Midterm Exam
Econometrics, PhD, Spring 2022
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1. **(40 points)** Suppose that x is distributed as $f\ (x, \alpha_0, \beta_0)$, where

$$f\ (x, \alpha, \beta)\ = \begin{cases} \alpha\beta x^{\beta-1} e^{-\alpha x^{\beta}} & if\ x \geq 0 \\ 0 & if\ x < 0 \end{cases}$$

and where (α_0, β_0) is known to lie in $[0.1, 10]\ \times\ [0.1, 10]$. Assume that we have access to an iid sample $(x_1, \ldots, x_n)$.

   i. Find the maximum likelihood estimator of (α_0, β_0), denoted $(\hat{\alpha}, \hat{\beta})$. (Hint: it may not be possible to solve for it analytically)
   ii. Show that $(\hat{\alpha}, \hat{\beta})$ is consistent. (Hint: Use the general theory given in class.)
   iii. Show that is $\left(\hat{\alpha}, \hat{\beta}\right)$ asymptotically normal and give its asymptotic variance. (Hint: Use the general theory given in class.)

2. **(30 points)** Consider the following model,

$$Y_i\ =\ min(U_i, X_i'\beta + \varepsilon_i) \quad \varepsilon_i \sim N(0, \sigma_{\varepsilon}^2)$$

and where we observe (Y_i, X_i, U_i).

   i. For what type of phenomenon might this model be useful?
   ii. Derive the likelihood function for the model.
   iii. Find $E(Y_i|X_i)$ and $E(Y_i|Y_i\ <\ U_i, X_i)$
   iv. Suppose that one only imposed a median restriction on the residuals so that $median(Y_i|X_i) = 0$. Show that,

$$median(Y_i|X_i)\ =\ \min\{U_i, X_i'\beta\}$$

   and using this suggest a method for estimating β that does not require any other distributional assumptions. (Hint: Draw a picture of the distribution of the latent variable $X_i'\beta + \varepsilon_i$ conditional on ε_i and determine what censoring does to the median of this distribution.)

3. **(30 points)** Consider a homogeneous nonstationary duration model with the hazard rate

$$\lambda(t) = \alpha t \quad \alpha > 0$$

Supposing we observe n completed spells of duration $t_1, t_2, \ldots, t_n$ and $N - n$ censored spells of duration $t_{n+1}, t_{n+2}, \cdots, t_N$, derive the MLE of α and its asymptotic variance.