Homework 2: Dynamic Optimization

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1. Consider the one time period problem we solved in the class. Lets change the production function to $zf(n)$. Here z is called total factor productivity. Find the first order condition to the agent's optimization problem. Check the second order condition. What happens to employment when z increases?

2. Now, suppose $U(c) = ac$ and $g(n) = \frac{bn^2}{2}$ find $\phi(\omega, A)$. Find the income and substitution effects associated with $\phi_\omega(\omega, A)$. Find $V(\omega, A)$ Is $V(\omega, A)$ increasing in both ω and A? Device a spreadsheet program to find the policy and value functions. Graph$\phi(\omega, A)$ and $V(\omega, A)$ first as functions of ω and then as functions of A. What can you deduce about the parameters a and b from observing the choice of labor input?

3. Now assume a two period time model, where agents work at time 1 and consume at both periods. So, the utility function is $u(c_1) - g(n1) + \beta u(c_2)$. People can save for consumption at period 2, which putting one unit into storage, yields R units at period 2. Therfore, the budget constraints are $c_1 = \omega n - s$, and $c_2 = sR$ that s is saving.

   (a) Find the first order conditions.

   (b) Find an expression for $\frac{dn}{d\omega}$

   (c) Suppose that lifetime utility is given by$U(c_1 - g(n_1)) + \beta U(c_2)$ with $g(n) = n^2/2$. Notice the placement of the brackets and the difference between this expression and the previous representation of preferences. In this case, find $\frac{dn}{d\omega}$, $\frac{dn}{dR}$, $\frac{ds}{dR}$

   (d) Try to formulate a version of the model where the agent consumes in both periods of life and works in both periods of life. What do the optimal choices of employment and saving depend on? How would you characterize the effect of a variation in the period 2 wage on period 1 employment?

4. Consider a discrete, cake eating problem. The consumer can either eat the cake or store it until the next period. The utility flow from eating a cake of size W is $u(W)$. Assume $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$. If the cake is not consumed in the current period then its size in the next period is given by $W_{t+1} = \rho W_t$. The discount factor β satisfies $0 < \beta < 1$

   (a) Assume $\rho = 1$, so the leftover does not destroy over time. Find the sequence of optimal consumption.

   (b) Now assume cake is perishable and $\rho < 1$. In this case compute the stream of consumption.

   (c) Discuss the saving behavior if ρ decreases. Compare the impact of income effect and substitution effect for different primitive values.

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