Homework 18: Idiosyncratic Shocks

Mohammad Hossein Rahmati *

August 11, 2016

1. Consider the following two period economy. Households preferences are $u(c_1, c_2) = ln(c_1) + \beta E(ln(c_2))$ where c_1 and c_2 denote consumption in period 1 and 2, respectively, and β is the discount factor. Income in second period is stochastic and can take values y^h or y^l(with equal probability and $y^l < y_1 < y^h$). Notice that the total income in the second period is fixed (no aggregate shock) i.e. $0.5y^l + 0.5y^h = y^2$ The households budget constraints in the two periods are

$$\begin{aligned} c_1 + a &= y_1 \\ c_2^i &= y^i + a(1+r) \end{aligned}$$

where a is the saving/borrowing of the household, and y^i is income of household i in the second period. Households also face a borrowing constraint in the first period

$$a \geq -\alpha \left(\frac{y^l}{1+r} \right)$$

i.e., they cannot borrow more than a fraction $\alpha > 0$ of the (discounted) lowest endowment in the next period.

   (a) Solve for the optimal consumption allocations $(c_1^*, c_2^{i,*})$ as a function of $(y_1, y^i, y^2, y^l, y^h, r, \beta)$ by first assuming that the borrowing constraint does not bind.

   (b) Characterize the parameters region where the borrowing constraint binds, i.e., state an inequality with α on the left hand side and $(y_1, y^2, y^l, y^h, r, \beta)$ on the right hand side. What are the optimal consumption choices when the borrowing constraint binds?

2. Assume that the consumer has CRRA period utility over consumption given by $u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}$ with $\gamma > 0$, discounts the future at rate $\beta < 1$ and faces a constant interest rate R. Assume that consumption is conditionally log-normal with mean $E_t ln(c_{t+1}) = \mu_t$ and variance ν_t

   (a) Show that the optimal consumption path follows

$$E_t(\Delta ln(c_{t+1})) = \frac{1}{\gamma} ln(\beta R) + \frac{1}{2}\gamma\nu_t$$

   (b) Based on the equation above, does the agent display precautionary saving behavior?

   (c) Suppose we tested the Permanent Income Hypothesis (in particular the statement that obly news accruing between t and $t+1$ affects the change in consumption $c_{t+1} - c_t$ by running the regression

$$\Delta c_{t+1} = \alpha_0 + \alpha_1 y_t + \varepsilon_{t+1}$$

*Sharif University of Technology, rahmati@sharif.edu

where y_t is past income and we found that α_1 is significantly different from zero. Based on your analysis above, could you say that this result means necessarily a rejection of the PIH?

3. Consider the "income fluctuation problem" of an agent with CARA utility. Precisely, assume that the agent is infinitely lived, discounts the future at the factor β, faces i.i.d. income shocks y_t, can save/borrow through a risk-free asset with constant gross interest rate R (ignore borrowing limits), and has period utility

$$u(c_t) = -\frac{1}{\sigma}e^{1-\sigma c_t}$$

Guess that the optimal consumption allocation takes the following form

$$c_t = B(Ra_t + y_t) + D$$

where B and D are constants and have to be determined.Solve for the consumption allocation in closed form (i.e., determine the two constants) and argue that the "precautionary saving motive" is constant across all agents, i.e., it is independent of the individual pair of state variables (a, y) Explain your answer.

4. Consider a stationary economy populated by a continuum of measure one of infinitely lived, ex-ante equal agents with preferences over sequences of consumption and leisure given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_{it}) + e^{\psi_{it}} \nu(1 - h_{it}) \right]$$

Agents face individual shocks to their preference for leisure ψ_{it}, which follow the stochastic process $\psi_{it} = \rho\psi_{i,t-1} + \eta_{it}$, with $\eta_{it} \sim N(0, \sigma_\eta)$ Agents can save but cannot borrow. Production takes place through the aggregate technology

$$C_t + K_{t+1} - (1 - \delta)K_t = AK_t^\alpha H_t^{1-\alpha}$$

where C_t, K_t and H_t are, respectively, aggregate consumption, aggregate capital, and aggregate hours at time t. Labor and asset markets are competitive and clear, every period, with prices ω_t and r_t respectively. The government taxes capital income at a fixed flat rate τ. Tax revenues are returned to agents as tax-exempt lump-sum transfers b. Households can evade taxes by choosing every period t the fraction of capital income ϕ_{it} to declare in their tax return, i.e., the fraction of capital income on which they pay taxes. Let x_{it} be the total undeclared taxes at time t. The government, knowing that agents may have evaded taxes at time $t-1$, at time t can monitor and perfectly verify the past period individual tax returns. Let π be the probability that, at time t the time $t-1$ tax return of a household is subject to monitoring. The household finds out whether her $t-1$ period tax return is monitored at the beginning of period t, i.e., before consumption decisions are taken. In the event the household is caught, at time t the tax agency gives her a fine equal to $z(x_{i,t-1}) > x_{i,t-1}$, where $x_{i,t-1}$ is the tax amount due from the past period, with $z(0) = 0$ and $z(x_{i,t-1}) > x_{i,t-1}$

(a) Write down the problem of the household in recursive form, making explicit the individual and the aggregate state variables.

(b) Write down the individual first-order necessary condition that characterizes the optimal tax evasion choice.

(c) Define a stationary recursive competitive equilibrium for this economy.

5. Consider an economy with a continuum (measure 1) of ex-ante identical consumers, each living for two periods. The consumers have utility given by

$$log(c_1) + \beta log(c_2)$$

where c1 and c2 are period 1 and period 2 consumption and β is the discount factor. In period 1, each agent is endowed with y units of output which can be either consumed, c_1, or invested, k. In period two, consumers receive income from the capital they saved in period 1 and from inelastically supplying their labor endowment to the market. The labor endowment of any given individual is random and it is independent across agents. Period-2 labor endowments can be either $1 - \epsilon$ or $1 + \epsilon$, with $0 < \epsilon < 1$; the probability that any agent's labor endowment is $1 - \epsilon$ is 1/2. Due to the independence of shocks across consumers, a law of large numbers operates so that also the fraction of agents with labor endowment in period 2 equal to $1 - \epsilon$ is 1/2. That is, there is no uncertainty about the period 2 aggregate labor endowment: the supply of labor is constant at 1. In the second period, output is produced by perfectly competitive firms which operate a standard Cobb-Douglas production function: they sell the output to consumers and rent the capital and the labor services from the same consumers at rates r and w, respectively.

(a) Write down the consumers problem and define a competitive equilibrium for this economy

(b) For the case $\epsilon = 0$ (no individual uncertainty) analytically solve for the competitive equilibrium. Argue that the equilibrium is efficient.

(c) Show that the equilibrium interest rate is a decreasing function of ϵ. (Hint. The key to show this is to show that individual investment k is increasing in ϵ)

(d) For the case $\epsilon > 0$ define the equilibrium risk faced by consumers as the ratio between equilibrium consumption in the high endowment state and equilibrium consumption in the low endowment state. Show that a) equilibrium risk is not affected by the equilibrium level of k and b) agents perceive that their individual risk is affected by their choice of k.

(e) For the case $\epsilon > 0$ is the equilibrium efficient (i.e. could a planner improve welfare simply by changing the level of k)? (You do not need to provide a formal argument, simply use your logic together with the results from point (d).

6. Consider a closed economy inhabited by a continuum of infinitely lived agents with common expected utility given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$u(c) = \frac{1}{1-\gamma} c^{1-\gamma}, \gamma > 0$$

where $\beta < 1$ is the discount factor and c is consumption. Each agent can trade a non contingent bond and faces the following budget constraint and borrowing constraint

$$a_{t+1} + c_t = a_t(1 + r) + y_t \qquad a_{t+1} = 0$$

Where r is the equilibrium interest rate and y_t is a stochastic endowment Assume that y_t can take the values $1 + \epsilon$ and $1 - \epsilon$ with probability 1/2 and is i.i.d across time and across people. Bonds are in 0 net supply and each agent starts with initial bond position equal to 0.

(a) Define a stationary competitive equilibrium for this economy and solve for the equilibrium consumption and asset distributions. Show that any interest rate belonging to the set $[r^*, \infty)$ is consistent with equilibrium. Solve for r^*

(b) Assume that at time t,unexpectedly, the government finances an investment that costs g. The investment is financed by issuing a bond which is repaid in full in period t + 1 using the proceeds from the investment (assume that additional proceeds from the investment are simply discarded). From time t+1 on the economy is the same as before period t.Plot aggregate consumption and r^* in periods t - 1, t, t + 1 and t + 2 (you don't have to solve for the exact values of aggregate consumption and r^* in t, t + 1, t + 2, just plot the qualitative changes relative to period t - 1)

(c) Argue that the allocation in the economy with the government investment weakly Pareto dominates the allocation of an economy without the investment. Argue that if the investment is large enough the the Pareto dominance becomes strict (i.e. every agent is strictly better off).