Homework 4: Money Models

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1. Consider our benchmark framework with monopolistic competition. Suppose that each period a fraction of firms $1-\theta$ gets to choose a path of future prices for their respective goods (a "price plan"), while the remaining fraction θ keep their current price plans. We let $\{P_{t,t+k}\}_{k=0}^{\infty}$ denote the price plan chosen by firms that get to revise that plan in period t. Firm's technology is given by $Y_t(i) = \sqrt{A_t} N_t(i)$. Consumer's period utility is given assumed to take the form $U(C_t, N_t) = C_t - \frac{N_t^2}{2}$, where $C_t \equiv [\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di]^{\frac{\epsilon}{\epsilon-1}}$. The demand for real balances is assumed to be proportional to consumption with a unit velocity, i.e., $\frac{M_t}{P_t} = C_t$. All output is consumed.

   (a) Let $P_t \equiv [\int_0^1 P_t(i)^{1-\epsilon} di]^{\frac{1}{1-\epsilon}}$ denote the aggregate price index. Show that, up to a first order approximation, we will have:

$$pt = (1-\theta) \sum_{j=0}^{\infty} \theta^j p_{t-j,t} \tag{1}$$

   (b) A firm i, revising its price plan in period t will seek to maximize

$$\sum_{t=0}^{\infty} \theta^k E_t \{Q_{t,t+k} Y_{t+k}(i)(P_{t,t+k} - \frac{W_{t+k}}{\sqrt{A_{t+k}}})\}$$

   Derive the first order condition associated with that problem, and show that it implies the following approximate log-linear rule for the price plan:

$$p_{t,t+k} = \mu + E_t\{mc_{t+k}^n\} \tag{2}$$

   for $k = 0, 1, 2, \cdots$ where $mc_t^n = \omega_t - \frac{1}{2} a_t$ is the nominal marginal cost.

   (c) Using the optimality conditions for the consumer's problem, and the labor market clearing condition show that the natural level of output satisfies $\bar{y}_t = -\mu + a_t$, and (ii) the (log) real marginal cost (in deviation from its perfect foresight steady state value) equals the output gap, i.e. $\hat{mc}_t = \tilde{y}_t$ for all t, where $y_t \equiv y_t - \bar{y}_t$

   (d) Using (1) and (2) show how one can derive the following equation for inflation:

$$\pi_t = \frac{1-\theta}{\theta} \tilde{y}_t + \frac{1-\theta}{\theta} \sum_{j=1}^{\infty} \theta^j E_{t-j}\{\Delta \tilde{y}_t + \pi_t\} \tag{3}$$

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(e) Suppose that the money supply follows a random walk process $m_t = m_{t-1} + u_t$, where $m_t \equiv log M_t$ and $\{u_t\}$ is white noise. Determine the dynamic response of output, employment, and inflation to a money supply shock. Compare the implied response to the one we would obtain under the standard new Keynesian Phillips curve, where $\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \tilde{y}_t$ (hint: use the fact that in equilibrium $y_t = m_t - p_t$ to substitute for $\tilde{y}_t$ in (3), in order to obtain a difference equation for the (log) price level)

(f) Suppose that technology is described by the random walk process $a_t = a_{t-1} + \varepsilon_t$, where where $a_t \equiv log A_t$, and $\{\varepsilon_t\}$ is white noise. Determine the dynamic response of output, the output gap, employment, and inflation to a technology shock. Compare the implied response to the one we would obtain under the standard new Keynesian Phillips curve, where $\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \tilde{y}_t$. (hint: same as above).

2. The technology available to a firm producing a differentiated good is give by

$$Y_t(i) = N_t(i)$$

where

$$N_t(i) \equiv [\int_0^1 N_t(i,j)^{1-\frac{1}{\eta_t}} dj]^{\frac{\eta_t}{\eta_t - 1}}$$

and where $N_t(i,j)$ denotes the quantity of type-j labor employed by firm i in period t. We assume that η_t follows an exogenous stochastic process. Household j is specialized in the supply of type of labor j. Each period the household sets the wage nominal wage for type j labor, $W_t(j)$, in order to maximize

$$U(C_t(j), N_t(j)) = \frac{C_t(j)^{1-\sigma}}{1-\sigma} - \frac{N_t(j)^{1+\phi}}{1+\phi}$$

subject to a budget constraint $P_t C_t(j) = W_t(j) N_t(j) + X_t$ (where X represents other terms not affecting the wage setting decision), and a demand schedule for its labor to be derived below.

(a) Show that the quantity of type j labor demanded by the typical firm will be given by

$$N_t(i,j) = (\frac{W_t(j)}{W_t})^{-\eta_t} N_t(i)$$

where $W_t \equiv [\int_0^1 W_t(j)^{1-\eta_t} dj]^{\frac{1}{1-\eta_t}}$ is an aggregate wage index.

(b) Derive the household's optimal wage setting rule. Show that it has a log-linear representation of the form:

$$w_t(j) = \mu_t^{\omega} + p_t + \sigma c_t(j) + \phi n_t(j)$$

where $\mu_t^{\omega} \equiv log(\frac{\eta_t}{\eta_t - 1})$

(c) Derive the associated Phillips curve when firms set prices a la Calvo. (hint: think about the appropriate definition of the natural rate of output to be used).