Homework 11: Real Business Cycle Model

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September 7, 2016

1. Consider the following static version of the Hansen (1985) indivisibility paper. There is a unit measure of ex-ante identical agents. There are only two possible number of hours per worker $h \in \{0, \bar{h}\}$, $\bar{h} < 1$, which implies there are only two possible levels of leisure an agent can derive utility from given the normalization that $1 = h + l$. Let preferences be given by $u(C, l) = (1 - \alpha) lnC + \alpha lnl$. The production technology is given by $y = zh^{\theta}$ with $\theta < 1$ where z is the state of technology.

   (a) Suppose that a planner has access to a randomization device which she can program such that $\pi = prob(h_t = \bar{h})$. Assume further she can set this device to be i.i.d. across all households, can see the outcome of the realization of the device in each case, and can enforce that outcome. Given the state of technology z, state the planner's problem using the notation that a realization of the lottery which entails the agent should work (should not work) is denoted C^e (C^u).

   (b) Show that it is optimal to provide full insurance with respect to the realizations of the lottery.

   (c) Characterize how the fraction of people working and consumption depends on the technology shock z. Under what conditions are some people unemployed? More specifically, what parameterization is necessary for π to be a well-defined probability?

   (d) Now suppose that the planner cannot see the outcome of the lottery but can receive messages from each agent about its realization. Will the full insurance scheme work? Specifically, will all agents truthfully reveal their realization? Show why or why not.

   (e) What will a planner do when there is private information? State the planner's programming problem subject to incentive compatibility constraints which state that for each realization the household's utility must be higher when they report truthfully than when they lie. What are the implications for compensation of workers versus those who are unemployed? (Hint: Focus on the implications of the incentive compatibility constraints.)

2. This problem studies a neoclassical growth model with "time-to-build", as in the famous article by Kydland and Prescott (Econometrica, 1982) awarded the Nobel Prize in Economics. Specifically, suppose that it takes two time periods to build and install new capital (rather than one period as in the standard growth model). Let s_{2t} denote new investment projects initiated in period t; s_{2t} is a choice variable at all points in time. The stock of partially completed investment projects in period t (i.e., investment projects one period from completion in period t) is denoted s_{1t}. The stocks of partially completed and new investment projects are related by $s_{1,t+1} = s_{2t}$. The capital accumulation equation reads: $k_{t+1} = (1 - \delta)k_t + s_{1t}$, where k_t is the stock of completed projects.

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The resource constraint is: $c_t + i_t = F(k_t)$, where investment $i_t = (1-\phi)s_{1t} + \phi s_{2t}$ and $\phi \in [0,1]$. In other words, starting a new investment project of size s is a commitment to invest resources ϕs in the first stage (or period) of its construction and resources $(1-\phi)s$ in the second stage of its construction. Finally, assume that F has the standard properties.

(a) Formulate the Bellman equation for the social planning problem in this economy, assuming that a typical consumer's preferences are given by: $\sum_{t=0}^{\infty} \beta^t u(c_t)$, where u has the standard properties. Be clear about what the state variables are and what the choice variables are.

(b) Derive the first-order and envelope conditions for this problem.

(c) Under the assumptions that $F(k) = k^\alpha$ and $\phi = 1$, derive an expression for the steady-state capital stock in terms of the structural parameters.

3. Consider a real-business-cycle model with variable capital utilization. There is a representative consumer whose preferences are given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t),$$

where u is strictly increasing and strictly concave. The aggregate resource constraint reads: $c_t + x_t = f(k_t, n_t, z_t, h_t) \equiv exp(z_t)(h_t k_t)^\alpha n_t^{1-\alpha}$. The variable $h_t \geq 0$ measures the "utilization level" of machines, and it is a choice variable at all points in time. Capital accumulates according to:

$$k_{t+1} = (1 - \delta(h_t))k_t + x_t,$$

where the function $\delta(h_t)$ is given by:

$$\delta(h_t) = \delta_0 + \delta_1 \frac{h_t^w}{w}$$

The parameters δ_0 and δ_1 are positive and the parameter w is greater than 1. Thus, the depreciation rate of capital in period t is an increasing and convex function of the utilization level h_t.

Individuals do not value leisure and supply labor inelastically. Without loss of generality, set labor resources n_t equal to 1. The productivity variable z_t is stochastic and evolves according to:

$$z_{t+1} = \rho z_t + \epsilon_{t+1},$$

where $\{\epsilon_{t+1}\}_{t=0}^{\infty}$ is an independent and identically distributed sequence of shocks drawn from a $N(0, \sigma_\epsilon^2)$ distribution and $|\rho| < 1$.

(a) Carefully define a recursive competitive equilibrium for this economy. Assume that consumers own the factors of production and rent their services to firms in every period in competitive markets. (Hint: Let the firm's production function be $F(k_t, n_t, z_t, h_t) \equiv f(k_t, n_t, z_t, h_t) + (1 - \delta(h_t))k_t$.)

(b) What is the deterministic steady-state value of the aggregate capital stock in the competitive equilibrium of this economy? (You need to find an equation that determines the steady-state capital stock in terms of primitives, but you do not have to solve it.)

(c) Explain how to use linearization methods to obtain an approximation to the stochastic behavior of the competitive equilibrium. You do not have to carry out any explicit computations, but you should provide a careful, detailed description of how to perform the required computations.

4. Consider a stochastic neoclassical growth model with the following structure:

- Each consumer has preferences of the form:

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left(c_t + B\frac{(1-n_t)^{1-\nu}-1}{1-\nu}\right)^{1-\sigma} - 1}{1-\sigma}$$

  where labor supply $n_t < 1$.

- The economy's technology is described by:

$$c_t + k_{t+1} - (1-\delta)kt = z_t k_t^{\alpha} n_t^{1-\alpha}$$

  where $log(z_t)$ is stochastic and evolves according to a stationary AR(1) process:

$$z_{t+1} = \rho z_r^{\rho} \epsilon_{t+1}$$

  with $log(\epsilon_t) \sim iid \quad N(0, \sigma_\epsilon^2)$

(a) Derive the Euler equation for the savings decision.

(b) Derive the first-order condition for the labor-leisure decision.

(c) Show that the first-order condition for the labor-leisure decision can be written as a simple function relating $log(1-n)$ to $log(w)$, where w is the wage rate.

(d) Use the following facts from the hypothetical economy Pekrland to calibrate all of the model's parameters except ρ and σ_ϵ^2:

   i. The average value of the capital-output ratio (in annual terms) is 2.

   ii. Pekrlanders work (on average) one-half of their total available time.

   iii. Experimental evidence on Pekrlanders' attitudes towards risk shows that they all have a coefficient of relative risk aversion equal to 2.

   iv. Capital's share of income is one-third.

   v. The average value of the investment-to-output ratio is 0.2.

   vi. Labor economists in Pekrland have found that, in a log-log regression of a typical Perklander's hours of leisure on the wage, the coefficient on the log of the wage is ?0.5: if wages go up by 1%, a typical Pekrlander works 0.5% more hours. You do not need to compute all of the parameters numerically, but you do need to describe how you would compute them.