Homework 17: International Finance

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August 11, 2016

1. Consider the economy in Backus-Kehoe-Kydland (International Real Business Cycles, JPE, 1992). It is a two country model. Each country indexed by $i = \{h, f\}$ standing for home and foreign, is represented by a representative consumer and a production technology. The countries produce the same good, and their preferences and technology have the same structure and parameter values. Although the technologies have the same form, they differ in two important respects: in each country (i) and production is subjected to country-specific technology shocks, and (ii) the labor input consists only of domestic labor, i.e., labor does not flow across countries, only goods do.We will solve the world planners problem. The planner maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\phi u(c_t^h, l_t^h) + (1-\phi) u(c_t^f, l_t^f) \right] \tag{1}$$

where c_t^i denotes consumption and l_t^i leisure at date t in country i. We are interested in the equal-weight case $\phi = 1/2$ We assume that

$$u(c_t^i, l_t^i) = \frac{(c_t^{\mu} l_t^{1-\mu})^{1-\gamma}}{1-\gamma}$$

with $\mu \in (0,1)$ and $\gamma > 0$. Production in country i takes place through

$$y_t^i = exp(z_t^i)(k_t^i)^{\theta}(1-l_t^i)^{1-\theta}$$

and world output is $y^h + y^f$ The world resource constraint is then

$$\sum_{i \in \{h,f\}} \left(c_t^i + x_t^i - exp(z_t^i)(k_t^i)^{\theta}(1-l_t^i)^{1-\theta} \right) = 0 \tag{2}$$

with x_t^i being gross investment and $n_t^i = y_t^i - c_t^i - x_t^i$ being net exports in country i Capital in country i evolves through $k_{t+1}^i = (1-\delta)k_t^i + x_t^i$. The productivity shocks follow the joint evolution

$$\begin{bmatrix} z_t^h \\ z_t^f \end{bmatrix} = A \begin{bmatrix} z_{t-1}^h \\ z_{t-1}^f \end{bmatrix} + V \begin{bmatrix} \epsilon_t^h \\ \epsilon_t^f \end{bmatrix}$$

where ϵ_t^i has mean zero and SD normalized to one. The planners problem is to maximize (1) subject to the aggregate resource constraint (2) and laws of motion for endogenous (capital) and exogenous (shocks) states. The choice variables for the planner are $\{c_t^i, c_{t+1}^i, l_t^i\}_{i \in \{h,f\}}$ Consider the following quarterly parameterization, where we think of the home country as the

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US and the foreign country as Europe. The values are, approximately, from the original BKK article. Set $\beta = 0.99$, $\mu = 0.34$, $\gamma = 2$, $\theta = 0.36$,$\delta = 0.025$ Finally, set:

$$A = \begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix}$$

and set the standard deviation of the innovations (the diagonal elements of V) equal to 0.01 for both countries. The correlation between innovations is set equal to 0.26. This implies a value for the covariance, and thus for the off-diagonal elements of V

(a) Solve for the steady state.

(b) Write a Dynare code to solve for the equilibrium of the same economy above.

(c) Simulate the model and compute the international cross-correlations between country h and country f for HP-filtered ($\lambda = 1600$) log output and log consumption.

(d) Plot the IRF of $(c^i, x^i, y^i, 1 - l^i, n^i)$ to a one-standard-deviation innovation in the home countrys technology shock. Plot each variable for both countries in a separate panel.

2. Consider a two-country two-good endowment model with complete markets. Both goods are traded. Country $i = 1$ has endowments $x_t(z^t)$ while country $i = 2$ has endowments $y_t(z^t)$. Each country consumes both goods and has a representative consumer with preferences

$$u(a^i, b^i) = \sum_{t=0}^{\infty} \sum_{z^t} \beta^t \pi(z^t)(a_t^i(z^t)^\theta b_t^i(z^t)^{1-\theta})^{1-\sigma}/(1-\sigma)$$

(a) Define a competitive equilibrium in this economy. Define the terms of trade in this environment.

(b) The competitive equilibrium can be solved using the social planners problem. First, set up the social planners problem with welfare weights λ_i. Taking weights as given, solve for the efficient allocation and the supporting prices. Next find the implied welfare weights associated with a market economy when country 1 has endowments $x_t(z^t)$ and country 2 has endowments $y_t(z^t)$.

(c) Solve for the terms of trade and the trade balance. How does the trade balance covary with the terms of trade? (Hint: what common factors do both the terms of trade and trade balances depend on?)

(d) Now assume that there is no international financial market and the trade has to be balanced every period. Solve for the competitive equilibrium allocations and terms of trade?

(e) Compare the results of question e with those of the complete markets economy. Provide the economic intuition for the results?

3. **Heterogeneous preferences for nontraded goods** Consider a dynamic stochastic exchange economy with I countries, each represented by a single agent, and $I + 1$ goods, one traded and I nontraded goods. Uncertainty is described by the usual event tree. In each state, the agent of country i consumes a_i units of the traded good and b_i units of her own nontraded good. Her endowments are ω_i and x_i respectively. Preferences are additive over time and across states, and agents have the same discount factor and probability assessments. Preferences in each state are given by $c_i = g_i(a_i, b_i)a_i^{\gamma_i} b_i^{1-\gamma_i}$ and utility function $u_i(c_i) = log(c_i)$. The resource constraints are $\sum_i a_i < \sum_i \omega_i = \omega$ for the traded good and $b_i \leq x_i$ for each nontraded good.

(a) Solve for a Pareto problem for optimal allocations and implicit prices.

(b) What is the price index p_i that corresponds to the aggregator g_i ?

(c) Use your answer to a to express p_j/p_i and c_j/c_i as functions of the endowments. Comment on the relation between them. Does heterogeneity in the γ_i s affect your answer.

4. **Limited Enforcement** Consider a two-country one-good pure exchange economy. Two countries have identical preference given by $E_0 \sum_t \beta^t ln(c_t)$. The following two income streams have equal probabilities of occurring

$$y_1 = \{2, 4, 2, 4, 2, 4, \cdots\} \qquad y_2 = \{4, 2, 4, 2, 4, 2, 4, \cdots\}$$

The uncertainty is revealed in period 0 and no uncertainty afterwards. Countries trade a complete set of arrow securities before the uncertainty is revealed.

(a) What is the efficient allocation where two countries have the same social weight?

(b) What is the ex post interest rate decentralize the efficient allocation?

(c) Suppose that each country has the option to walk away from the contracts anytime, what will be the extra conditions that the constrained efficient allocations have to satisfy?

(d) Assume $\beta = 0.5$, is the efficient allocation in part 1 constrained-efficient?

(e) What about $\beta = 0.9$?

(f) Assume $\beta = 0.5$. Compute the constrained efficient allocation that can be supported as a competitive equilibrium in this economy? What is the ex post interest rate?

5. Consider the recursive formulation of the one-sided limited commitment problem. The Small Open Economy receives a stochastic endowment stream $y_t \in Y$, where Y is a finite set with minimal value $\underline{y}$ and maximal value $\overline{y}$, and y_t is iid over time. Let $B(\nu)$ denote the value of the risk-neutral representative foreign lender given that the SOE enjoys value ν. The efficient allocation solves the following Bellman equation:

$$\begin{aligned} B(\nu) \quad &= max_{\{c(y'),\omega(y')\}} E[y' - c(y') + R^{-1}B(\omega(y'))] \\ s.t. \quad & \qquad E[u(c(y')) + \beta\omega(y')] = \nu \\ & \omega(y') \geq \underline{V}(y') = u(y') + \beta\underline{V} \quad for \quad all \quad y' \end{aligned}$$

where $\underline{V} \equiv Eu(y)/(1 - \beta)$

(a) Show that B is strictly decreasing on the domain $\nu \geq u(\underline{y}) + \beta\underline{V}$

(b) Show that the Bellman operator is a contraction

(c) Show that $B(\nu)$ is concave. Hint: let $u(y')$ be the choice variable rather than $c(y')$, and define $C(u)$ as the inverse utility function, so the flow payment to the lender is $y' - C(u(y'))$. Now the objective is strictly concave and the constraints are linear in the choice variables.

(d) Show that if $\beta R = 1$, ν converges to $u(\overline{y}) + \beta\underline{V}$. Is the convergence monotonic and does ν reach its long-run level in finite time?

(e) Suppose that y takes two values, $y \in \{\underline{y}; \overline{y}\}$. Assume $\beta R < 1$. Depict the policy function for ν for each realization of y and argue that v will eventually converge to a stationary (ergodic) distribution bounded by $u(\underline{y}) + \beta\underline{V}$ and $u(\overline{y}) + \beta\underline{V}$.

(f) Show how the efficient allocation can be decentralized with state contingent assets and portfolio constraints.