Homework 7: Sequential and Recursive Competitive Equilibrium

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Optional to hand in [*]

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1. Consider an exchange economy with two (types of) consumers. Type-A consumers comprise fraction λ of the economy's population and type-B consumers comprise fraction $1 - \lambda$ of the economy's population. Each consumer has (constant) endowment w in each period. A consumer of type i has preferences over consumption streams of the form $\sum_{t=0}^{\infty} \beta_i^t u(c_t)$. Assume that $1 > \beta_A > \beta_B > 0$: type-A consumers are more patient than type-B consumers. Consumers trade a one-period riskfree bond in each period There is no restriction on borrowing except for a no-Ponzi-game condition. Assume that each consumer has zero bonds in period 0.

   (a) Carefully define a sequential competitive equilibrium for this economy.

   (b) Show that this economy has no steady state: in particular, show that the type-B consumers become poorer and poorer over time and consume zero in the limit.

2. This problem studies a neoclassical growth model with an externality in production. Leisure is not valued and the (representative) consumer has time-separable preferences with discount factor $\beta \in (0, 1)$. Consumers, who own the factors of production, are endowed with k_0 units of capital in period 0 and with one unit of time in each period. There is a large number of identical profit-maximizing firms each of which has the following production technology:

$$f(k, n, \bar{k}) = A k^\alpha n^{1-\alpha} \bar{k}^\gamma + (1 - \delta)k$$

   where k is the amount of capital rented by the firm, n is the amount of labor hired by the firm, $\bar{k}$ is the aggregate capital stock, δ is the rate of depreciation of capital. The parameters satisfy: $0 < \gamma < 1 - \alpha$, $0 < \alpha < 1$, and $0 < \delta \le 1$. Thus there is a productive externality from the rest of the economy: a higher aggregate capital stock increases the productivity of each firm. A typical (small) firm takes the aggregate capital stock as given when choosing its inputs.

   (a) Carefully define a sequential competitive equilibrium for this economy.

   (b) Carefully define a recursive competitive equilibrium for this economy.

   (c) Find a second-order difference equation that governs the evolution of the aggregate capital stock in competitive equilibrium. (Hint: Find a typical consumer's Euler equation and then impose equilibrium conditions.) Use this equation to find an expression for the steady-state aggregate capital stock in competitive equilibrium.

(d) Display the Bellman equation for the social planning problem in this economy. The planner internalizes the externality in production: his production technology is

$$h(\bar{k}, n) = f(\bar{k}, n, \bar{k}) = A\bar{K}^{\alpha+\gamma}n^{1-\alpha} + (1-\delta)\bar{k}$$

Is the competitive equilibrium allocation Pareto optimal? (Hint: Compare the planner's Euler equation to the second-order difference equation that you found in part (c).)

(e) Now introduce a government that subsidizes savings at a proportional rate τ and finances these subsidies by means of a lump-sum tax on consumers. The investment subsidy is constant across time but the lump-sum tax varies over time so as to balance the government's budget in every period. Define a recursive competitive equilibrium for this economy.

(f) For what subsidy rate τ is the competitive equilibrium steady-state aggregate capital stock equal to the steady-state aggregate capital stock in the planning problem?