Homework 6: Zero Lower Bound

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1. Consider a new Keynesian model with government purchases g_t and shocks Δ_t to the interest rate facing households

$$\tilde{y_t} = -\frac{1}{\sigma}(i_t + \Delta_t - E_t[\pi_{t+1}] - r_t^n) + E_t[\tilde{y_{t+1}}]$$

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa \tilde{y_t}$$

Monetary policy is set according to an interest rate rule but also faces a zero lower bound

$$i_t = max[0; \rho + \phi_\pi \pi_t + \phi_y \tilde{y_t}]$$

The natural real rate and natural output are given by

$$r_t^n = \rho - \sigma(1-\Gamma)E_t[\Delta g_{t+1}]; \qquad y_t^n = \Gamma g_t$$

The interest spread shock Δ_t can take on two values $\Delta_L; \Delta_H$ with $\Delta_H = 0$ and $\Delta_L > 0$. The economy starts in the L state. With probability α it stays in the L state. With probability $1-\alpha$ it transitions to the H state. Once it enters the H ("normal") state it stays there forever. Suppose that $g_H = 0$. We are interested in calculating economic outcomes as a function of fiscal policy g_L in the L ("crisis") state.

   (a) To begin with, suppose that $g_L = 0$. Solve for the equilibrium values of inflation, the output gap, and the nominal interest rate in the L state.

   (b) Now explain how an increase in government purchases to some $g_L > 0$ would affect inflation, expected inflation, output and the nominal and real interest rates in the L state. Explain how your answers depend on the size of the interest spread Δ_L. How large is the government purchases multiplier? Does this value depend on the the size of the fiscal stimulus?

   Now consider the possibility that government purchases g_L persist at an elevated level after the crisis has abated. To be specific, imagine that there are three states, $L; S; H$. The economy starts in the L state with $\Delta_L > 0$. Suppose the initial interest spread Δ_L is sufficiently high that the ZLB is binding in the L state. With probability α the economy transitions to the S state. In the S ("transitional") state, the crisis is over $\Delta_S = \Delta_H = 0$. In the S state, with probability λ the fiscal stimulus continues with government purchases $g_S = g_L > 0$. With probability $1 - \lambda$ the fiscal stimulus ends with $g_S = g_H = 0$.

   (c) Let $\pi_S; \tilde{y_S}$ iS denote the equilibrium values of inflation, the output gap and the nominal interest rate in the S state. Following the same logic as in part (a), solve for these equilibrium values as a function of the size of the fiscal stimulus $g_L > 0$. Explain how your answers depend on whether the fiscal stimulus continues $g_S = g_L$, or not $g_S = g_H$.

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(d) Using your results from part (c), now solve for the equilibrium values of inflation, the output gap and the nominal interest rate in the initial L state given that with probability λ the fiscal stimulus continues after the crisis has abated. How does the government purchases multiplier compare to the one you found in part (b)? How does the multiplier vary with the probability of the stimulus continuing? Explain.

[Hint: this question is based on Woodford's article "Simple Analytics of the Government Expenditure Multiplier" American Economic Journal: Macroeconomics. 3(1): 1-35, especially Section IV.B]

2. Consider the following Diamond-Dybvig model with aggregate uncertainty about household type. In particular, while all agents are ex-ante (i.e. $t = 0$) identical, there are two equally likely events $\omega \in \{H, L\}$ at $t = 1$ such that when $\omega = H$ there is a high probability that any given agent is an early consumer and when $\omega = L$ there is a low probability she is an early consumer. Specifically, each agent faces an iid preference shock $\theta \in \{1, 2\}$ that is realized at $t = 1$ which depends on the aggregate state such that the probability of being an early consumer is given by $prob(\theta = 1|\omega = H) = \pi_H = \frac{3}{4}$ and $prob(\theta = 1|\omega = L) = \pi_L = \frac{1}{4}$. Otherwise, all other aspects of the environment are identical as that presented in class. In particular, early consumers have preferences $u(c_1) = \sqrt{c_1}$ and late consumers have preferences $u(c_2) = \sqrt{c_2}$. There are two storage technologies: a long term productive technology

$t = 0$	$t = 1$	$t = 2$
-1	1	R

and a pillow technology

$t = 1$	$t = 2$
-1	1

in which storage is privately observable.

(a) State and solve the planner's problem when the aggregate state and type are observable (First Best)

(b) State and solve the planner's problem when the aggregate state but not type is observable.

(c) Explain why a deposit contract cannot implement the planner's problem. (Note: a deposit contract is simply time contingent, but not state contingent).

(d) Devise a resource feasible government deposit insurance scheme that improves upon the autarkic solution.