Homework 15: Money Search

Mohammad Hossein Rahmati *

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1. Consider the following search model of money. Time is discrete and there is a continuum of agents with population normalized to 1. Any particular agent specializes in the production of one service (a nonstorable good) but likes other services in an interval of size $x \in (0, 1)$. She derives utility $u(q) = q^{1/2}$ from consuming $q \in R_+$ units of the service provided it falls in her desired interval. An agent discounts the future at rate $(1+r)^{-1}$. There is a constant disutility $-q$ to producing q units of a service. Production and consumption occur at the end of the period (and hence should be appropriately discounted). At the beginning of time, a fraction of agents $M \in (0, 1)$ are randomly given one unit of currency. Currency is indivisible and can be stored only one unit at a time. Agents are exogenously matched in the following way. Agents with money (we will term them buyers) are randomly matched in pairs with agents without money. Thus, the probability that a buyer is matched with a seller whose good she desires is $x(1 - M)$. Also, the probability that a seller is matched with a buyer who wants her good is xM. Every agent's trading history is private information. Finally, assume that buyers submit take-it-or-leave-it offers (which amount to a trade of 1 unit of money for Q units of the seller's service).

   (a) Taking the quantity of services bargained for Q as given, write down the value functions for a buyer $V_b(Q)$ and a seller $V_s(Q)$ respectively .

   (b) Taking the value functions V_b and V_s as given, what is the value of Q from the buyer's take-it-or-leave-it offer? To answer this question, proceed as follows. What condition assures that a seller accepts money in exchange for the production of his services? In particular, what is the seller's utility if he accepts the offer (produces the service and obtains the unit of currency)? What is the seller's utility if he rejects the offer (and goes back into the search pool)? Under what conditions on Q then will the seller accept the offer? Hence, if the buyer is trying to get as much services as possible, what value of Q will she demand from the seller?

   (c) Define a monetary equilibrium.

   (d) Does a monetary equilibrium exist? If so, under what conditions on r, x, and M? Do any other equilibria exist?

   (e) Does the price level vary with increases in M?

   (f) If we define ex-ante welfare as $W = MV_b + (1 - M)V_s$, how is welfare affected by changes in the money supply?

2. **Based on Kiyotaki, Wright [1993]** Consider the following environment where the goods and money are indivisible. The exogenous parameter $0 < x < 1$ equals the proportion of commodities that can be consumed by any given agents and x also equals the proportion of

*Sharif University of Technology, rahmati@sharif.edu

agents that can consumes any given commodities. One unit of consumption yields $U > 0$, while consuming other commodities or money yields zero utilities. A fraction of M of the total agents at each period own money while $1 - M$ are producing goods or own commodities .Money and commodities are costlessly storable. There is a production sector. That is, once an agent consumes enter in production sector and during one time could produce one unit of output with probabilities of $\alpha > 0$. In exchange sector, agent who has just produced looks for other agent to trade. Traders in the exchange sector meet pairwise and with probability $\beta > 0$ find other traders. The exchange take place if and only if it is mutually agreeable, that is, if and only if both agents are at least as well off after the trade. Also there is a transaction cost $0 < \epsilon < U$, that must be paid by the receiver whenever any real commodity is accepted in trade.In the exchange sector two types of agents, commodities trader and money traders, exist. Let μ denote the fraction of trader in the exchange sector who are money trader, so that a trader located at random has money with probability μ and a real commodity with probability $1 - \mu$. Let Π denote the probability that a commodities trader accepts money and let π be the best response of the representative individual. Let V_j denote the value function for the individual in state $j = 0, 1, m$ indicates that he is a producer, a commodity trader or the money trader, respectively.

(a) Assume that we do not have double coincide problem. write the Bellman's Equations. In the rest of problem assume that double coincide matching is possible.

(b) For this case write the Bellman's Equations.

(c) Assumes that N_0 and N_1 and N_m denote the number of producer, commodity trader or the money trader, respectively, in the steady state.Find the implicit function for the μ as a function of M and Π. Is the μ is increasing with respect to M? what about Π? or show that it is indeterministic?

(d) What is the value of Π which there exist mixed strategy? The equilibrium is called mixed-monetary equilibrium. (hint: what happen if $\pi < x$, what is the best response of commodity traders.)

(e) For simplicity assume that $\alpha \to \infty$ thus the production is instantaneous and $N_m = M$, $N_1 = 1 - M$ and $\mu = M$. Assume $x < \frac{1}{2}$, find the value μ^0 which maximize the welfare function.

3. **Based on Lotz, Shevchenko, Waller [2007]** Assume the following environment where the money and goods are indivisible. Agents discount future at rate r. Agents meet each other with probability α and the probability an agent can produce one's desired consumption good is x. There is no double coincide of wants. The agents are (ex ante) two types, $i = H, L$, whose measure are given by μ_H and μ_L, respectively. Type i agents get utility $u_i(q)$ from consuming q and incur disutility $c_i(q)$ from producing q units. We assume that $\dot{u}_i > 0$, $\ddot{u}_i < 0$, $u_i(0) = 0$, $\dot{c}_i > 0$, $\ddot{c}_i > 0$, $\frac{\dot{u}_i(0)}{\dot{c}_i(0)} = \infty$ for all i, j. When goods are indivisible we assume that $u_i(1) = U_i$ and $c_i(1) = C_i$ where $U_i > C_H > C_L$ for $i = H, L$ and $\frac{C_h}{C_L} > \frac{U_H}{U_L}$.
Let M be the fraction of agents with money and they are constraint to hold no more than one unit of indivisible money. Let m_H denote the fraction of high types holding a unit of money and m_L denotes the same for the low type. Thus the fraction of money holder in each types differs from each other. Now we introduce the lottery in the economy. [1] We introduce two probability. If the match happen between buyer i and seller j, the buyer enjoy from utility U_i with probability λ_{ij} while pay the unit indivisible money with probability τ_{ij}. Similarly, assume that the seller j accepts to sell the unit of good to buyer i, then he will incur cost

[1] Remember the indivisible lobar Hansen (1985) that to resolve the noncovexity of labor introduce the lottery to number of agent participate in the labor market.

C_j with probability λ_{ij} and give money with probability τ_{ij}. Assume that V_i^1 denotes the stationary value function of the agent holding one unit of money and V_i^0 denotes the agents without money

(a) Consider the case where buyer has full bargaining power and take all the surplus. Thus when a type $i (i = H, L)$ buyer meets a type $j (j = H, L)$ seller, he makes a take-it-or-leave-it offer to the seller. The offer consist of the pair $(\lambda_{ij}, \tau_{ij})$. Write the bargaining problem and find the $(\lambda_{ij}, \tau_{ij})$ as a function of value functions.

(b) Prove that if $\tau_{LH} > \tau_{HL}$ then $m_H > M > m_L$.

(c) Define the stationary Bellman Equation and find the stationary value function. You can assume that $\rho = \frac{r}{x\alpha}$ For simplicity in the rest of problem assume that $\tau_{ij} = \tau_j > 0$ and $\lambda_{ij} = \lambda_j > 0$. Thus the lotteries prices only depend on the seller's type.

(d) It is important question that is it possible that money and lottery exist in equilibrium simultaneously. Show that for $\rho \in (0, \rho_1)$ and $M \in (0, \bar{M}_1]$, a unique monetary equilibrium exists with $\tau_H, \tau_L < 1$ and $\lambda_H, \lambda_L = 1$.