Homework 14: Labor Search

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1. Imagine a firm got a worker and it can produce one unit of output per period for 10 periods (there is a zero interest rate). If the firm does not operate today, it loses its license and is out. The worker is risk neutral, can either take the job or run a hot dog operation what yields 0.25 units per period for five years and sick for another 5 years during which she will be on disability insurance collecting .1 units of output.

   (a) What is the minimum wage that the worker would accept.

   (b) If the firm had to pay an entrance fee to open, what would be the maximum fee under which the firm would enter.

   (c) Pose a wage that is the bargaining solution with the firm having twice the weight than the worker.

   (d) Briefly describe what could happen if there was a possibility of multiple entry of firms. How could the free entry condition be applied?

2. Consider the decision problem of a single worker who has preferences given by $\sum_{t=0}^{\infty} \beta^t u(c_t)$ where c_t denotes consumption, β is a discount factor and u is strictly concave. At the beginning of each period the worker is either unemployed or employed. If unemployed, the worker receives a wage offer from a distribution $G(w)$ and if employed, gets a new offer from a distribution $F(w)$. These offers are constant for the duration of the jobs. These draws are i.i.d. Jobs disappear with probability δ at the end of each period. The worker cannot save or borrow and if unemployed receives a benefit b.

   (a) Set up the decision problem as a dynamic program.

   (b) Show that the solution to the dynamic program is of the reservation wage form.

   (c) Assume that $b = 0$, $u(0) = 0$ and that G and F are both the same with the supports of both distributions given by $[0, \overline{w}]$. What jobs if any does the worker reject?

3. Consider a continuous-time equilibrium matching model with the following features:

   - Workers and firms have linear utility and discount at a net rate r.
   - There is a constant-returns-to-scale matching function $M(u; v)$ describing the number of matches taking place at each instant as a function of the unemployment and vacancy rates, respectively. It is increasing in both arguments.
   - Firms can enter for free but there is a cost c of posting vacancies. Firms enter until the profit (net of the entry fee) is zero.

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- There is a $[0, 1]$ continuum of workers. A fraction ϕ of the workers are in couple relationships; the rest are single.
- All unemployed workers receive unemployment compensation b.
- Workers in couple relationships are less unhappy being at home than are single workers. The cash value of being at home of single workers is normalized to zero; the cash value of being at home of workers in couple relationships is $h > 0$.
- When firms search for workers, they cannot tell if workers are in couple relationships or single: there is "undirected" search.
- Firm-worker pairs are exogenously terminated at a rate σ
- A firm-worker pair produces p units of consumption.
- At every instant, matched workers and firms Nash bargain; workers obtain a share β of the surplus and firms obtain a share $1 - \beta$.
- The economy is in a steady state.

Complete the following tasks. Assume throughout that the primitives are such that workers prefer working to not working.

(a) Describe, separately for workers in couple relationships and single workers, two equations determining the value functions of the unemployed worker and of the employed worker as a function of the probability for an unemployed worker of finding a job, denoted λ_ω, the net interest rate r, the separation rate σ, (in the case of the workers in couple relationships) the cash value of being unemployed at home h, and the wage.

(b) Describe two equations determining the value functions of the vacant firm and of the matched firm as a function of the probability for a vacant firm of

finding any worker, denoted λ_f , the probability that the worker is in a couple relationship, ϕ, the net interest rate r, the separation rate σ, the productivity p,the vacancy posting rate c, and the wage w paid by the firm to the worker.

(c) Define the two total surpluses, S_c and S_s, for both kinds of matches a firm can end up in: one with a worker in a couple relationship and one with a single worker. Solve for these as a function of market tightness $\theta \equiv \frac{u}{v}$, using the matching function to derive λ_f and λ_ω.

(d) Combine the results above with the free-entry condition to derive one equation in one unknown: θ

(e) Find expressions for the wages of the two kinds of workers as a function of primitives and of market tightness.

(f) What is the effect on market tightness of an increase in the fraction of workers who are in couple relationships?

(g) What is the effect on unemployment of an increase in the fraction of workers who are in couple relationships?

(h) Suppose the government introduces a welfare program transferring wealth from unemployed workers who are in couple relationships to unemployed single workers so that their unemployment incomes (unemployment benefit plus the value of being at home plus/minus transfers/taxes) are equalized. What is the effect of this welfare program on the rate of unemployment?

4. Individuals meet firms at an exogenous rate λ. Once they have met the firm, the value of the match is revealed to the worker and the firm, which is denoted by θ. The matching distribution is given by $G(\theta)$. If a match θ generates an employment contract, the worker is paid a wage $w(\theta)$ and the job is dissolved at an exogneous rate η. The "threat point" of the potential employee is the value of continuing search, denoted V_n, and the threat point of the employer is 0. The value of the wage is determined using a Nash bargaining framework, so that

$$w(\theta, V_n) = argmax_w (V_e(w) - V_n)(\frac{\theta - w}{\rho + \eta}),$$

where η is the instantaneous discount rate, and $\rho + \eta$ is the effective discount rate of the firm.

   (a) Write down the labor market dynamics generated by this model in as much detail as possible. In particular, find $V_e(w)$, $w(\theta, V_n)$, and V_n. What is the probability density function of accepted wages? What is the steady state unemployment rate?

   (b) Workers and firms can increase the value of the match by purchasing health insurance. The instantaneous price of health insurance is $\phi > 0$. If health insurance is purchased the match the total value of the match improves and becomes $a\theta$, where $a > 1$ [due to the increased healthiness of the worker]. Then the Nash bargaining problem becomes

$$(w, d)(\theta, V_n) = argmax_{w,d} (V_e(w) - V_n)(\frac{ad\theta - w - d\phi}{\rho + \eta}),$$

   where d equals 1 if health insurance is purchased and equals 0 if not, and we have assumed that the firm actually pays the health insurance premium directly to the insurance company. Describe the equilibrium outcomes associated with this model as you did in part (a). In equilibrium, will some jobs be covered by health insurance and others not? If so, what proportion of jobs will be covered by health insurance, and what is the relationship between the presence of health insurance and wages? Provide some intuition for your results.