Homework 12: Computation of Real Business Cycle Models

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1. Using MATLAB, you are to obtain decision rules by the method of undetermined coefficients suggested in Christiano (2002) (his code and his examples on my website) for the economy in Hansen (1985). In particular, the problem you are to solve is to choose $\{C_t \geq 0, \pi_t \in [0,1], K_{t+1} \geq 0\}_{t=0}^{\infty}$ to solve

$$max \quad E\left[\sum_{t=0}^{\infty} \beta^t((1-\alpha)log(C_t) + \alpha\pi_t ln(1-\bar{h})\right]$$

subject to

$$C_t + K_{t+1} = Z_t K_t^{\theta}(\pi_t \bar{h})^{1-\theta}) + (1-\delta)K_t$$

and

$$Z_t = (1-\rho) + \rho Z_{t-1} + \epsilon_t$$

where K_0 is given, ϵ_t are i.i.d. $N(0, \sigma_\epsilon)$, and the steady state level of technology $\bar{Z} = 1$. You are to use the calibration that we considered in class. In particular,

$$\theta = 0.36 \quad \rho = 0.95 \quad \sigma_\epsilon = 0.007$$

$$\delta(quar) = 0.015 \quad \bar{h} = 0.53 \quad \beta(quar) = 0.9921 \quad \alpha = 0.666$$

   (a) Follow the steps we did in class to get decision rules: (i) Find the steady state; (ii)Linearize the equations characterizing an equilibrium (i.e. first order conditions and resource feasibility) around the steady state from the first step ; (iii) Posit linear decision rules as a function of the state variables (i.e. (2.7) or (3.7) in Christiano, which for this specific application is (4.6) and (4.7)) and solve the implied system of equations.

   (b) Graph impulse response functions of unemployment and output to a unit innovation in technology.

   (c) Simulate artificial data to obtain standard deviations and correlations between output and consumption, hours, etc. To do so, generate time series of shocks ϵ_t drawing from MATLAB's random number generator (i.e. let $e = Normrnd(0, .007, 200, 1)$ generates a vector of 200 elements (about the number of quarters in post WWII data) drawn from a Normal distribution with mean 0 and standard deviation 0.007). Use this vector to build up a time series of deviations $\hat{z}_t$ (take $\hat{z}_{t1} = 0$, its steady state value) from (2). Generate time series for $1\hat{\pi}_t$ and $\hat{y}_t$ from the decision rules given the state variables $\hat{z}_t$ and $\hat{k}_t$ with $\hat{k}_{t-1} = 0$. For this sample, calculate the second moments. Do this 100 times and calculate the mean and standard deviation of each of these sample second moments. Use this to calculate a table like that (Table 1) in Hansen.

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2. Use the Tauchen method introduced in the class and discretize the shock process Z_t in the question 1 to two states. (Code in my website you need only to run it)

3. (No need to handout) This problem introduces you to dynamic programming in an infinite horizon growth model on the computer. In particular, you are to modify the MATLAB program that have been placed on my website called question3.m and question3.f95. These programs use value function iteration to solve for the decision rule $K_{t+1}(K_t)$ in a non-stochastic setting. You are to modify these programs to add uncertainty over technology shocks, both in matlab and in fortran. Then you are to compare the savings in speed. Specifically, assume that households have log preferences, the production technology satisfies $Y_t = Z_t K^\theta$ t where $\theta = 0.36$, and capital depreciates at rate $\delta = 0.025$. We will assume technology shocks follow a 2 state Markov Process. The transition matrix is calibrated to NBER business cycle data where we take an expansion to be an instance of a positive technology shock and recession to be an instance of a negative technology shock. The transition matrix is given by

$$\Pi = \begin{bmatrix} 0.977 & 0.023 \\ 0.074 & 0.926 \end{bmatrix}$$

where, for instance, $prob(Z_{t+1} = Z^g | Z_t = Z^g) = 0.977$. As for modifying the program, you must expand the state space to add technology shocks from the set $\zeta = Z^g = 1.25, Z^b = 0.2$. I chose these values in order to satisfy that $\bar{Z} = 1$.To see this, note that the note that implies an invariant distribution over the two states of $\bar{p}^g = 0.763$ and $\bar{p}^b = 0.237$. In that case, I chose $Z^g = 1.25$ and solved for $\bar{Z} = \bar{p}^g Z^g + \bar{p}^b Z^b$.

   (a) State the dynamic programming problem.

   (b) Plot the value function over K for each state Z. Is it increasing (i.e. is $V(K_{i+1}, Z) \geq V(K_i, Z)$ for $K_{i+1} > K_i$)? Is it "concave" (in the sense that $V(K_{i+1}, Z) > V(K_i, Z)$ is decreasing)?

   (c) Is the decision rule increasing in K and Z (i.e. is $K'(K_{i+1}, Z) > K'(K_i, Z)$ for $K_{i+1} > K_i$ and is $K'(K, Z^g) > K'(K, Z^b)$)? Is saving increasing in K and Z (to see this, plot the change in the decision rule $K'(K, Z) - K$ across K for each possible exogenous state Z)?