Homework 6: Sequential and Recursive Competitive Equilibrium

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1. Consider a two-consumer, two-period exchange economy with date-0 trading. Type-1 consumers are endowed with 2 units of the consumption good in period 0 and 0 units of the consumption good in period 1; type-2 consumers are endowed with 0 units of the consumption good in period 0 and 2 units of the consumption good in period 1. The consumption good is not storable. Both types of consumers have the same preferences: $log(c_0^i) + \beta^i log(c_1^i)$, where c_t^i is the consumption of a type-i consumer in period t. Let the measures (fractions) of the two types of consumers both be equal to one-half.

   (a) Find the equilibrium consumption allocation and the equilibrium (relative) price of period-1 consumption goods. (Adopt the normalization that the period-0 consumption good is the numeraire: that is, the price of the period-0 consumption good is 1.) Express the equilibrium allocation and the equilibrium price in terms of $\beta^1 \quad \beta^2$. Explain intuitively why type-1 consumers have higher consumption than type-2 consumers in equilibrium.

   (b) Find the allocation of consumption chosen by a social planner who maximizes:

   $$\alpha[log(c_0^1) + \beta^1 log(c_1^1)] + (1-\alpha)[log(c_0^2) + \beta^2 log(c_1^2)]$$

   subject to the resource constraints: $c_t^1 + c_t^2 = w_t^1 + w_t^2$, for $t = 0, 1$. The weight $\alpha \in (0, 1)$. Your answer should depend on α and β^i.

   (c) An allocation in this economy is Pareto efficient if, one, it is feasible (i.e., it satisfies the resource constraints); and, two, it has the property that any reallocation that makes one (type of) consumer better off makes the other (type of) consumer worse off. Show that the set of solutions to the planning problem (as the weight α varies) coincides with the set of Pareto efficient allocations in this economy. (Hint: In this economy, the set of Pareto efficient allocations is the set of feasible allocations for which the intertemporal marginal rates of substitution of the two types of consumers are equated.)

   (d) Find the value of α for which the allocation chosen by the planner is identical to the competitive equilibrium allocation that you computed in part (a). Your answer should depend only on β. Explain intuitively why the social planner puts more weight on type-1 consumers than on type-2 consumers.

   (e) Show that the value of α that you computed in part (d) is equal to:

   $$\frac{\frac{1}{\lambda^1}}{\frac{1}{\lambda^1} + \frac{1}{\lambda^2}}$$

   , where λ^i is the marginal utility of period-0 consumption for a type-i consumer in competitive equilibrium. Give an intuitive explanation for this result.

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2. Consider an exchange economy with two consumers named A and B. The two consumers have identical preferences: they each value consumption streams according to $\sum_{t=0}^{\infty} \beta^t log(c_t)$. Consumer is endowment of consumption goods is $\{w_t\}_{t=0}^{\infty}$, $i = A, B$. Consumption goods are perishable (i.e., they cannot be stored and used for consumption in future periods).

   (a) Carefully define a competitive equilibrium with date-0 trading for this economy.

   (b) Suppose that $w_{At} = 4$ for all t and $w_{Bt} = 1$ for all t. Find the competitive equilibrium allocations and prices.

   (c) Suppose now that the endowments fluctuate deterministically: consumer A's endowment stream is $\{4, 1, 4, 1, 4, 1, \cdots\}$ and consumer B's endowment stream is $\{1, 4, 1, 4, 1, 4, \cdots\}$. Find the competitive equilibrium allocations and prices.

   (d) In parts (b) and (c) there is no variation in the aggregate endowment across time. Suppose that, as in part (b), consumer A's endowment is 4 in every period but that consumer B's endowment fluctuates: his endowment stream is $\{\frac{1}{2}, 2, \frac{1}{2}, 2, \frac{1}{2}, 2, \cdots\}$. Find the competitive equilibrium allocations and prices.

   (e) The social planning problem for this economy is:

$$\max_{c_{At}{}_{t=0}^{\infty}, c_{Bt}{}_{t=0}^{\infty}} \left\{ \alpha^A \sum_{t=0}^{\infty} \beta^t log(c_{At}) + \alpha^B \sum_{t=0}^{\infty} \beta^t log(c_{Bt}) \right\}$$

   subject to the resource constraint $c_{At} + c_{Bt} = w_{At} + w_{Bt}$ for all t. The numbers $\alpha^A \& \alpha^B$ are called Negishi weights. For each of the pairs of endowment streams in parts (b), (c), and (d), show that the consumption allocation chosen by the planner coincides with the allocation that arises in competitive equilibrium, provided that the weight α^i is set equal to the inverse of consumer i's marginal utility of income in competitive equilibrium.

   (f) Carefully define a competitive equilibrium with sequential trading for this economy. Use your results from parts (b), (c), and (d) to determine the equilibrium interest rates for each pair of endowment streams. In addition, for each case determine how each consumer's asset holdings vary over time (assume that each consumer starts with zero assets in period 0).

   (g) For endowment in part (c), Find the transfer payment necessary to implement the allocation $(c_t^A, c_t^B) = (2.5, 2.5)$ as an equilibrium with transfer.

   (h) Find the welfare weights $\alpha^A \& \alpha^B$ such that the transfer payment are equal to zero.

3. Consider a neoclassical growth model with logarithmic felicity function, Cobb-Douglas production function $F(\bar{k}, l) = A\bar{k}^{\alpha} l^{1-\alpha}$, full depreciation of the capital stock in one period (the rate of depreciation is equal to 1), and inelastic labor supply (leisure is not valued). In this problem, you will solve explicitly for the recursive competitive equilibrium of this economy. Assume that the economy is decentralized.

   (a) Suppose that aggregate capital evolves according to $\bar{k}' = G(\bar{k}) = sF(\bar{k}, 1)$. (You will verify the validity of this conjecture below.) Find explicit formulas for the value function $v(k, \bar{k})$ and the decision rule $\bar{k}' = g(k, \bar{k})$ of a "small" (or typical) consumer who takes the law of motion for aggregate capital as given. The functions v and g depend on s as well as on primitives of technology and preferences. (Hint: Guess that $v(k, \bar{k}) = a + blog(k + d\bar{k}) + elog(\bar{k})$ and then find expressions for the unknown coefficients a, b, d, and e in terms of the structural parameters α and β and the behavioral parameter s.)

(b) Find the competitive equilibrium value of s by imposing the consistency condition $G(\bar{k}) = g(k, \bar{k})$. Verify that the resulting law of motion for aggregate capital solves the planning problem for this economy. Display v and g for the equilibrium value of s.

(c) How does an increase in aggregate capital affect the savings behavior and the (indirect) utility of a typical consumer (holding fixed the consumer's own holdings of capital)?

(d) How does the equilibrium utility of a typical consumer vary with aggregate capital (taking into account that the consumer's own holdings of capital equal aggregate capital in equilibrium)?