Homework 6: Time Inconsistency and Optimal Taxation

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1. Consider a version of the environment studied by Stokey (1989) that was taught in class. Let (x, X, y) be the choice variables available to a representative agent, the market as a whole, and a benevolent government, respectively, where $x, X \in \chi = \{x_L, x_H\}$ and $y \in Y = \{y_L, y_H\}$. Let per period payoffs to the government be denoted $u(x_i, X_j, y_k)$. In the case where $x_j = X_j$ let payoffs be given as in the table The values of $u(x_i, X_j, y_k)$ not reported in the table are such

$u(x_j, X_j, y_k)$	X_L	X_H
y_L	0^*	20
y_H	1	10^*

that the competitive equilibria (i.e. ones where $x = X = h(y)$) are the outcome pairs denoted by the asterisk. For completeness assume that if an agent deviates, she gets exactly what the average person gets in all cases except one (where x_L, X_H, y_L). In particular, we assume $u(x_L, X_H, y_L) > 20$ and for all other cases she gets the average payoff (i.e. $(x_j, X_j, y_k) \geq u(x_i, X_j, y_k)$ for $i \neq j$, for $k = \{L, H\}$). Therefore, a weakly dominant strategy is to do what the average person does.

   (a) Define a Ramsey plan and a Ramsey outcome for a one-period economy. Find the Ramsey outcome.

   (b) Define a Nash equilibrium (in pure strategies) for the one-period economy.

   (c) Show that there exists no Nash equilibrium (in pure strategies) for the one period economy. Consider an infinitely repeated version of the economy.

   (d) Define a subgame perfect equilibrium (SPE).

   (e) Find the value to the government associated with the worst subgame perfect equilibrium (this is similar to a minmax problem).

   (f) Assume that the discount factor is $\delta = (1/10)^{1/20} = 0.8913$. Determine whether infinite repetition of the Ramsey outcome is sustainable as a SPE. If it is, display the associated SPE.

   (g) Find the value to the government associated with the best subgame perfect equilibrium.

   (h) Find the outcome path associated with the worst subgame perfect equilibrium.

   (i) Find the one period continuation value ν_1 and outcome path associated with the one-period continuation strategy σ^1 that induces adherence to the worst subgame perfect equilibrium.

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(j) Find the one period continuation value ν^2 and outcome path associated with the one-period continuation strategy σ^2 that induces adherence to the first period outcome σ^1 you found in part (i)

(k) Proceeding recursively, define ν_j and σ^j , respectively, as the one-period continuation value and continuation strategy that induces adherence to the first period outcome of σ^{j-1}, where (ν_1, σ^1) were defined in part (i) Find ν_j for $j = 1, 2, \cdots$ and the associated outcome paths.

(l) Find the lowest value for the discount factor for which repetition of the Ramsey outcome is an SPE.

2. Consider an economy populated by a large number of identical people who live from period 0 through T (where T is possibly infinite). In each period a non storable consumption good is produced from labor $Y_t = N_t$. Part of the output is used for government consumption and the other part for private consumption. The per capita level of government consumption G_t is exogenously given and varies over time. The national income identity is $C_t + G_t = N_t$ where C_t and N_t denote per capita private consumption and labor input. The government raises revenue from a proportional tax on labor income τ_t with after-tax income given by $(1 - \tau_t)N_t$. The government also issues public debt in the form of one period discount bonds with face value of one unit of the consumption good. Let B_t denote public debt outstanding (so that $B_{t+1} > 0$ is the amount of government borrowing undertaken at time t to finance government expenditure above tax revenue). To accomodate the possibility of default, assume the government can levy a tax δ_t on outstanding debt (i.e. if $\delta_t = 1$, the government is said to have defaulted on its debt). Let q_t denote the price of debt issued in period t for redemption in $t+1$. Of course, the price of such discount bonds will depend on rates of return on private bonds and expectations of future government default. Let D_t denote the stock of private debt outstanding and $r_{t,t+1}$ the rate of return (in terms of the consumption good) of private debt between t and $t+1$. Let the terminal conditions be given by $B_{T+1} = D_{T+1} = 0$. Finally, let household preferences be given by $\sum_{t=0}^{T} \beta^t U(C_t, N_t)$ and let the government's objective function simply be to maximize the welfare of the representative household.

(a) Write the government budget constraint.

(b) Write the household optimization problem.

(c) Define a competitive equilibrium.

(d) Use an arbitrage argument to price new government debt off of the return on private debt.

(e) Define a Ramsey equilibrium.

(f) Show that in a Ramsey equilibrium, for any two dates with the same government expenditure, the government chooses the same tax policy. Hint: to do so, specify the implementability constraint for this problem.

(g) Show that that if $B_0 > 0$, the government sets $\delta_0 = 1$. What if $B_0 < 0$?

(h) Show that it is possible to construct a Ramsey equilibrium in which the government does not tax debt in $t = 1$ to T

(i) Assume that $B_t \geq 0$ (i.e. the government never owns claims on private households) and that there is no commitment technology? What are the implications for government taxation in both finite horizon and infinite horizon problems? What implications does this have for balanced budget amendments?

3. Consider the following optimal inflation tax problem. A government must finance an initial amount of exogenous government expenditure $g_0 > 0$ by issuing money M_t^s or nominal debt D_t. Thereafter, $g_t = 0$, for $t \geq 1$. Technology is given by $y_t = n_t$ where y_t is a nonstorable good and n_t is the fraction of time devoted to labor. Households are endowed with 1 unit of time. Households have preferences given by $\sum_{t=0}^{\infty} \beta^t [u(c_t) + \nu(l_t)]$ where c_t is consumption and l_t is leisure. Utility is strictly increasing, concave, continuously differentiable, and satisfies the Inada condition. Households must use money to buy a fraction fc_t worth of consumption goods and the remaining fraction $(1-f)c_t$ can be purchased on credit (i.e. by issuing private bonds). The fraction $f \in (0,1]$ is exogenously given. The nominal gross rate of return on private or government bonds between t and $t+1$ is denoted R_t and the price level is denoted p_t. The timing in any period is as follows: (i) Loans are repaid (i.e. $R_{t-1}B_{t-1}$), new money holdings $M_t \geq 0$ and bond holdings B_t are chosen; (ii) the shopper goes to the goods market while the worker goes to the production site and is paid nominal wages $w_t n_t$; (iii) the shopper and worker come home to consume and count their earnings. Assume that $M_{-1}^s + D_{-1}R_{-1} = 0$

   (a) Formulate the household's problem.

   (b) Define a competitive equilibrium

   (c) Is it possible to construct a competitive equilibrium where the cash-in-advance constraint never binds, interest rates are constant, and the limiting amount of real debt is zero?

   (d) Define a Ramsey equilibrium where the government commits to choosing a policy $\pi = \{g_t, M_t^s, D_t\}_{t=0}^{\infty}$

   (e) How would you solve for a Ramsey Equilibrium? What properties do you think an optimal government policy might have? What does your answer to part c imply about implementing the Friedman rule in this setting?

4. Consider the following two period problem where the government can tax capital gains and/or bequests to fund an exogenous amount of government expenditure. There is a unit measure of households (HHs) who make a portfolio choice at $t = 0$ and a bequest decision at $t = 1$. Their preferences are given by $u(c) + \alpha u(b)$ where $c \geq 0$ is consumption at $t = 1, b \geq 0$ are bequests for one's children chosen at $t = 1$, and $\alpha > 0$ measures the relative importance of bequests vs own consumption. Assume $u'(.) > 0$, $u''(.) < 0$, and $u'(0) = \infty$. Each HH receives an endowment $\omega > 0$ at $t = 0$ and at that time must either make a storage decision in tax-free assets $a \geq 0$ yielding gross return equal to 1 at $t = 1$ or a productive asset $k \geq 0$ yielding $R > 1$ which is proportionately taxed at rate $\delta in[0,1]$ in period $t = 1$. Assume that if HHs are indifferent between storing in a or k, they save in k. Then at $t = 1$, HHs choose how much to consume $c \geq 0$ or leave bequests $b \geq 0$. At $t = 1$, HH's have to pay τb for any bequests they leave where $\tau \in [0,1]$. Taxes are used to finance government expenditure G at $t = 1$.

   (a) Assume government announces a tax package (δ, τ) at $t = 0$ and can commit to it. Define a Ramsey equilibrium. Be explicit about the HH's choice problem. What are the optimal capital gains δ taxes? How do bequests react to changes in τ? Do HHs leave bequests? Are bequest taxes 0 or 100% with commitment?

   (b) Assume that there is no commitment on the part of the government with respect to its announced tax package. That is, after households choose their portfolio in $t = 0$, the government chooses taxes in $t = 1$, and then households choose their bequests. Define and solve for a subgame perfect equilibrium. What are equilibrium (δ, τ)? Compare tax revenues in part (a) and (b).