amount of debt a household borrows. Nonetheless, the alien debt doesn't need to be balanced within a county and should be cleared worldwide. The rest of economy is similar to 3.1. & 3.2. Define $\bar{r}^f(\bar{d})$ as its steady state. Find the "IS" and Phillips curves in this economy. Answer the same questions as 3.2.1, 3.2.2, 3.2.3, 3.4. Think out of box when you are thinking on 3.2.3 in this part!! Discuss the difference between an Autarky economy and bond economy?

4. (30 points) Question 3 introduce alien shocks to household optimization. The underlying assumption is that alien goods are "final" goods. We know that this is not necessary true! Lets use alien good in this question as an intermediate goods. To make it simple, I assume households just consume final goods, therefore:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_{h,t}^{1-\gamma}}{1-\gamma} - \frac{l_t^{1+\psi}}{1+\psi} \right]$$

$$c_{h,t} + \frac{b_{t+1}}{1 + r_t} = b_t + W_t l_t - \tau_t \bar{\tau}$$

4.1. (5 points) Assume final producer employ alien goods ($y_{f,t}$)to produce final goods. To do so, she should export $y_{f,t}/\zeta_t$ of her production to import its intermediary. This costs her $P_t y_{f,t}/\zeta_t$. Therefore, her production function combined with trade balance is:

$$Y_t + y_{f,t}/\zeta_t = \left[\left(\int_0^1 y_{j,t}^{(1/\mu)} dj \right)^{(\mu/\sigma)} + y_{f,t}^{(1/\sigma)} \right]^{\sigma}$$

Solve the final producer demand function assuming that the shadow price on its recourse constraints is the aggregate price index (P_t).

4.2. (15 points) Solves for the Philips curve assuming the same monopolistic intermediary firms as discussed in 3.2.

4.2.1 How does expected depreciation in domestic currency affect current inflation?

4.2.2 How does expected depreciation in domestic currency affect current alien goods demand? If you think this part of results is not what you think, in words, how would you change set up the model to get what you want?

4.2.3 How does current and expected future depreciation in domestic currency vary real wages?

4.3. (10 points) Solve for optimal monetary policy in response to $zeta_t$? Discuss whether ZLB limits is a constraint in this economy? Why? (Hint: Use the standard welfare function we discussed in the class).