Homework 2: Cash-in-Advanced and Search

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1. We will consider a cash-in-advance environment to study "Unpleasant Monetarist Arithmetic". The technology is given by $y_t = n_t$ where $n_t \in [0, 1]$ is fraction of hours worked. Preferences are given by $\sum_{t=0}^{\infty} \beta^t \{U(c_t) - \gamma n_t\}$ where $U(c_t) = \frac{c_t^{1-\theta}}{1-\theta}$. Households can hold money and/or government bonds. Let M_{t+1} and B_{t+1} be money (dollars) and nominal bonds (claims to dollars next period) held by households between t and $t+1$. The government expands or contracts the money supply at a constant rate μ according to $M_{t+1} = (1+\mu)M_t$. Let p_t be the dollar price of consumption goods and q_t be the consumption good price of a bond (so that $p_t q_t$ is the dollar price of a bond). Let τ_t be real lump sum taxes/transfers the government levies to help pay for its constant real expenditure g on goods. Assume that only money M_t accumulated last period can be used to purchase consumption goods c_t this period at price p_t.

   (a) Write down the household's budget constraint in nominal terms. Transform it into real terms (e.g. let $m_t = \frac{M_t}{P_t}$).

   (b) Write down the household's cash-in-advance constraint in nominal terms. Transform it into real terms.

   (c) Write down the government's budget constraint in nominal terms. Transform it into real terms.

   (d) Define a competitive equilibrium.

   (e) Characterize a stationary competitive equilibrium where agents take $\tau_t = \tau$ as given. Under what conditions does the cash-in-advance constraint bind? Under the assumption that the c-i-a constraint binds, characterize real seignorage revenue in a steady state (i.e. defined as $\frac{\bar{M}_{t+1} - \bar{M}_t}{P_t} = \mu m_t = \mu m$ in a steady state). Are there values of θ such that an increase in μ leads to a decrease in real seignorage? That is, can the monetary authority be on the wrong side of the seignorage "Laffer" curve?

   (f) Suppose there is an equilibrium where $B_t = B$ and M_t satisfies the government budget constraint for a constant money growth rate μ and all other government policy variables (i.e. τ_t) constant, taking as given M_0 and B_0. Now analyse the effect of an open market sale of bonds, defined as a decrease in the money supply at $t = 1$ to say $\widehat{M}_1$ accompanied by an increase in $B_t = \widehat{B}$ for $t \geq 1$ with the other fiscal government policy variables (i.e. τ) the same as before. This issue is known as "Unpleasant Monetarist Arithmetic". Hint: Increase B in the $t = 0$ and $t \geq 1$ government budget constraints separately.

2. Consider the following search model of money. Time is discrete and there is a continuum of agents with population normalized to 1. Any particular agent specializes in the production of one service (a nonstorable good) but likes other services in an interval of size $x \in (0, 1)$. She

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derives utility $u(q) = q^{1/2}$ from consuming $q \in R_+$ units of the service provided it falls in her desired interval. An agent discounts the future at rate $(1+r)^{-1}$. There is a constant disutility $-q$ to producing q units of a service. Production and consumption occur at the end of the period (and hence should be appropriately discounted). At the beginning of time, a fraction of agents $M \in (0,1)$ are randomly given one unit of currency. Currency is indivisible and can be stored only one unit at a time. Agents are exogenously matched in the following way. Agents with money (we will term them buyers) are randomly matched in pairs with agents without money. Thus, the probability that a buyer is matched with a seller whose good she desires is $x(1-M)$. Also, the probability that a seller is matched with a buyer who wants her good is xM. Every agent's trading history is private information. Finally, assume that buyers submit take-it-or-leave-it offers (which amount to a trade of 1 unit of money for Q units of the seller's service).

(a) Taking the quantity of services bargained for Q as given, write down the value functions for a buyer $V_b(Q)$ and a seller $V_s(Q)$ respectively .

(b) Taking the value functions V_b and V_s as given, what is the value of Q from the buyer's take-it-or-leave-it offer? To answer this question, proceed as follows. What condition assures that a seller accepts money in exchange for the production of his services? In particular, what is the seller's utility if he accepts the offer (produces the service and obtains the unit of currency)? What is the seller's utility if he rejects the offer (and goes back into the search pool)? Under what conditions on Q then will the seller accept the offer? Hence, if the buyer is trying to get as much services as possible, what value of Q will she demand from the seller?

(c) Define a monetary equilibrium.

(d) Does a monetary equilibrium exist? If so, under what conditions on r, x, and M? Do any other equilibria exist?

(e) Does the price level vary with increases in M?

(f) If we define ex-ante welfare as $W = MV_b + (1-M)V_s$, how is welfare affected by changes in the money supply?

3. **Based on Kiyotaki, Wright [1993]** Consider the following environment where the goods and money are indivisible. The exogenous parameter $0 < x < 1$ equals the proportion of commodities that can be consumed by any given agents and x also equals the proportion of agents that can consumes any given commodities. One unit of consumption yields $U > 0$, while consuming other commodities or money yields zero utilities. A fraction of M of the total agents at each period own money while $1-M$ are producing goods or own commodities .Money and commodities are costlessly storable. There is a production sector. That is, once an agent consumes enter in production sector and during one time could produce one unit of output with probabilities of $\alpha > 0$. In exchange sector, agent who has just produced looks for other agent to trade. Traders in the exchange sector meet pairwise and with probability $\beta > 0$ find other traders. The exchange take place if and only if it is mutually agreeable, that is, if and only if both agents are at least as well off after the trade. Also there is a transaction cost $0 < \epsilon < U$, that must be paid by the receiver whenever any real commodity is accepted in trade.In the exchange sector two types of agents, commodities trader and money traders, exist. Let μ denote the fraction of trader in the exchange sector who are money trader, so that a trader located at random has money with probability μ and a real commodity with probability $1-\mu$. Let Π denote the probability that a commodities trader accepts money and let π be the best response of the representative individual. Let V_j denote the value function for the individual in state $j = 0, 1, m$ indicates that he is a producer, a commodity trader or the money trader, respectively.

(a) Assume that we do not have double coincide problem. write the Bellman's Equations. In the rest of problem assume that double coincide matching is possible.

(b) For this case write the Bellman's Equations.

(c) Assumes that N_0 and N_1 and N_m denote the number of producer, commodity trader or the money trader, respectively, in the steady state.Find the implicit function for the μ as a function of M and Π. Is the μ is increasing with respect to M? what about Π? or show that it is indeterministic?

(d) What is the value of Π which there exist mixed strategy? The equilibrium is called mixed-monetary equilibrium. (hint: what happen if $\pi < x$, what is the best response of commodity traders.)

(e) For simplicity assume that $\alpha \to \infty$ thus the production is instantaneous and $N_m = M$, $N_1 = 1 - M$ and $\mu = M$. Assume $x < \frac{1}{2}$, find the value μ^0 which maximize the welfare function.