HW1: Utility Function

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1.The class of HARA utility functions is given by

$$U(c) = \frac{1-\mu}{\mu}\left(\frac{\alpha c}{1-\mu} + \omega\right)^{\mu}$$

where α, ω, μ are parameters.

a) Show that the following utility functions are special cases of HARA, that is, show that for particular choices of the parameters α, ω, μ the HARA utility function becomes the

(a) Linear utility function $U(c) = ac$ where $a > 0$ is a parameter.

(b) Quadratic utility function $U(c) = -\frac{1}{2}(c - \bar{c})^2$ where $\bar{c} > 0$ is the so called bliss point and a parameter.

(c) CRRA utility function (note that this function diers from the CRRA specification in the question above just by a constant $\frac{-1}{1-\sigma}$)

$$U(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

where $\sigma > 0$; $\sigma \neq 1$ is the coecient of relative risk aversion and a parameter.

(d) CARA utility function

$$U(c) = -e^{-\gamma c}$$

where $\gamma > 0$ is the coeficient of absolute risk aversion and a parameter.

b) Show that for a particular choice of α, ω, μ a utility function equivalent to the HARA utility function equals log-utility $U(c) = ln(c)$. Two utility functions U and V are said to be equivalent if

$$V(c) = gU(c) + h$$

for some $g > 0, h$.

c) Show that the HARA utility function has a coecient of absolute risk aversion of the form

$$\gamma(c) = \frac{-U''(c)}{U'(c)} = \frac{1}{dc + e}$$

where d; e are functions of the parameters α, ω, μ This property gives the HARA utility function is name, as the coecient of absolute risk aversion for this class of utility functions is characterized by a generalized hyperbola.

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d) Give necessary and sucient conditions for the parameters α, ω, μ so that the HARA utility function has

   (a) Constant relative risk aversion

   (b) Constant absolute risk aversion.