Homework 4: Labor Search

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November 1, 2018

1. Imagine a firm got a worker and it can produce one unit of output per period for 10 periods (there is a zero interest rate). If the firm does not operate today, it loses its license and is out. The worker is risk neutral, can either take the job or run a hot dog operation what yields 0.25 units per period for five years and sick for another 5 years during which she will be on disability insurance collecting .1 units of output.
   (a) What is the minimum wage that the worker would accept.
   (b) If the firm had to pay an entrance fee to open, what would be the maximum fee under which the firm would enter.
   (c) Pose a wage that is the bargaining solution with the firm having twice the weight than the worker.
   (d) Briefly describe what could happen if there was a possibility of multiple entry of firms. How could the free entry condition be applied?

2. Consider the decision problem of a single worker who has preferences given by $\sum_{t=0}^{\infty} \beta^t u(c_t)$ where c_t denotes consumption, β is a discount factor and u is strictly concave. At the beginning of each period the worker is either unemployed or employed. If unemployed, the worker receives a wage offer from a distribution $G(w)$ and if employed, gets a new offer from a distribution $F(w)$. These offers are constant for the duration of the jobs. These draws are i.i.d. Jobs disappear with probability δ at the end of each period. The worker cannot save or borrow and if unemployed receives a benefit b.
   (a) Set up the decision problem as a dynamic program.
   (b) Show that the solution to the dynamic program is of the reservation wage form.
   (c) Assume that $b = 0$, $u(0) = 0$ and that G and F are both the same with the supports of both distributions given by $[0, \overline{w}]$. What jobs if any does the worker reject?

3. Consider a continuous-time equilibrium matching model with the following features:
   - Workers and firms have linear utility and discount at a net rate r.
   - There is a constant-returns-to-scale matching function $M(u; v)$ describing the number of matches taking place at each instant as a function of the unemployment and vacancy rates, respectively. It is increasing in both arguments.
   - Firms can enter for free but there is a cost c of posting vacancies. Firms enter until the profit (net of the entry fee) is zero.

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- There is a $[0, 1]$ continuum of workers. A fraction ϕ of the workers are in couple relationships; the rest are single.
- All unemployed workers receive unemployment compensation b.
- Workers in couple relationships are less unhappy being at home than are single workers. The cash value of being at home of single workers is normalized to zero; the cash value of being at home of workers in couple relationships is $h > 0$.
- When firms search for workers, they cannot tell if workers are in couple relationships or single: there is "undirected" search.
- Firm-worker pairs are exogenously terminated at a rate σ
- A firm-worker pair produces p units of consumption.
- At every instant, matched workers and firms Nash bargain; workers obtain a share β of the surplus and firms obtain a share $1 - \beta$.
- The economy is in a steady state.

Complete the following tasks. Assume throughout that the primitives are such that workers prefer working to not working.

(a) Describe, separately for workers in couple relationships and single workers, two equations determining the value functions of the unemployed worker and of the employed worker as a function of the probability for an unemployed worker of finding a job, denoted λ_ω, the net interest rate r, the separation rate σ, (in the case of the workers in couple relationships) the cash value of being unemployed at home h, and the wage.

(b) Describe two equations determining the value functions of the vacant firm and of the matched firm as a function of the probability for a vacant firm of

finding any worker, denoted λ_f , the probability that the worker is in a couple relationship, ϕ, the net interest rate r, the separation rate σ, the productivity p,the vacancy posting rate c, and the wage w paid by the firm to the worker.

(c) Define the two total surpluses, S_c and S_s, for both kinds of matches a firm can end up in: one with a worker in a couple relationship and one with a single worker. Solve for these as a function of market tightness $\theta \equiv \frac{u}{v}$, using the matching function to derive λ_f and λ_ω.

(d) Combine the results above with the free-entry condition to derive one equation in one unknown: θ

(e) Find expressions for the wages of the two kinds of workers as a function of primitives and of market tightness.

(f) What is the effect on market tightness of an increase in the fraction of workers who are in couple relationships?

(g) What is the effect on unemployment of an increase in the fraction of workers who are in couple relationships?

(h) Suppose the government introduces a welfare program transferring wealth from unemployed workers who are in couple relationships to unemployed single workers so that their unemployment incomes (unemployment benefit plus the value of being at home plus/minus transfers/taxes) are equalized. What is the effect of this welfare program on the rate of unemployment?

4. Individuals meet firms at an exogenous rate λ. Once they have met the firm, the value of the match is revealed to the worker and the firm, which is denoted by θ. The matching distribution is given by $G(\theta)$. If a match θ generates an employment contract, the worker is paid a wage $w(\theta)$ and the job is dissolved at an exogneous rate η. The "threat point" of the potential employee is the value of continuing search, denoted V_n, and the threat point of the employer is 0. The value of the wage is determined using a Nash bargaining framework, so that

$$w(\theta, V_n) = argmax_w (V_e(w) - V_n)(\frac{\theta - w}{\rho + \eta}),$$

where η is the instantaneous discount rate, and $\rho + \eta$ is the effective discount rate of the firm.

   (a) Write down the labor market dynamics generated by this model in as much detail as possible. In particular, find $V_e(w)$, $w(\theta, V_n)$, and V_n. What is the probability density function of accepted wages? What is the steady state unemployment rate?

   (b) Workers and firms can increase the value of the match by purchasing health insurance. The instantaneous price of health insurance is $\phi > 0$. If health insurance is purchased the match the total value of the match improves and becomes $a\theta$, where $a > 1$ [due to the increased healthiness of the worker]. Then the Nash bargaining problem becomes

$$(w, d)(\theta, V_n) = argmax_{w,d} (V_e(w) - V_n)(\frac{ad\theta - w - d\phi}{\rho + \eta}),$$

   where d equals 1 if health insurance is purchased and equals 0 if not, and we have assumed that the firm actually pays the health insurance premium directly to the insurance company. Describe the equilibrium outcomes associated with this model as you did in part (a). In equilibrium, will some jobs be covered by health insurance and others not? If so, what proportion of jobs will be covered by health insurance, and what is the relationship between the presence of health insurance and wages? Provide some intuition for your results.

5. Consider an economy in which workers accumulate human capital while working. A worker currently working at a job produces a single nonstorable good according to h_t^α where h_t is human capital and $0 < \alpha < 1$. Jobs disappear at the end of the period with probability δ. if the job does not disappear, the worker's human capital next period is $h_{t+1} = \gamma h_t$ where $\gamma > 1$. If the job disappears, the worker becomes unemployed. Unemployed workers receive one new job offer in each period. The offers are parameterized by a random variable z which is uniformly distributed between 0 and 1 and is i.i.d. over time. A worker who accepts a job at t has human capital $h_t = z h_{t-1}$. That is, part of the human capital $(1 - z)h_{t-1}$ disappears forever. Workers are risk-neutral and maximize $\sum \beta^t c_t$. Assume no borrowing or lending is possible.

   (a) Set up the worker's problem as a dynamic program.

   (b) Prove that unemployed workers have a reservation strategy of the form $z(h_{t-1})$ where they accept all offers greater than $z(h_{t-1})$ and reject all others.

   (c) What can you say about $z(h_{t-1})$? Is it linear in h_{t-1}?

6. Consider the McCall search model with a mass 1 of risk neutral individuals with discount factor equal to β and an exogenously given stationary distribution of wages $F(w)$. Assume that there is no unemployment benefit, so unemployed workers receive zero wage. Once a worker finds and accepts a job, he will be employed in this job until the job is destroyed exogenously, which happens with independent probability equal to s in every period. Once the job is destroyed,

the individual returns to the unemployment pool. Suppose that at $t = 0$ all workers start out as unemployed.

(a) Show that, provided that the worker never quits, the value of a worker who accepts a job at the wage w is given by

$$v^a(w) = w + \beta[(1 - s)v^a(w) + sv]$$

where v is the value of an unemployed (searching) worker. Explain the intuition for this equation. Will the worker ever quit a job (unless there is an exogenous separation)?

(b) Write down the dynamic programming recursion that characterizes the optimal behavior of an unemployed worker. Be speci?c about he assumptions you are making in writing this recursion (and justify these assumptions). Derive an expression for the value of an unemployed worker, v.

(c) Find the reservation wage of the individual. Explain intuitively why this is constant over time. (Hint: use the fact that at the reservation wage R, the worker is indifferent between accepting the job and continuing to search, and combine this with the expression for v obtained in b).

(d) Find the the law of motion of unemployment. Why is unemployment not necessarily constant? Where does it converge to? Provide an interpretation of the limiting value unemployment in terms of separations and job creation.

(e) What happens to reservation wages and the unemployment process when s increases?

(f) Define the notion of "second-order stochastic dominance" What happens when $F(w)$ shifts to a new distribution $\tilde{F}(w)$ that has the same expected wage but second-order stochastically dominates F? Provide an intuition for this result.

7. Consider a discrete time model of on-the-job search, where w is the wage and $b > 0$ is the income and leisure value for unemployed worker and ρ is the discount rate. Let p^u be the offer probability while the individual is unemployed and $p^e > 0$ is the offer probability while the individual is working. Let $F(w) = CDF$ of the wage distribution, and $f(w) = density$ of the wage distribution.

   (a) Provide the Bellman equations for the search model and the closed form solution for worker optimal decision rule for the general wage density function. Prove that when $p^u < p^e$ the reservation wage is less than b. Explain the result.

   (b) Suppose that $c(s)$ is a convex cost function ($c(0) = 0$, $c^{'}(s) > 0$, $c^{''}(s) > 0$) of search given the effort level s while the individual is unemployed and employed. For each level of search effort s the offers probability is given by $p^u(s) = \frac{e^{\alpha s}}{c+e^{\alpha s}} = p^e(s)$, where c and α are some parameters. Rewrite the individual decision problem for job acceptance and effort of search as the individual unemployed and employed. Derive the conditions of optimal search efforts and job acceptance optimal decisions.

   (c) Add to the model in (b) the job destruction rate δ. Consider an employed individual on his first job after unemployment. Using the model explain the expected wage of the individual on his second job, third job, etc..

   (d) Using part (c), prove that probability of job transition is decreasing with the wage.

   (e) (advanced) Suppose that you have data on individuals where for each individual i the data starts at the date of school completion (say, high school). The data is on duration of unemployment, duration on the job and the wage on each accepted job, for 200 months for each individual. Write the likelihood function for the duration data and the accepted wage and discuss the identification of the model's parameters (define the parameters).

8. Consider the problem faced by each of a large number of inventors. An inventor can either choose to invent a new product at a cost of b or not. If the inventor chooses to invent, the quality of the invention is random. Let z denote the quality of an invention where z is drawn independently across inventors and time from a distribution $F(z)$. The quality of the invention is also the per period profits from the invention. Once an invention is produced, the inventor must manage the product if he wants to sell it. While managing, an inventor has no time to invent. With probability p, an invention becomes worthless, and the inventor can go back to his first love, inventing new products. Assume investors are risk neutral and discount future profits.

   (a) Set up a typical inventor's problem.

   (b) Define a stationary equilibrium for the economy.

   (c) Suppose the costs b fall. What happens to the fraction of those engaged in invention in a stationary equilibrium?

   (d) Suppose the distribution F changes in a mean-preserving fashion. What happens to the fraction of those engaged in invention in a stationary equilibrium?

9. Consider the following infinite-horizon model. A worker who is employed begins each period with a wage, say w. The worker can either work at that wage or search and receive unemployment benefits b on the current period. If the worker chooses to work, he is employed at wage w in the following period with probability δ and unemployed with probability $1 - \delta$. An unemployed worker who searches receives a wage offer from a distribution $F(w)$. Wage offers are i.i.d. over time. The worker's preferences are $\sum \beta^t u(c_t)$ where c_t denotes consumption and u is an increasing function. Assume no borrowing or lending.

   (a) Set up the worker's decision problem as a dynamic program.

   (b) Show that the solution to the worker's problem is of the reservation wage form. Characterize the reservation wage. Show that the reservation wage is increasing on the level of the unemployment benefit.

   (c) Calculate the probability that an unemployed worker is unemployed for N periods in a row as a function of the reservation wage and show that this probability is decreasing in N. How does this probability vary with unemployment benefits b? Justify your answer.

   (d) Suppose now that there are a large number of workers who all face the same problem as above. Define the unemployment rate as the fraction of all workers who are unemployed. How does the unemployment rate vary with b?

10. (**Programming**) In the Salop model, individuals search for a job in different "labor markets" where each labor market is characterized by a different probability of getting a job offer in a period of search. Once an offer is received in a given market, the individual must either take it, move on to another market to continue search, or stop searching altogether. Consider a situation in which there exists 10 separate markets. In each market the wage offer distribution (F) is the same, as is the cost of search c. The markets only differ in the probability of getting a wage offer in each period of search. In market 1, the offer probability is .1, in market 2 the offer probability is .2, etc. Thus

$$p_m = 0.1m, m = 1, ..., 10,$$

where m indicates the market number. As Salop demonstrates, given that the individual searches at all, she should begin by searching in the best market, which is the one in which the offer probability is highest. If she doesn't receive an acceptable offer in that market, she

should consider searching in the next best market. This process continues until all the markets have been exhausted, or until the value of nonparticipation is greater than the value of search in the best remaining market.
Perhaps the easiest way to solve this problem is to use backwards recursion, thus we first consider what happens in the worst market the individual faces [which will be the last one in which she could search]. Now in the worst market, the probability of getting an offer is 0.1. If an offer is received and not accepted in this market, the individual must stop searching altogether since there will be no markets left in which to search. Thus the value of not accepting an offer in this case is equal to the value of nonparticipation for the rest of the individual's life, which we will normalize to 0 [i.e., $V_0 = 0$]. Then write

$$V_1 = -c + \beta[0.1 \int \max[\frac{w}{1-\beta}, 0]dF(w) + .9V_0].$$

Given values of c, β, and F, this expression can be solved for V_1. If the expression is less than 0, than the individual will not search in the first market, and the value of V_1 should be set to the value of nonparticipation [which is 0].
The value of searching in the second market is similarly

$$V_2 = -c + \beta[.2 \int \max[\frac{w}{1-\beta}, V_1]dF(w) + .8V1].$$

Once again, if V_2 is less than 0,the individual would never search in the second market, and V_2 should be reset to the value of nonparticipation, which is 0. This process is repeated until we get to the 10th market. The value of search here is

$$V_{10} = -c + \beta[\int \max[\frac{w}{1-\beta}, V_9]dF(w).$$

You are to write a program (e.g., GAUSS, MATLAB, FORTRAN) to solve the above problem when $c = 1$ and $\beta = .8$. Further assume that w is distributed according to a uniform on the interval $[0, 10]$, that is,

$$F(w) = \frac{w}{10}, w \in [0, 10]$$

$$f(w) = .1, w \in [0, 10]$$

(a) Solve for the reservation wage sequence $w_1^*0, w_9^*, \cdots$ for all the markets in which the agent would actually search. For all such markets, compute the probability of getting an acceptable offer, as well as the expected value of an accepted wage offer in market s when an individual (optimally) searches in market s.

(b) Compute the conditional probability of leaving unemployment after t periods given unemployment through t periods [i.e., the discrete time hazard function] for this model when $t = 1, 2, 3$. Provide a characterization of the hazard for any arbitrary t, $t = 1, 2, \cdots$.

(c) Say the number of markets was countably infinite and that in each the probability of receiving an offer in a period was .8 [i.e., $p_m = .9, m = 1, 2, \cdots$]. Solve for the reservation wage in this case [continue to assume that $\beta = .8$ and $c = 1$], the discrete time hazard, and the expectation of the accepted wage distribution.

(d) Consider a simple generalization of the discrete time model. Let the times between offers be negatively exponentially distributed, so that the time to contacting the jth firm in the search process is given by t_j , which has a p.d.f. given by

$$g_j(t) = \lambda_j exp(-\lambda_j t), \lambda_j > 0 \forall j,$$

and without loss of generality assume that $\lambda_1 = .9 > \lambda_2 = .8 > ... > \lambda_{10} = 0$. Replace the discrete time discount factor β with a rate of time preference parameter $\rho > 0$. Let the wage offer distribution continue to be fixed at a known F, and let the cost of search remain at $c = 1$, but interpreted as a rate. Find a value of ρ such that no individual will search at more than 5 firms.

(e) Does the model in (d) produce the "discouraged worker" phenomenon? If not, think of how it might be generalized to do so.