Oil Extraction and Drilling

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Extraction of exhaustible resources: Hotelling 1931, Devarjan & Fisher 1981, Pyndick 1982

Ordering: Chakravorty, Roumasset, Tse. "Endogenous Substitution among Energy Resources and Global Warming", Journal of Political Economy, 1997

Drilling and Exploration: Hendricks, Kenneth, and Robert H. Porter. "The timing and incidence of exploratory drilling on offshore wildcat tracts." The American Economic Review (1996)

Introduction

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Introduction

- Hotelling (1931) had two purposes:
  - to assess policy debates arising out of the conservation movement
  - to develop a dynamic theory of natural resources
- Established "Hotelling rule": price of exhaustible resource must grow at the rate equal to the rate of interest

$$p_t = p_0 e^{r^t}$$

Introduction

- He showes competitive resource owner depletes at a socially optimal rate
- So, conservatives pleas for public intervention are not justified.
- Intuition: the present value of a unit extracted must be the same in all periods if there is to be no gain from shifting extraction among periods.
- Therefore, undiscounted prices must grow at the rate of interest

Model,1931

- Demand is $q = f(p, t)$
- if T time of final exhaustion $\int_0^T q dt = \int_0^T f(p_0 e^r t, t) dt = 0$
- So, the nature of solution depends on $f(p, t)$
- In competition, because of arbitrage, $p = p_0 e^{rt}$

State Interference

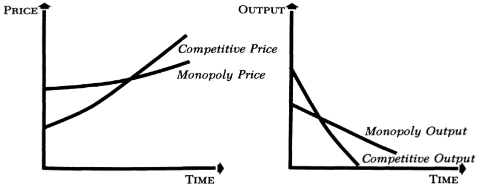
- Maximizing "total utility": $u(q) = \int_0^q p(q)dq$
- Discount "future enjoyment": $V = \int_0^T u[q(t)]e^{-rt}dt$
- Since $\int_0^T adt$ is fixed, so it should change unit increment in q
- F.o.c.: $\frac{d}{dq}u[q(t)]e^{-rt}$ should be constant
- By $u(q) = \int_0^q p(q)dq$,so pe^{-rt} should be constant.
- Thus, the same result as competition and $p = p_0 e^{rt}$

Extensions

- Other questions in the same framework:
  - Monopoly
  - Extraction costs that increase with cumulative production
  - Demand influenced by cumulative production of durables like gold and diamonds
  - Fixed investments (like mine)
  - A severance tax and a tax on the value of a mine
  - Uncertainty in demand and supply.

Monopoly and Rate of Depletion

- How does monopoly affect price and output paths?
- Marginal revenue (not necessary prices) grows at the rate of interest.
- Monopoly: price will rise less rapidly; depletion will be retarded



Monopoly and Rate of Depletion

- Hotelling shows
- Resources exhausted in finite periods under competition if demand be finite in zero price.
- Monopolist exhausts resources in finite time if marginal revenues is finite as quantity approaches zero.
- So, likely monopolist exhaust resources in infinite time if MR goes to infinity at $Q = 0$.
- Hotelling numerical examples also indicates monopolist exhausts resources in a longer period.

Monopoly and Rate of Depletion

- Recent research show it depends on demand curve.
- If elasticity is decreasing as quantity increases, monopolist depletes more slowly
- If demands shift overtime and becoming more elastic, the same results.
  - benefits from early inelastic demands and restricts output.
  - otherwise the opposite is true.
- What is OPEC: oligopolistic or a dominant seller and a competitive fringe. (next topic we will cover OPE topics)

Monopoly and Rate of Depletion

- The monopoly problem $y = pq$:

$$max \quad J = \int_0^\infty qp(q)e^{-rt}dt$$
$$s.t. \quad \int_0^\infty qdt = a$$

- calculus of variation $e^{-rt}\frac{d}{dq}(qp) - \lambda = 0$ and $e^{-rt}\frac{d^2}{dq^2}(pq) < 0$
- So, $y' = \frac{d}{dq}(pq) = p + q\frac{dp}{dq} = \lambda e^{rt}$
- compared with competitive case, we have additional term $q\frac{dp}{dq}$
- Also need to find T, and Lagrangian will be $\lambda(T, a)$,other boundary condition $q = 0$ for $t = T$

Example

- if p be finite value K as q approaches zero.(qdp/dq is finite):

$$\frac{d(pq)}{dq} = Ke^{r(t-T)}$$

- Demand function:

$$p = (1 - e^{-Kq})/q = K - K^2q/2 + K^3q^2/6 - \cdots$$

- So q approches zero, p approaches K
- With $y = pq = I - e^{-Kq}$ then $y' = Ke^{-Kq} = \lambda e^{rt}$ so

$$q = (Log(K/\lambda) - rt)/K$$

Example

- putting $q = 0$ for $t = T$ then $\log(K/\lambda) = rT$

$$a = \int_0^T (log(K/\lambda) - rt)dt/K = r\int_0^T (T - t)dt/K = rT^2/2K$$

- So: $T = \sqrt{2Ka/r}$ and $log(K/\lambda) = \sqrt{2Kra}$
- Therefore, $q = r(\sqrt{2Ka/r} - t)/K$
- The optimal decision was: $y'(p) = p(q) + qp'(q)$
- So, duration of monopoly is finite or infinite according as y' is finite or infinite when $q \to 0$

Effects of cumulative Production

- Hotelling:owner's profit depends on production, and amount of cumulative production (or stock remaining in the ground)

  1. extraction costs increase as the mine goes deeper
  2. demand for resources and like gold and diamond (durables) affected by cumulative stock in circulation

- How model: "net price" (average profit) function of cumulative production.

- Recent papers model cost and demand as a function of past production.

Effects of cumulative Production

- The price rise at the rate of interest less the percentage increase in cost caused by adding to the stock of cumulative production
- Pricing must consider increase in extraction cost in deferring the extraction.

Effects of cumulative Production

- Lets first Hotelling modeling: p is a function of x (amount extracted) so $q = dx/dt$
- Value of mine $\int_0^\infty p(x,q,t)qe^{-rt}dt = \int_0^\infty f(x,q,t)dt$
- Monopoly problem $\frac{df}{dx} - \frac{d}{dt}\frac{df}{dq} = 0$
- Second order in x because $q = dx/dt$ so two boundary conditions $x = 0$ for $t = 0$ and "Transversality Condition"
- TC:$f - q\frac{\partial f}{\partial q} = 0$ or $q^2\frac{\partial p}{\partial q} = 0$

Example

- Assume $p = \alpha - \beta q - cx + gt$
- The difference question would be:

$$2\beta \frac{d^2x}{dt^2} - 2\beta r \frac{dx}{dt} - crx = -grt + g - \alpha r$$

- Roots of auxiliary equation are real and of opposite signs. (m positive and $-n$ negative root)
- Because $m - n = r$ then

$$x = Ae^{mt} + Be^{-nt} + gt/c - 2\beta g/c^2 - g/cr + \alpha/c$$

$$q = Ame^{mt} + Bne^{-nt} + g/c$$

- You can use boundary conditions and solve it. (your homework to do the math)

Extension

- One important extension is depreciation by Pindyck & Levhari
- Demand $D(Q) = f(Q)y(t)$ where Q stock of resource and $y(t) = e^{\alpha t}$ constant proportional growth
- Cost of 1 unit of resource stock $= rp - \dot{p} + \delta p$
- Equates cost with marginal value provides differential equation for price:

$$\dot{p} = -f(Q)e^{\alpha t} + (r - \delta)p$$

Extension

- Producer problem with the assumption $C''(q) > 0$

$$max_q \quad \int_0^\infty [pq - C(q)]e^{-rt}dt$$

$$s.t. \quad \dot{Q} = q - \delta Q \quad Q(0) = 0 \quad \dot{X} = q \quad X \leq X_0 \quad q \geq 0$$

$$\dot{p} = -f(Q)e^{\alpha t} + (r - \delta)p$$

- Solution in the form $H = pqe^{-rt} - C(q)e^{-rt} + \lambda q$
- **U-shaped** results: the competitive market price falls initially as the stock in circulation increases, and later rise as the stock decreases

Uncertainty

- Hotelling raised the question of the impact of uncertainty on depletion rate.
- But, he just focused on uncertainty in exploration. Who find reserves exclude competitors from access
- So, excessive levels of exploratory activity
- Other aspect, knowledge of oil discovery in a track give information for the neighbor track.
- deficient investment on exploration.

Uncertainty

- Two categories: uncertainty in demand and supply
- Uncertainty in estimates of reserves:depletes in slower rates (Kemp 1976)
  - conservative depletion policy to avoid running out of resources unexpectedly
- Costly exploration+ Poisson discovery process $\rightarrow$ cyclical price (Arrow, Chang 78)
- Uncertain future price, less uncertain near future $\rightarrow$ faster depletion (Weinstein, Zechhauser 75)
- Pyndick 1980: two sources of uncertainty
  - future demand
  - total reserve
- Results: Hotelling r-percent rule applies

Model

- Market demand: $p = p(q,t) = y(t)f(q)$, $f'(q) < 0$
- $y(t)$ stochastic: $\frac{dy}{y} = \alpha dt + \sigma_1 dz_1 = \alpha dt + \sigma_1 \varepsilon_1(t)\sqrt{dt}$
- $\varepsilon_1(t)$ serially uncorrelated normal random variable with zero mean and unit variance (dz_1 Wiener process)
- Uncertainty over demand grows with time horizon+continuous
- Reserve: $dR = -qdt + \sigma_2 dz_2 = -qdt + \sigma_2 \varepsilon_2(t)\sqrt{dt}$
- q rate of production.
- Effective reserve initially expected: $R_e = \int_0^T q(t)dt = R_0 + \sigma_2 \int_0^T dz_2$
- R_e random variable with mean R_0 and variance $\sigma_2^2 T$, notice because of demand uncertainty T is also random
- R_0: volume of resource that could be produced today if there were no capacity constraints on the rate of production.
- R_0 will update over time.

Recall Random Process

- Limiting form as $h \to 0$ of discrete-time difference equation $[y(t+h) - y(t)]/y(t) = \alpha h + \sigma_2 \varepsilon_1(t)\sqrt{h}$
- Then $E(dy/y) = \alpha dt$
- and $var(dy/y) = \sigma_1^2 dt$
- $y(t)$ is lognormally distributed, with $E_0[log y(t)/y(0)] = (\alpha - 1/2\sigma_1^2)t$
- and $var\{log[y(t)/y(0)]\} = \sigma_1^2 t$
- Expected value of demand remains stationary if $\alpha = \sigma_1^2/2$
- Positive deterministic drift if $\alpha > \sigma_1^2/2$

Producer Problem

- Production problem

$$max_q E_0 \int_0^T [y(t)f(q) - C_1(R)]qe^{-rt}dt = E_0 \int_0^T \Pi_d(t)dt$$

- subject to stochastic process for demand and reserve
- subject to $R \geq 0$ and $t = T$ when $\Pi_d(t)/q = 0$
- $C_1(R)$ average production cost
- Competitive: then $f(q) = \overline{f}$ exogenous
- Monopoly: $f(q)$ function of his own production
- Important assumption: independence $E[\varepsilon_1(t)\varepsilon_2(t)] = 0$

Solution of the Model

- Stochastic dynamic programming

$$J = J(y, R, t) = maxE_t \int_t^T \Pi_d(\tau) d\tau$$

- Optimality condition

$$\begin{aligned} 0 &= max_q[\Pi_d(t) + (1/dt)E_t dJ] \\ & max_q[\Pi_d(t) + J_t - qJ_R + \alpha y J_y + \frac{1}{2}\sigma_1^2 y^2 J_{yy} + \frac{1}{2}\sigma_2^2 J_{RR}] \end{aligned}$$

- Therefore F.O.C. w.r q is $\partial\Pi_d/\partial q = J_R$

Solution of the Model

- Replace F.O.C. intro Bellman equation gives:

$$\frac{\partial \Pi_d}{\partial R} + J_{Rt} - qJ_{RR} + \alpha y J_{Ry} + \frac{1}{2}\sigma_1^2 y^2 J_{Ryy} + \frac{1}{2}\sigma_2^2 y^2 J_{RRR} = 0$$

- Which can also written as:

$$\frac{\partial \Pi_d}{\partial R} + (1/dt)E_t d(J_R) = 0$$

- Take differential operator $(1/dt)E_t d()$

$$(1/dt)E_t d(\partial \Pi_d/\partial q) = (1/dt)E_t d(J_R)$$

- combine above two equations, Euler Equation

$$(1/dt)E_t d(\partial \Pi_d/\partial q) = -\frac{\partial \Pi_d}{\partial R}$$

Solution of the Model

- Rewrite this in general form (use $\Pi_d[T] = 0$)

$$\frac{\partial \Pi_d(t)}{\partial q} = -E_t \int_t^T \frac{\partial \Pi_d(tau)}{\partial R} d\tau$$

- Marginal profit from selling one unit of reserves should just equal the expected sum of all discounted future increases in profit that would result from holding that unit in the ground (thereby reducing production costs).

Solution of Competitive Model

- Consider competitive: $\frac{\partial \Pi_d}{\partial q} = \frac{\Pi_d}{q} = [p(q,t) - C_1(R)]e^{-rt}$
- If substitute:

$$-r[p - C_1(R)] + (1/dt)E_t dp - (1/dt)E_t dC_1(R) = -(\partial \Pi_d / \partial R)e^{rt}$$
$$= qC_1'(R)$$

- Use Ito's lemma

$$dC_1(R) = C_1'(R)dR + 1/2C_2''(R)(dR)^2$$

- Substitute this, and $E_t(dR) = -qdt$,$E_t[(dR)^2] = \sigma_2^2 dt$

$$(1/dt)E_t dp = r[p - C_1(R)] + \frac{1}{2}\sigma_2^2 C_1''(R)$$

Solution of Monopoly Model

- Monopoly: $\frac{\partial \Pi_d}{\partial q} = [MR - C_1(R)]e^{-rt}$

$$(1/dt)E_t dMR = r[MR - C_1(R)] + \frac{1}{2}\sigma_2^2 C_1''(R)$$

- Social optimal: (consider only reserve uncertainty)

$$max_q E_0 \int_0^T [u(q) - C_1(R)q]e^{-rt}dt = E_0 \int_0^T U_d(t)dt$$

- define $u'(q) = \phi$

$$(1/dt)E_t d\phi = r[\phi - C_1(R)] + \frac{1}{2}\sigma_2^2 C_1''(R)$$

The Effects of Uncertainty

- Price stochastic
- Expected rate of competitive price differs from certainty only if $C_1(R)$ nonlinear
- Suppose $C''(R) > 0$, because of convexity fluctuations that cancel out each other in expected value, have increase extraction costs.
- Producers speed up the rate of production
- So by equations, relative to fixed reserve case, price begins lower and rises more rapidly.
- Constant average production cost (linear in R), expected rate of change of price is as in certainty

Another Extension- Exploration to Reduce Uncertainty

- Decision:production q and exploratory activity ω

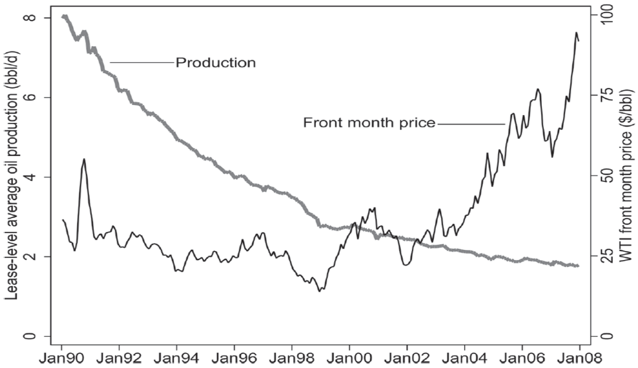
$$\max_{q,\omega} E_0 \int_0^T [p(q)q - C_1(R)q - C_2(\omega)]e^{-rt}dt = E_0 \int_0^T \Pi_d(t)dt$$

- s.t. $dR = -qdt + \sigma(K)dz$ and $dK = g(\omega)dt$
- K stock of knowledge that is produced by exploratory activity

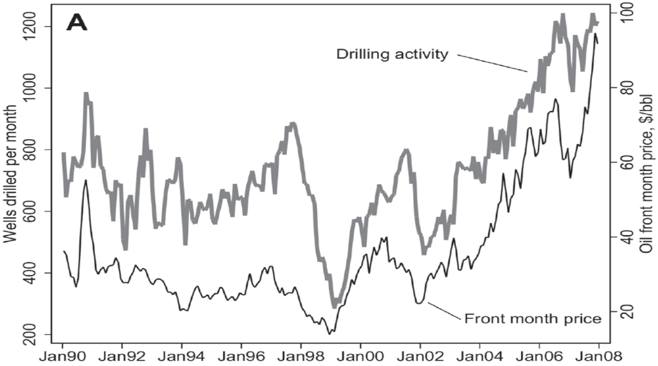
Empirical Test of Hotelling-Anderson et al. 2018, JPE

- Its mapping into empirical work is limited
- Focused on testing the "Hotelling rule" that resource prices (values) should rise at the rate of interest often finding that the rule fails to hold
- This paper using micro facts and show that
  - Production from existing wells declines asymptotically toward zero & unresponsive to oil price shocks (intensive margin)
  - Inconsistent with Hotelling prediction that freely allocate extraction across different periods without constraint
  - Drilling of new oil wells & rental price of drilling rigs respond strongly to oil price shocks (extensive margin)

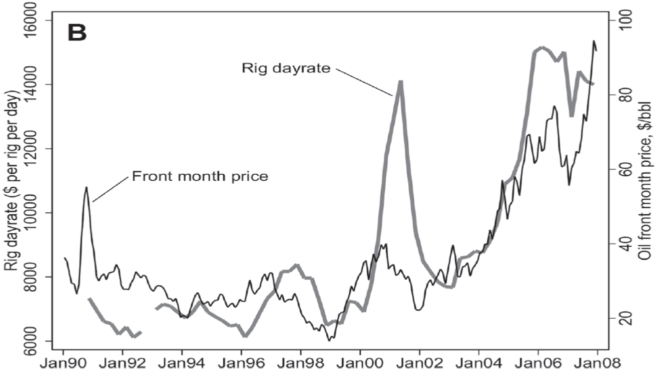
Crude oil prices and production fromexisting wells inTexas



Number of new wells drilled versus crude oil spot prices



Dayrates for rigs versus crude oil spot prices



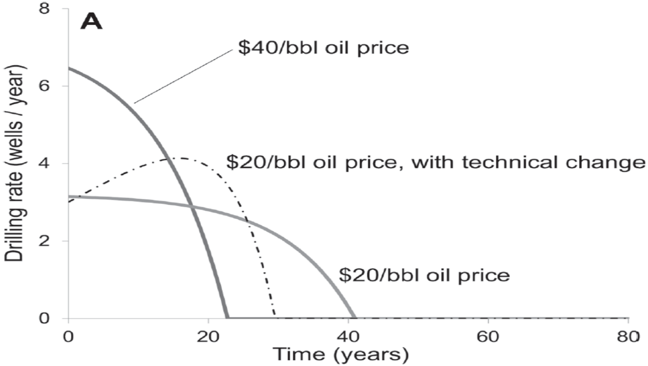
Recasting Hotelling as a Drilling Problem-Planner problem

- Objectve: $\max_{F(t),a(t)} \int_{t=0}^{\infty} e^{-rt}[U(F(t)) - D(a(t))]dt$
- Subject to: $0 \leq F(t) \leq K(t)$ (F: rate of oil flow, K: capacity)
- $\dot{R}(t) = -a(t)$ (a: rate new wells are drilled, R:wells that remain untapped is state variable)
- $\dot{K}(t) = a(t)X - \lambda F(t), a(t) \geq 0, R(t) \geq 0$
- $D(a(t))$ drilling cost at rate a, X maximum flow from a newly drilled well
- Hamiltonian of the planner's maximization problem

  $$H = U(F(t)) - D(a(t)) + \theta(t)[a(t)X - \lambda F(t)] + \gamma(t)[-a(t)] + \phi(t)[K(t) - F(t)]$$

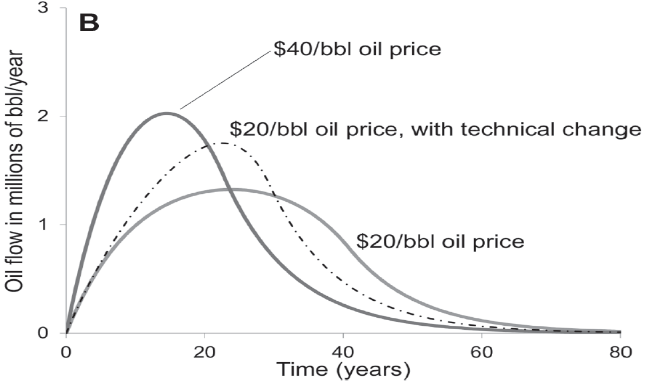
- $\theta(t)$ and $\gamma(t)$ are the constant variables on the two state variables $K(t)$ and $R(t)$, and $\phi(t)$ is the shadow cost of the oil flow capacity constraint

Drilling rates, model simulation after estimation with Texas data



Reserve is fixed,

Oil production, model simulation, pick oil production



Conclusion

- Production from preexisting wells steadily declines over time and does not respond to oil price shocks, even when the oil price is anticipated to rise (temporarily) faster than the rate of interest
- Drilling of new wells and drilling rig rental rates strongly covary with oil prices
- Local oil-producing regions and fields exhibit production peaks
- Steady technological progress in a local region can cause the rates of drilling and production to both steadily increase
- Following an unanticipated positive demand shock, the oil price will jump up on impact but can then gradually fall

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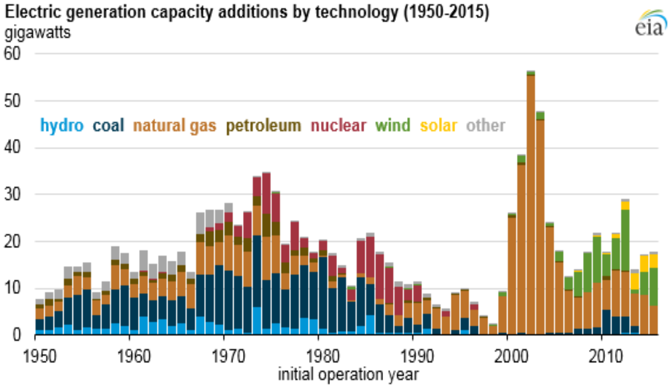
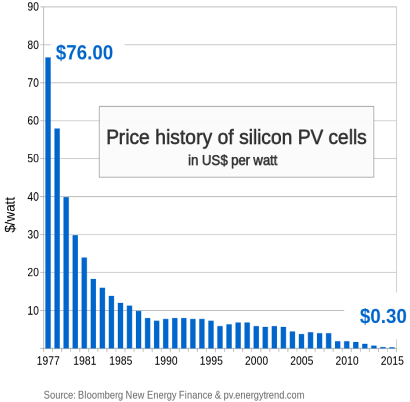
Electric generation capacity additions



- Declining price of gas
- Federal tax credit, states mandates to generate renewable.
  - due to tax credit and no fuel, wind drive price negative off-peak

Swanson's law (founder of SunPower Corporation)

- Solar cell prices fall 20% for every doubling of capacity



- In 2015, A new PPP agreement at 3.87 cents/kWh levelised

Herfindahl (1967)

- Extended the Hotelling model by considering many resources with different unit costs of extraction
- "least cost first" principle: extraction must be ordered by cost, with the cheapest resource used first.
- Extensions of Herfindal:
  - General equilibrium setting (Kemp, Long 1980; Lewis 1982)
  - Presence of setup costs (Gaudet, Moreaux, Salant 2001)
  - Heterogeneous demands (Chakravorty, Krulce 1994)
  - Extraction rate is constrained (Amigues et al. 1998; Holland 2003).

Introduction-Paper

- We come back to topic of global warming later (add taxation)
- This paper because a nice application of Hotelling modeling and show you how to estimate a long-run effect in energy
- Up to this paper, theory of resource extraction focused on extraction of a single good with homogeneous demand for it.
- This paper: simultaneous extraction of different resources (oil, gas, ...) and multiple demands (transportation, residential, ..)
- Estimate reserves and energy demand data for world economy.

Introduction

- Global warming ignore price-induced energy conservation
  - endogenous substitution between alternative energy sources
  - cost-saving improvements in extraction technology
  - rapidly declining cost of solar-powered electricity generation
- This paper
  - a multiple-resource
  - multiple-demand framework
  - a simulation model that yields the optimal extraction path
- Results: if historical rates of cost reduction in the production of solar energy are maintained, more than 90 percent of the world's coal will never be used.
- Global temperatures will rise by only about 1.5-2 degrees centigrade by the middle of the next century and then decline steadily to preindustrial levels, even without carbon taxes.

Literature-Extraction

- Resource extraction: mostly on a theory of resource extraction in which there is a single, homogeneous demand for the resource.(Hotelling 1931; Dasgupta and Heal 1974)
- Ordering: examine the order of extraction of different grades of an exhaustible resource when there is a single demand function for the resource. (Solow and Wan 1976; Kemp and Long 1980)
- Nordhaus (1973) divided the energy sector into transportation, residential/commercial heating, industrial heating, and electricity sectors.
  - limited to examining the optimal path of resource extraction and its associated resource prices in the developed world and in the Middle East

Literature-Global Warming

- Using a top-down growth-theoretic framework.
- Assume a certain exogenous relationship between growth in gross domestic product (GDP) and the level of greenhouse gas emissions (Nordhaus 1991, Peck and Teisberg 1992)
- Exception: a model accounts for the economy-wide impacts of rising energy costs. (Manne, Mendelsohn, and Richels 1993)
  - considers alternative sources of energy supply
  - production is a fixed fraction of remaining reserves
  - resource prices are fixed exogenously.
- This paper use this framework in the global market to simulate the effects of technological change in reductions in the cost of the backstop technology on resource extraction and global warming.

Model

- I resources (e.g., oil, coal, natural gas, and solar energy)
  $i = 1, \cdots, I$
- J energy demand sectors (e.g., electricity, industrial heating, residential/commercial heating, and transportation)
  $j = 1, \cdots, J$
- Except for solar energy, all the other I resources are exhaustible.
- Solar energy is a backstop technology, denoted by b

Model

- Each energy sector faces a downward-sloping demand $D_j(.)$
- The process of conversion from a resource to an end use (demand) involves heat loss,
- $v_{ij} \in [0, 1]$ efficiency of conversion of resource i into demand j
- $q_{ij}(t)$ extraction of resource i for use in demand j
- $d_{ij}(t)$ as the net or delivered energy of resource i into demand j from $q_{ij}(t)$

$$d_{ij}(t) = v_{ij}.q_{ij}(t)$$

Model

- The aggregate stock of resource i at time t is given by $Q_i(t)$.
- Cost of energy is sum of extraction and conversion
- c_i marginal cost of extraction of resource i.
- z_{ij} The conversion cost of resource i into end use j (amortized capital + operation + maintenance)
  - Capital: a gasoline car for conversion of oil into transportation
- Efficiency factor v_{ij} and conversion cost z_{ij} are resource and demand specific: $I \times J$ matrix.
- Extraction cost of the backstop technology is zero
- Positive conversion costs to each end use given by $z_{bj}, j = 1, \cdots, J$

Model

- Discount rate: r
- Sum of conversion and extraction costs: $\omega_{ij} = c_i + z_{ij}$
- Maximize discounted producer plus consumer surplus:

$$\max_{d_{ij}(t)} \int_0^{\infty} e^{-rt} \left[\sum_{j=1}^{J} \int_0^{\sum_{i=1}^{I} d_{ij}(t)} D_j^{-1}(\theta) d\theta - \sum_{j=1}^{J} \sum_{i=1}^{I} \frac{\omega_{ij}}{v_{ij}} d_{ij}(t) \right] dt$$

$$s.t. \quad \dot{Q_i}(t) = - \sum_{j=1}^{J} \frac{d_{ij}(t)}{v_{ij}} \quad given \quad z_{bj}$$

- $q_{ij}(t)$substituted by using $d_{ij}(t) = v_{ij}.q_{ij}(t)$

Model

- Hamiltonian

$$H = \sum_{j=1}^{J} \int_{0}^{\sum_{i=1}^{I} d_{ij}(t)} D_j^{-1}(\theta) d\theta - \sum_{j=1}^{J} \sum_{i=1}^{I} \frac{\omega_{ij}}{v_{ij}} d_{ij}(t) - \sum_{i=1}^{I} \lambda_i(t) \sum_{j=1}^{J} \frac{d_{ij}(t)}{v_{ij}}$$

- Price of end use j is $P_j(t) \equiv D_j^{-1}\left[\sum_{i=1}^{I} d_{ij}(t)\right]$

Model

- With necessary conditions:
- F.o.c d_{ij}: $P_j(t) \leq \frac{\omega_{ij}+\lambda_i(t)}{v_{ij}} \quad i = 1, \cdots, I \quad j = 1, \cdots, J$
  if $<$ then $d_{ij}(t) = 0$
  - determines which resources are being used for which demand
  - the price of the resource must be equal to the efficiency-adjusted sum of the extraction cost, the conversion cost of that resource for that demand, and the scarcity rent of the resource.
  - the left-hand side is the marginal benefit and the right-hand side the marginal cost.

Model

- Constraint:
  $\dot{Q_i}(t) = -\sum_{j=1}^{J} \frac{d_{ij}(t)}{v_{ij}} \quad i = 1, \cdots, I \quad j = 1, \cdots, J$
  - The stock of any resource will be depleted by the quantity extracted aggregated over all demands.
- Hamiltonian: $\dot{\lambda}_i(t) = r\lambda_i(t) \quad i = 1, \cdots, I$
  - Hotelling's rule (the scarcity rent for an exhaustible resource will rise at the rate of interest) holds for each of I resources
  - All grow by r, so the relative order of scarcity rents for the different resources is determined by their values at the initial time period

Model

- $P_j(T_j) = \frac{z_{bj}}{v_{bj}} \quad j = 1, \cdots, J$
  where T_j switch points for transition to the backstop fuel for demand sector j.
  - usual condition for transition to the backstop technology
  - the price in each demand sector j is exactly equated to the backstop price at the endogenously determined time T_j
- Difficult to draw precise analytical conclusions on patterns of resource extraction
- Chakravorty and Krulce (1994) obtain a solution for the simple case $(I = 2,\ J = 2)$
- This paper solve numerically

Demand and Supply Parameters

- As Nordhaus (1979): four demand sectors ("specific" electricity, industry, residential/commercial, transportation)
- "Specific" electricity: include sectors indirectly met by conversion to electricity
- Energy resources are oil, coal, natural gas
- Other resources (nuclear, hydro, geothermal, wind energy) not included; instead netted out of the demand for petrochemical and solar energy sources.

Demand and Supply Parameters

- Solar energy (backstop technology) with zero extraction costs but with nonzero costs of conversion
- Another backstop candidate is nuclear fusion (Furth 1995)
- Use solar because nuclear is still at an experimental stage whereas solar energy is already commercially viable
- Various technologies convert solar energy: biomass, wind turbines, solar-powered heat engines, and photovoltaic cells.
- Paper considers only photovoltaic technology, because "application of modern manufacturing techniques is expected to bring down the cost to less than 10 cents per KW early in the next century (Hoagland 1995)".
- Commercialization by building a 100-megawatt plant in 1997 (Enron and Amoco)

Demand Equations

- Cobb-Douglas sectoral annual demand functions

$$D_j = A_j P_j^{\alpha_j} Y^{\beta_j}$$

- α_j and β_j the price and income elasticities of demand
- P_j is the price of delivered energy service j
- Y is the aggregate income or output level(GDP of the world economy)

Demand Equations

- The empirical model is formulated as a discrete-time model.
- Each time period to be L years
- the annual growth rate of GDP (g_t) is constant within each $L - year$ time period t.

$$
\begin{aligned}
D_{jt} = & A_j P_{jt}^{\alpha_j} \left(\frac{Y_0}{1+g_1} \right)^{\beta_j} \\
& \times \left[(1+g_t)^{\beta_j} + (1+g_t)^{2\beta_j} + \cdots + (1+g_t)^{L\beta_j} \right] \\
& \times \left[(1+g_1)(1+g_2)\cdots(1+g_{t-1}) \right]^{L\beta_j}
\end{aligned}
$$

Demand Equations

- Define

$$\begin{aligned}\gamma_{jt} =& A_j \left(\frac{Y_0}{1+g_1}\right)^{\beta_j} \\ &\times \left[(1+g_t)^{\beta_j} + (1+g_t)^{2\beta_j} + \cdots + (1+g_t)^{L\beta_j}\right] \\ &\times \left[(1+g_1)(1+g_2)\cdots(1+g_{t-1})\right]^{L\beta_j}\end{aligned}$$

- Aggregation implies that P_j, is constant within each $L-year$ time period

$$P_{jt} = \left(\frac{D_{jt}}{\gamma_{jt}}\right)^{1/\alpha_j}$$

- This inverse demand function can be substituted into the maximization problem

Demand Equations

- For α_j and β_j from Nordhaus (1979)

Final Demand Sector	Price Elasticity α_i	Income Elasticity β_i
Electricity	−.65	.92
Industry	−.52	.76
Residential/commercial	−.79	1.08
Transportation	−1.28	.81

Demand Equations

- A_j is computed from world GDP, energy consumption, and the prices of energy resources for a particular base year, using demand function
- The year 1990 is chosen as the base year, for which $Y_0 = \$20,209$ billion and world aggregate date:

Energy Resource	Price ($/mmBtu)	World Consumption (Billion mmBtu)
Petroleum	3.73	108.04
Coal	2.08	41.67
Natural gas	2.35	42.56
Electricity	...	22.67

Energy Balance 1990

- Disaggregate based on OECD shares information.

	Residential/ Commercial*	Industry	Transportation	Electricity[†]
		B. World (Billion mmBtu) **Consumption**		
Oil	18.02	20.71	69.31	12.89
Coal	5.15	36.50	.022	46.22
Natural gas	22.58	19.96	.027	16.55
Total	45.75	77.17	69.359	75.66
Multiply by efficiency factor:		C. World Delivered Energy (Billion mmBtu)		
Oil	14.41	14.50	20.79	4.07
percentage:	(39.95)	(26.84)	(99.93)	(17.95)
Coal	3.60	25.55	.0056	13.78
	(9.99)	(47.3)	(.03)	(60.79)
Natural gas	18.02	13.97	.0082	4.82
	(50.06)	(25.86)	(.04)	(21.26)
Total (D_{ij})	36.03	54.02	20.8038	22.67

Demand Equations

- Compute the price of delivered energy from the prices of the fuels using the weights derived from panel B.

	Residential/ Commercial	Industry	Transportation	Electricity
Oil	\$7.26	\$2.14	\$74.86	\$3.44
Coal	\$2.27	\$5.71	\$0.03	\$10.87
Natural gas	\$5.12	\$1.42	\$0.03	\$3.18
Weighted price (P_{ij})	\$14.64	\$9.27	\$74.93	\$17.49

- Demand equations:

$$electricity : D_1 = 0.015927 P_1^{-0.65} Y^{0.92}$$

$$industry : D_2 = 0.091866 P_2^{-0.52} Y^{0.76}$$

$$residential/commercial : D_3 = 0.006730 P_3^{-0.79} Y^{1.08}$$

$$transportation : D_4 = 1.699235 P_4^{-1.28} Y^{0.81}$$

Conversion Efficiency

- No good documentation.
- Simplify calibration by choosing representative activities for each sector
  - Light-duty vehicles for transportation
  - Stove heating for the residential/commercial sector
  - Industrial process heating for the industrial sector
  - Electricity generation for "specific" electricity.
- From various sources efficiency values

	Residential/ Commercial	Industry	Transportation	Electricity
Oil	.8	.7	.3	.3157
Coal	.7	.7	.25	.2983
Gas	.8	.7	.3	.2913
Solar	.1275	.1275	.1275	.15

Conversion Costs

- The conversion cost of a resource into an end use per unit of delivered energy:

$$conversion \quad cost \quad \left(\frac{z_{ij}}{v_{ij}}\right) = \frac{annualized \quad capital \quad cost + operation + maintenance \quad cost}{energy \quad consumption \times efficiency \quad factor(v_{ij})}$$

- Where

$$annualized \quad capital \quad cost = K\frac{s(1+s)^m}{(1+s)^m - 1}$$

- K is the total capital cost of a conversion technology, s is the rate of interest, and m is the lifetime of the capital stock.

Conversion Costs

- Examples of how does compute conversion costs
  - Residential/commercial sector
    - Oil: stove heating usually use liquefied petroleum gas (LPG) as fuel (end use)
    - cost of converting oil to LPG by the price difference between natural gas and LPG.
    - Coal: cost of coal gasification as conversion cost.
    - Solar: first converting solar energy into electricity then cost of transforming electricity into a specific end use.
- The matrix of conversion costs:

	Residential/ Commercial	Industry	Transportation	Electricity
Oil	$13.50	$2.64	$62.48	$7.35
Coal	$19.71	$9.10	$107.74	$10.90
Gas	$7.29	$2.13	$72.32	$6.87
Solar:				
Electricity generation	$86.12	$86.12	$86.12	$73.20
Other costs	$10.43	$1.59	$126.95	$0.00
Total	$96.55	$87.71	$213.07	$73.20

Extraction Costs

- Estimate the continuous extraction cost equations functions of cumulative extraction using historic data:

$$oil : c_{oi}(t) = 0.1774e^{0.000217Q_{oil}(t)} \quad R^2 = .960$$

$$coal : c_{coal}(t) = 0.2827e^{0.00000743Q_{coal}(t)} \quad R^2 = .997$$

$$gas : c_{gas}(t) = e^{0.8908-[3264.7/Q_{gas}(t)]} \quad R^2 = .992$$

- For computational simplicity changes to step function:

Resource	Grade I	Grade II	Grade III
Gas	\$0.92 (6,683.98)		
Oil	\$0.60 (11,242.67)	\$3.47 (4,916.13)	
Coal	\$0.65 (225,622.35)	\$2.37 (121,354.2)	\$5.08 (82,068.59)

Growth of GDP

- Demand for energy to increase over time because of the growth in income
- Asia-Pacific region alone are growing at the rate of 5-6% per year,world average is closer to 2 %
- Global GDP growth rates for 1965-80 and 1980-90 were 4.0 and 3.2 %
- Paper assumed growth in 1990 at 3.0 percent, decreasing at the rate of 10 % every decade.
- Annual rate of discount of 2% over all periods

Simulation

- The programming that guesses the six scarcity rents (three grades for coal, two for oil, one for natural gas) in the initial time period.
- Solar energy is available in infinite supply, so its scarcity rent is zero.
- Since scarcity rents rise at the rate of interest, their paths are completely determined by the initial guesses.
- Resources are allocated to each demand at any instant of time by comparing their prices and choosing the one with the least price.

Counterfactuals

- **The Baseline Model (BASE)**
  - the model is run with all the parameters fixed over time.
- How does technological Change affect long run?
  - New horizontal oil-drilling techniques enlarged the stock of resources that can be extracted at any given cost
  - combined cycle technology in natural gas engines made it competitive relative to oil
- This paper: R & D that decrease in the cost of solar energy conversion to electricity.
- Ahmed 1994: the cost of solar electricity from 25 ¢/kWh ($73.2 /mmBtu) to 4 ¢/kWh in three decades.

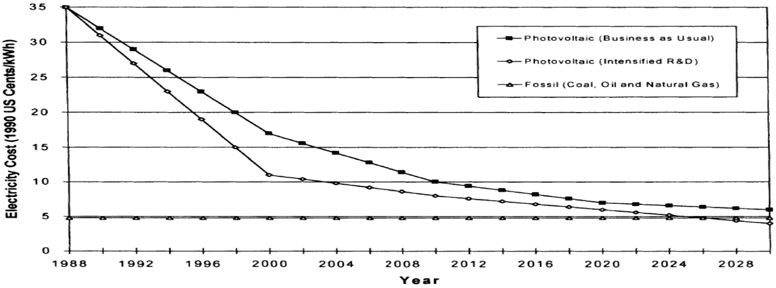
Counterfactuals

- There are two issues that determine future projected costs of energy technologies.
  1. size or the rate of growth of the market
  2. the extent of R & D support.

- Cody and Tiedje (1992): cost of photovoltaic electricity from 40¢/kWh in 1988 to 7¢-12¢/kWh in 2010, because size of market

Counterfactuals

- Department of Energy (1990): because of R & D:



- Two scenarios: business-as-usual and intensified R & D
- Paper sets a lower bound for the conversion cost of solar energy at 20 ¢/kWh

Counterfactuals

- **Decreasing Cost of Solar Energy (DCSE50)**
- Intensified R & D case, based on Ahmed
- Costs solar energy would decrease at an approximate rate of 50 percent per decade
- Conversion cost would drop to reach 4¢per /kWh in about four decades.

Counterfactuals

- **Decreasing Cost of Solar Energy (DCSE30)**
- A "pessimistic" business-as-usual scenario: 30% reduction in costs per decade
- DCSE50 and DCSE30 assume reductions in conversion costs involve only the solar energy to electricity cost component.
- The cost of electricity to end use cost has no reason to decrease
- For the transportation sector (the electric car)
  - 40% of the total conversion cost fixed;
  - 60% is expected to decrease at the rate of 50% per decade
  - Electric car will cost the same as gasoline car in 30 years.

Counterfactuals

- **An Across-the-Board Decrease in Conversion Costs (DCC)**
- All conversion costs decrease at an equal rate over time.
- Conversion costs for each resource decrease to 40% of their present levels, at the rate of 50% per decade.
- The cost of solar energy is expected to decline as in DCSE50.

Counterfactuals

- **DCC with a Carbon Tax (DCCT100 and DCCT200)**
- Most carbon tax experiments choose tax to achieve a given carbon emission target at each time period.
- For instance, annual reductions of 1-2% from the 1990 base.
- Tax rates vary by geographical region and increase over time.
- Tax rates varying between $20 per ton of carbon in the initial years to $2,000 per ton in future periods
- Paper: the effect of a flat tax of $100 per ton and $200 per ton of carbon on the DCC case.
- A flat tax of $100 per ton of carbon raise coal prices by $70 per ton or 300%, increase oil prices by $8 per barrel.

Counterfactuals

- **DCSE30 with a Carbon Tax of $100 per Ton (DCSE30T100)**
- A carbon tax of $100 per ton is imposed on DCSE30.
- **BASE with Carbon Taxes of $100 per Ton and $200 per Ton (BASET100 and BASET200)**
- BASE model is run with uniform carbon taxes to examine the impact of taxation under a worst-case scenario of no technological change.

Energy Resource Use-Base

BASE					DCSE50					Carbon Emissions (Billion Tons)	
Period	Electricity	Transportation	Residential	Industrial	Period	Electricity	Transportation	Residential	Industrial	Year	BASE
1	Coal	Oil	Gas	Oil	1	Coal	Oil	Gas	Oil	1995	7.1691
2	Coal	Oil	Gas	Oil	2	Coal	Oil	Gas	Oil	2005	8.8716
3	Coal	Oil	Gas	**Oil**	3	Coal	Oil	Gas	Oil	2015	10.6948
4	Coal	Oil	Gas	**Coal**	4	**Coal**	**Oil**	Gas	Oil	2025	13.3516
5	Coal	Oil	Gas	Coal	5	**Solar**	**Solar**	Gas	Oil	2035	15.6003
6	Coal	Oil	Gas	Coal	6	Solar	Solar	**Gas**	Oil	2045	17.8627
7	Coal	Oil	**Gas/Coal**	Coal	7	Solar	Solar	**Solar**	**Oil**	2055	21.7682
8	Coal	Oil	Coal	Coal	8	Solar	Solar	Solar	**Solar**	2065	25.2134
9	Coal	Oil	Coal	Coal	9	Solar	Solar	Solar	Solar	2075	27.9959
10	Coal	**Oil/Coal**	Coal	Coal	:	:	:	:	:	2085	32.0235
11	Coal	Coal	Coal	Coal	:	:	:	:	:	2095	35.458
12	Coal	Coal	Coal	Coal	19	Solar	Solar	Solar	Solar	2105	38.2686
:	:	:	:	:	20	Solar	Solar	Solar	Solar	2115	40.8508
:	:	:	:	:						2125	43.1531
28	Coal	Coal	Coal	Coal						2135	45.129
29	**Coal**	Coal	Coal	Coal						2145	46.7385
30	**Solar**	Coal	Coal	Coal						2155	47.9489
31	Solar	**Coal**	Coal	Coal						2165	48.7361
32	Solar	**Solar**	Coal	Coal						2175	49.0859
33	Solar	Solar	Coal	Coal						2185	48.0842
34	Solar	Solar	Coal	Coal						2195	46.6957
35	Solar	Solar	**Coal**	Coal						2205	46.4315
36	Solar	Solar	**Solar**	Coal						2215	45.7866
37	Solar	Solar	Solar	**Solar**						2225	44.7728
:	:	:	:	:						2235	43.4116
:	:	:	:	:						2245	41.7336
53	Solar	Solar	Solar	Solar						2255	39.7551
54	Solar	Solar	Solar	Solar						2265	38.1977
55	Solar	Solar	Solar	Solar						2275	36.4222

370 years to move completely to solar energy

Results-BASE

- Electricity sector: coal is used exclusively until the transition to the backstop.
- Transportation sector: relies on oil until it is exhausted, then moves to coal.
- Industry: oil is used in for a short period, followed by coal.
- Residential/commercial heating: Natural gas is exclusively used, replaced by coal on exhaustion.
- Comparative advantages of oil in transportation, gas in heating, coal in electricity and industry

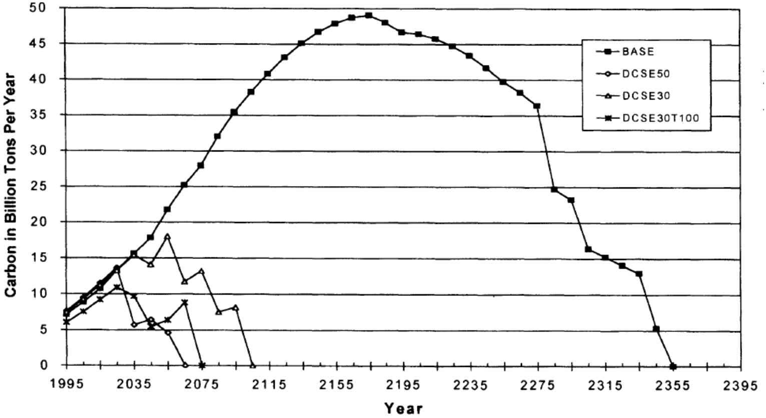
Results-BASE

- BASE emissions peak in the year 2175 (49 billion tons) and then decline
- Fossil fuels gets successively exhausted and is replaced by solar energy
- Because of the slow atmospheric absorption of greenhouse gases, global temperatures continue to rise by a maximum of 6 degrees until the year 2275
- Abundant coal reserves emerge as a backstop when oil is exhausted

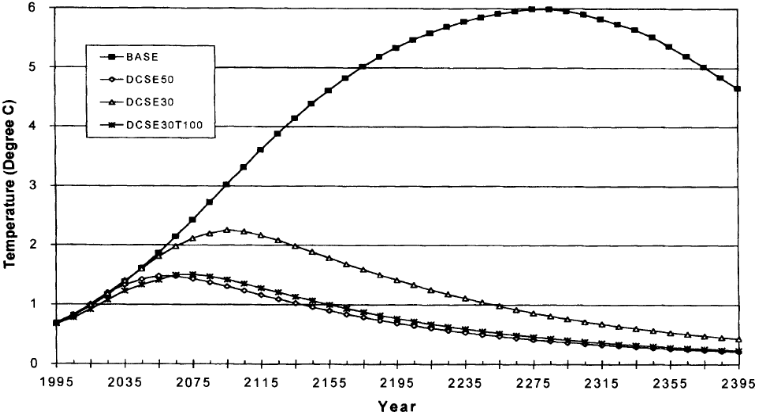
Results-DCSE50

- Rapid technological change in solar energy (DCSE50)
- Interesting specialization of resources: coal in electricity generation, oil in transportation and industrial, natural gas in the residential sector.
- Direct transition from oil to solar energy in transportation and industry.
- Carbon emissions peak around 2025 at 13 billion tons, and temperature rises by 1.5 degrees and declines after 2055
- Under these rates of reductions in solar energy costs, the global mean temperature bounces back to the 1995 level in the year 2195.

Worldwide Carbon Emission-4 Scenarios



Change in Global Mean Temprature-4 Scenarios



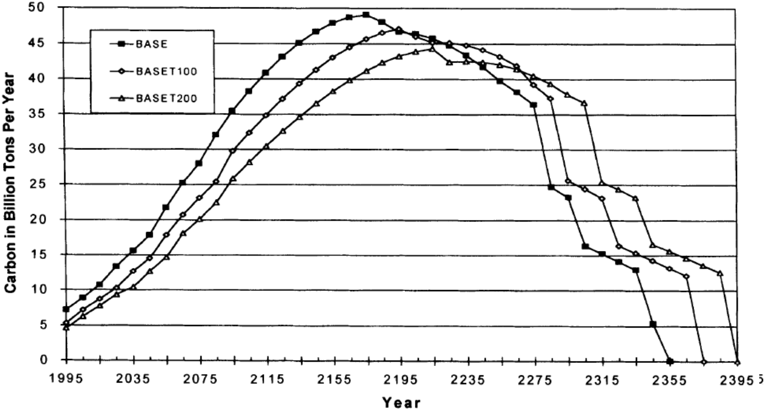
Results-DCSE30

- With more "conservative" estimates of technological change (DCSE30)
- Carbon emissions peak in 2055, that is, 20 years later than in DCSE50
- Solar energy takes over in all sectors by 2105, 40 years late compared to the optimistic case.
- Temperature peaks in 2095, and it takes 320 years to return to the 1995 level (100 years in DCSE50)
- The maximum level of aggregate emissions is about 18 billion tons reached in the year 2055.

Results-DCSE30T100

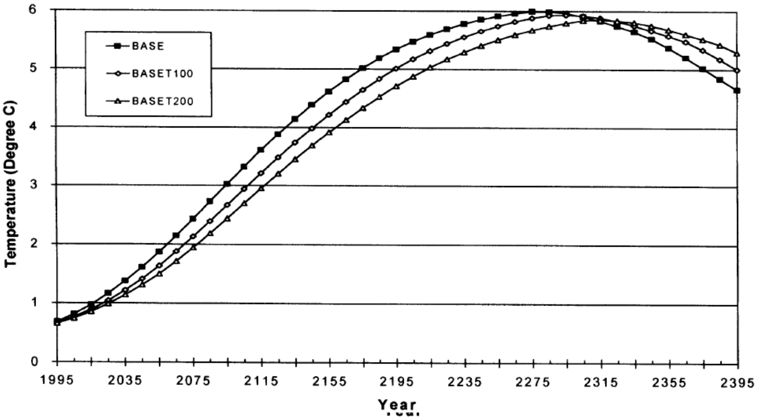
- Conservative technological change + uniform carbon tax of \$100 per ton (DCSE30TO00) will be similar to the case of rapid technological change (DCSE50).
- Of course the growth and distributional implications of such a tax, which implies that coal and oil prices would go up by \$70 per ton and \$8 per barrel, respectively, may be serious.
- A carbon tax of \$100 per ton in the United States alone is expected to raise nearly \$200 billion

Emission: Carbon Tax+BASE Model



Therefore flat carbon taxes will only postpone global warming and reduce it somewhat. Need a more complex tax structure.

Temperature: Carbon Tax+BASE Model



Resource Use

- Under BASE, all resources are consumed.
- Under DCSE30, only 8% of the world's estimated coal reserves are exhausted.
- Under DCSE50, only 1.5% of the coal is used.
- Oil and natural gas, however, are both completely exhausted in all three situations.

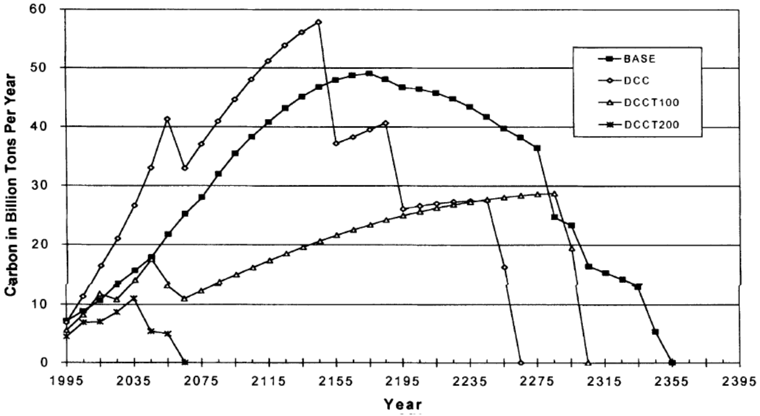
Loss in World GDP

- Loss in GDP from a rise in temperature using a relationship given by Nordhaus (1992).
- Maximum percentages of world GDP loss within the first 100 years are 0.32 %, 0.74 %, and 1.3 %, respectively, for the three models DCSE50, DCSE30, and BASE.
- Beyond this 100-year horizon, the annual GDP loss will continue to rise only for the BASE model (with peak at 5.2%) in the year 2285.

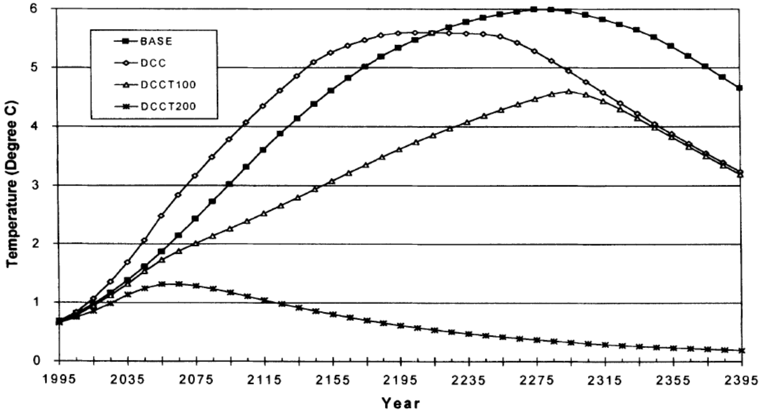
Results-DCC

- DCC (across-the-board reduction in all conversion costs) carbon emissions is higher than BASE over the next 150 years.
- The maximum level of aggregate emissions reached is also higher (58 billion tons)
- A reduction in all conversion costs reduces the price of energy, increasing energy consumption in immediate future.
- Emissions drop abruptly as fossil fuels are exhausted and each sector moves to solar energy.
- Carbon taxes of $200 per ton reduce emissions substantially.
- So, magnitude of cost reductions in the backstop technology relative to fossil fuels, matters

Emission: DCC



Temperature: DCC



Heterogeneity in Polluting

- Another important paper is
  Chakravorty, Ujjayant, Michel Moreaux, and Mabel Tidball.
  "Ordering the extraction of polluting nonrenewable resources."
  The American Economic Review 98.3 (2008): 1128-1144

- In earlier studies resources were differentiated by cost alone.

- In this paper, resources are differentiated only by their pollution characteristics, which affect their extraction.

- Two resources, one more polluting than the other.

- Environmental regulation is imposed through a cap on the stock of pollution (Kyoto Protocol)

Result-Two Polluting Resources

- When the economy is already at its allowable stock of pollution, the clean natural gas is used first and use of the dirty coal is postponed to the future.
- When the economy starts from below the ceiling and accumulates pollution, coal may be used first and use of the clean natural gas is postponed.
- The optimal strategy is to benefit from natural dilution by building the stock of pollution as quickly as possible.
- Only when natural gas is abundant is it used before coal.
- "preference reversal": coal may be used for a period of time, then natural gas, and finally coal for another time period.

Hotelling Model with Two Polluting Resource

- Utility $u(.)$, two non-renewable resources $i = 1, 2$

$$\max_{x_1, x_2, y} \int_0^{\infty} \{u\left(\sum_i x_i + y\right) - c_r y\} e^{-rt} dt$$

- θ_i pollution generated unit of the resource i, and X_i^0 its given initial stock.($\dot{X}_i = -x_i$)
- Abundant renewable nonpolluting backstop resource with unit cost $c_r > 0$ with rate of extraction y
- Aggregate stock of pollution denoted by $Z(t)$ and natural decay $\alpha > 0$ then $\dot{Z} = \sum_i \theta_i x_i - \alpha Z$
- Regulatory level $\overline{Z}$

Ordering the Extraction, Why Important?

- Why is this topic important for Iran?
- What if you have multiple resources, multiple demand market and restrictions to export!
- Iran example: natural gas, oil, power; domestic market versus exports

Table of Content

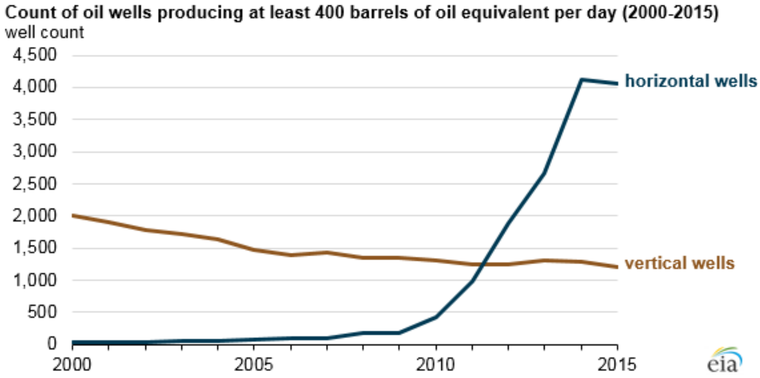
Extraction of exhaustible resources: Hotelling 1931, Devarjan & Fisher 1981, Pyndick 1982

Ordering: Chakravorty, Roumasset, Tse. “Endogenous Substitution among Energy Resources and Global Warming”, Journal of Political Economy, 1997

Drilling and Exploration: Hendricks, Kenneth, and Robert H. Porter. "The timing and incidence of exploratory drilling on offshore wildcat tracts." The American Economic Review (1996)

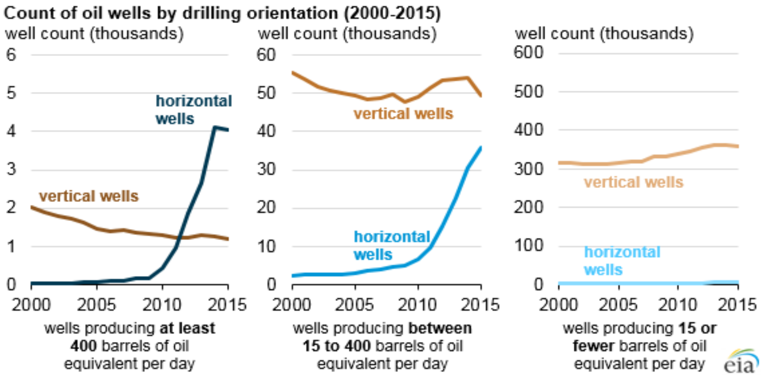
Drilling Technology

- Lets before we study paper look at how well horizontal drilling compared with vertical



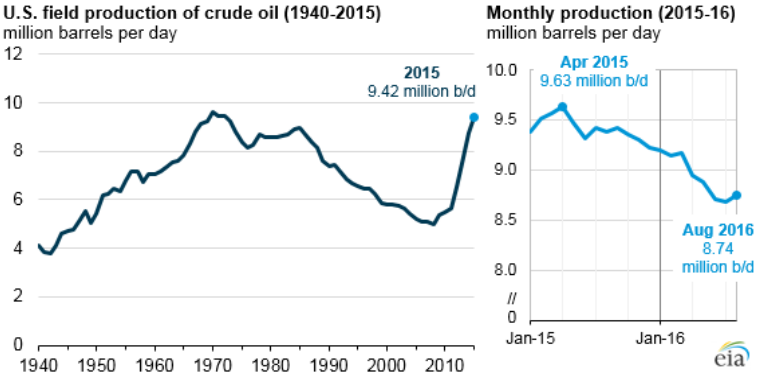
Drilling Technology

- Lets before we study paper look at how well horizontal drilling compared with vertical



Drilling Technology

- Probably one of the most important element in high oil production in US



Introduction

- This paper is an empirical study of learning and strategic delay in exploratory drilling.
- Data: drilling on federal land off the coasts of Texas and Louisiana between 1954 and 1990
- U.S. federal government sell the oil and gas rights to thousands of parcels (tracks) of its offshore land.
- Tracts each cover an area of five thousand acres on average.
- The typical sale involves more than a hundred tracts.

Introduction

- Most sales are wildcat sales (tracts with no prior exploratory drilling)
- Firms have access only to seismic information and as a consequence face considerable uncertainty.
- The rights to the tracts are sold individually using a first-price sealed-bid auction, and ownership of tracts in an area is typically distributed among several firms.
- Tracks within an area often share common geological features, and a subset may be located over a common pool.
- Ex post value of nearby leases will be correlated, information externality associated with exploratory drilling.

Introduction

- Information externalities generates a free-rider problem.
- Most leases have a relatively small number of neighbors in an area of information spillover
- When their owners must decide whether to initiate drilling.
- The lease term, limited to the exploration phase, is 5 years.
- If no exploratory drilling by the end of the lease term, ownership of the lease reverts to the government.

Introduction

- The exploration decision is costly,millions of dollars.
- Uncertainty: only half of the tracts that were explored yielded positive revenues (commercial)
- Revenues are variable: standard deviation of logarithm of discounted revenues on productive tracts is approximately 1.5
- As a result, information is valuable and incentive to delay its drilling

Paper Contribution

- Firm's decision of when to drill its lease(s) as a game of timing (war of attrition). (Hendricks, Kovenock [1989])
  - Firms with marginal leases prefer to wait and learn more about drilling outcomes of other leases in the area.
  - If all waits, better to drill earlier, avoid time cost of delay.
  - Equilibrium: a probability distribution over drilling times. (high probability of both leases being drilled in the last period)
- This paper:evidence for war of attrition model
  - Number of tracts drilled and the hazard rate declining by number of quarters that the lease has been held
  - In the last quarters both rates increase dramatically.
  - U-shaped pattern over the term of the lease.
  - Result suggests that firms behaved noncooperatively.
  - If coordinate: drilling would have ended earlier

Paper Contribution

- Initially, a tract is more likely to be explored the more the lease owner bid to acquire it
- But as time progresses, bid levels are decreasingly accurate predictors of whether drilling will be initiated.
- Instead, firms increasingly reliant on the information generated by post-sale drilling activity in the local geographic area.
- If lease holdings in an area are relatively asymmetric across firms, drilling is less likely to be delayed.
- These results are consistent with noncooperative behavior.

Literature

- Hendricks et al. (1987), tracts off the coasts of Texas and Louisiana, document that 29% of wildcat leases expired without any wells being drilled.
- The Probit results indicate that the decision to abandon tracts is rational using date-of-sale and post-sale information.
- Literature on social learning: Bolton, Christopher[1993] infinite-horizon games, N gamblers, each owning identical machines, have to decide individually in each period whether to play the machine with unknown payoff.
- The gamblers obtain information about machine by observing the outcome of their own plays and other gamblers.
- Exploratory drilling is a finite version of this game.

Literature

- Empirical work on the effect of the free rider problem (information externality) on equilibrium rates of experimentation is almost nonexistent.
- An exception is the recent work on the adoption of a new variety of high yielding cotton by Besley, Case (1994).

Theory

- Prior to a lease sale, firms can only conduct seismic surveys.
- Seismic is noisy signals about the likelihood of finding oil and gas on the tracts.
- Firms use this information to determine whether and how much to bid for individual leases
- All bids have to be submitted by a certain date, bid and bidders are public information afterwards.
- Each firm can use the bidding information to update its beliefs to making any drilling decisions.
- As drilling outcomes on nearby leases become public information, undrilled firms will revise their beliefs.

Model

- Log Size of the deposit (X) is a random draw from a normal distribution with mean e^{θ}, precision h.
- These parameters are fixed within an area but can vary across areas.
- The firms know $h(=1)$ but not θ
- They learn about θ through surveys and drilling.
- A firm must pay fixed costs c to initiate a drilling program

Model

- Size of the deposit has to exceed some minimum level to be worth developing.
- Value leases($\pi(x)$)= present value of revenues net of royalty payments and the costs of developmental
- Lease terms are of length T periods, discount factor β
- Net present value of a lease that is drilled $= \beta^t(\pi(x) - c)$
- Bids from the sale provide information on the sizes of the deposits.

Model

- N = number of tracts receiving bids in the area.
- Sale information relevant for lease $i = s_i \sim N(x_i, 1/\tau_i^2)$
- Surveys are less informative than drilling outcomes ($\tau_i < 1$)
- Firms use the signals to update their beliefs about θ and deposit sizes.

Model

- Bayes rule for density function of each firm's beliefs θ is normal with mean μ, precision ρ

$$\mu = \sum_{i=1}^{N} s_i \tau_i (1+\tau_i)^{-1}/\rho$$

$$\rho = \sum_{i=1}^{N} \tau_i/(1+\tau_i)$$

- The firms' beliefs about x_i conditional on $(S_1, cdots, S_N)$ are described by a normal distribution with mean $(\mu + \tau_i s_i)/(\tau_i + 1)$ precision $\rho(\tau_1+1)^2/[\rho(\tau_1+1)+1]$
- Suppose leases are ordered by their signals and the k highest signal leases are drilled with outcomes $(x_1, \cdots, x_k)$. Let x denote the average discovery size.

Results

- **LEMMA 1:** Conditional on $(x_i, ..., x_k, s_1, \cdots, s_N)$, beliefs about θ are given by a normal distribution with precision $\rho_k = k + \sum_{i=k+1}^{N} \tau_i/(1+\tau_i)$ and mean

$$\mu_k = \left[k\bar{x} + \sum_{i=k+1}^{N} s_i \tau_i (1+\tau_i)^{-1} \right] / \rho_k$$

- Posterior mean of θ is a weighted average of average discovery size and the sum of appropriately weighted signals on the tracts that have not been drilled.
- Once x_i is known, s_i is redundant information.

Results

- **LEMMA 2:** Conditional on $(x_1, ..., x_k, s_1, \cdots, s_N)$, beliefs about $X_i, i = k+1, \cdots, N$ are normal with precision $\rho_k(1+\tau_i)^2/[1+\rho_k(1+\tau_i)]$ and mean $(\mu_k + \tau_i s_i)/(l + \tau_i)$.
- Drilling outcomes across leases are not perfectly correlated.
- Leases are heterogenous (differ not only in expected deposit size but also in their informational value)
- Drilling outcomes on leases with low precision generate more information about θ than outcomes on leases with high precision
- Expected lease values increase with average discovery size.

Model to Data

- These features do not require normality.
- Exploration histories possess sufficient statistics that are easily constructed from data:
  - μ_k weighted average of discovery sizes
  - ρ_k the number of wells drilled in the area
  - τ_ii can be measured by the number of bids submitted on lease i

A War of Attrition

- Firm has to decide when to drill its lease.
- Leases with very optimistic signals are drilled immediately.
- Costs of delay dominant probably of new signal to do otherwise.
- Leases with NPV< 0 wait, but their option value of new information may justify drilling later.
- Most firms made strategic decision to drill: NPV> 0 but not enough to drill immediately, wait to observe others' outcome.
- Not sure when other will drill

A War of Attrition

- Assume only 2 leases
- A (behavioral) strategy for each firm specifies the probability of drilling each period as a function of the state of the world
- Other firm not-drilled: the same belief
- Drilled and found a deposit of size x, so updates θ using Bayes rule
- Then, drill immediately if the expected value of the lease is positive and to let the lease expire otherwise.
- Subgame perfect equilibrium

A War of Attrition

- (μ, ρ) state of information θ. Expected value:

$$v_i(\mu, \rho) = \int (\pi(x_i) - c) \phi(x_i; (\mu + \tau_i s_i)/(\tau_i + 1), \rho(\tau_i + 1)^2/(\rho(\tau_i + 1) + 1)) dx_i$$

- $\phi(, ; \theta, h)$ density of a normal distribution
- If no one drilled previously, the expected payoff to firm i from drilling leases: $V_i(\mu, \rho)$.

A War of Attrition

- If firm j drill at t, the state of information on θ changes to $(y, \rho + (\tau_j + 1)^{-1})$

$$y = [\mu\rho + (x_j - s_j\tau_j(1+\tau_j)^{-1})]/[\rho + (1 + tau_j)^{-1}]$$

- From firm i's perspective in period t (j outcome not observed yet)

$$W_i(\mu, \rho) = \int max[0, V_i(y, \rho + (\tau_i + 1)^{-1})] \times \phi(y; \mu, \rho + \rho^2(\tau_j + 1))dy$$

- Note that W_i is greater than V_i

A War of Attrition

- The game is solved recursively.
- In the last period, not any drilling yet, firm i drill if and only if $V_i > 0$
- $T-1$, no prior drilling, drilling payoff V_i
- If waits, its rival's probability of drilling is q_j^{T-1}, then the expected payoff is $\beta[q_J^{T-1} W_i + (1-q_j^{T-1}) max(0, V_i)]$.
- Firm i is indifferent between drilling and waiting iff
$$q_j^{T-1} = (1-\beta) max(0, V_i) / \beta(W_i - max(0, V_i)) = q_j^*$$
- $q_j^* < 1$ if $V_i > 0$ and is less than βW_i (war of attrition)

A War of Attrition

- War of attrition: the payoffs from letting the other firm drill first, exceed the payoffs from leading, and the latter declines with time.
- $q_j^* > 1$: gains from waiting are insufficient & drill immediately
- β close to 1 and s_i or τ_j is sufficiently large, the expected payoff to firm: V_i
- Suppose $V_i > 0$ and less than βW_i, for both leases. Then, in period $T-2$, the same as from waiting in period $T-1$,
- Hence, in equilibrium with same reason,$q_j^t = q_j^*$ for $t = 1, 2, \cdots, T-1$ and $q_j^T = 1$

A War of Attrition

- Mixed strategy equilibrium is delay and duplication.
- Leases may also be drilled simultaneously instead of sequentially
- If a single owner
  - at least one lease is always drilled if either $V_1 + \beta W_2 > 0$ or $V_2 + \beta W_1 > 0$
  - no leases are drilled $V_1 < 0$ and $V_2 < 0$

A War of Attrition

- How likely is this outcome for OCS (Outer Continental Shelf) leases?
- The amount of time required to initiate and complete a drilling program is 3 months.
- So, discount rate $\beta = 0.99$
- V_i is approximated average of discounted revenues less royalty payments and drilling costs $=$\$3.65 million (in 1972 \$)
- The difference between W_i and V_i is equal to drilling costs times the probability that the first well drilled in the area is a dry hole.

A War of Attrition

- The cost of an exploratory well offshore leases is about \$1.5 million and the hit rate is 1/2.
- Then $q^* = 0.05$. The length of the typical lease is 5 years or $T = 20$
- Thus probability that neither lease is drilled until the last period is 0.15.

A War of Attrition

- How does q^* vary with respect to the underlying parameters?
- Higher values of μ and s_i causes expectations about θ, x_i, V_i to increase,
- The gain from waiting, $W_i - V_i$, decreases
- Higher values of ρ means that firm i is more certain about the value of θ
- The effect of this reduction on V_i and $W_i - V_i$ depends on $\pi(x)$.

A War of Attrition

- As ρ gets large, the likelihood that x_j can move expectations about θ significantly goes to zero.
- A similar argument applies to τ_i
- In each case, the informational value of x_j becomes small and implies that firm i should not wait to drill its lease

A War of Attrition-Extensions

- The above calculations ignore heterogeneity & assume only 2 tracts
- firms possess private information + delay: signals that a firm is not very optimistic (Wilson (1984))
- Hendricks and Kovenock (1989) analyze a common value setting
- More tracts & lease holders: Firm i may wish to drill one of its leases in an early period in order to encourage subsequent drilling by other firms (Bolton and Harris)
- Asymmetries in number of leases lead to less delay (Hendricks and Porter [1993])

Empirical Implications

1. In the absence of coordination, drilling patterns on most leases should exhibit delay and duplication.
2. Tracts with very high signal values may be drilled immediately
3. Tracts with lower signal values are not likely to be drilled until the end of the lease tenure, if at all
4. There should also be a significant decrease in the quality of tracts drilled in the last period compared to preceding periods.
5. The pace of drilling activity should be higher in area-cohorts that are perceived to be more valuable

Drilling Behavior

- Wildcat sales of tracts off the coasts of Texas and Louisiana:
  - fixed bonus bids
  - royalty payments: 1/6 of revenues
  - the high bid was accepted
- Many of the tracts (usually half) do not receive any bids.
- There are 6,178 wildcat tracts in the full sample, which includes tracts sold from 1954 until March 21, 1990.

Data

- Data: wildcat tracts off the coasts of Texas and Louisiana that were auctioned between 1954 and 1979
- In the sample period, 2,255 tracts received bids.
- The mean winning bid on the
  - 602 unexplored tracts is \$2.86 million
  - 2,255 wildcat tracts is \$6.07 million.
- Abandonment of a tract, without conducting exploratory drilling, no lease after expiration
- Average drilling costs on the 897 unproductive tracts are \$1.52 million

Data

- Data :
  - the dates lease was put up for sale
  - its location and acreage
  - which firms bid and the value of their bids
  - the number and date of any wells that were drilled
  - monthly production of oil, gas, condensate, and miscellaneous through 1990 if any oil or gas was extracted.
- Other sources: compute drilling costs, ex post discounted revenues and costs for each tract

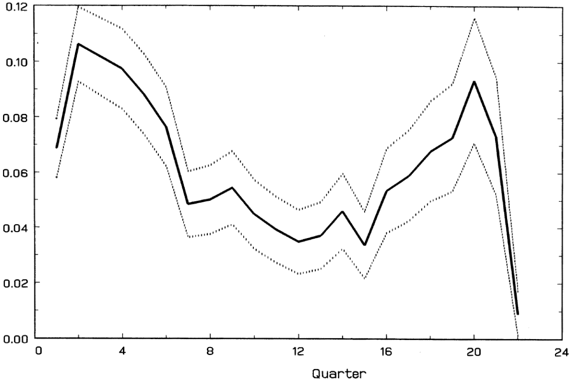
The Aggregate Hazard Rates

- Hazard rates from lease granted to first drilling.
- In 75 cases, exploratory drilling began after the 5-year lease horizon, paper classifies these tracts as being never drilled.
- U-shaped pattern in the number of tracts drilled in a given quarter
- Hazard rate declines monotonically until quarter 12, slowly increases after that, and then jumps up
- 24.3% of the 2,255 tracts were never explored
- Risk set$_t$ $=$ risk set$_{t-1}$-number drilled$_{t-1}$
- Hazard rate$_t$$=$ number drilled$_t$/risk set$_t$

Quarterly Hazard Rates 1954-1979

Quarter	Risk set	Number drilled	Hazard rate	Standard error
1	2,255	155	0.0687	0.0053
2	2,100	223	0.1062	0.0067
3	1,877	191	0.1018	0.0070
4	1,686	164	0.0973	0.0072
5	1,522	134	0.0880	0.0073
6	1,388	106	0.0764	0.0071
7	1,282	62	0.0484	0.0060
8	1,220	61	0.0500	0.0062
9	1,159	63	0.0544	0.0067
10	1,096	49	0.0447	0.0062
11	1,047	41	0.0392	0.0060
12	1,006	35	0.0348	0.0058
13	971	36	0.0371	0.0061
14	935	43	0.0460	0.0069
15	892	30	0.0336	0.0060
16	862	46	0.0534	0.0077
17	816	48	0.0588	0.0082
18	768	52	0.0677	0.0091
19	716	52	0.0726	0.0097
20	664	62	0.0934	0.0113
21	602	44	0.0731	0.0106
22	558	5	0.0090	0.0040
23	553	5	0.0090	0.0040
Never	2,255	548	0.2430	

Hazard Rates for Exploratory Drilling: 1954-1979



The Aggregate Hazard Rates

- If H_t, is the hazard rate, R_t:size of the risk set
- then the variance of the hazard is $H_t(1 - H_t)/R_t$
- The standard error of the difference in hazard rates over time can then be approximated by the square root of the sum of the individual variances.
- The increase in the hazard rate between quarters 19 and 20 is significant, with a t statistic of 2.34.

The Aggregate Hazard Rates

- Tracts with negative expected values should not be part of the risk set.
- Such tracts are likely to represent an increasing fraction of the risk set as positive value tracts are drilled and eliminated from the set.
- This implies that the empirical hazard rate should decrease in periods 2 through $T-1$, even ignoring heterogeneities across area-cohorts

Area-Cohort Hazard Rates and Heterogeneity

- Paper uses an exogenous classification provided by the government which divides the offshore region off the coasts of Texas and Louisiana into 51 separate geographical areas.
- All tracts within a given area are considered to be potential neighbors.

- Too broad a classification since the typical area contains hundreds of tracts.
- So, paper focuses on area-cohorts
- In any given sale, tracts in a particular area tend to be clustered.
- There are 270 area-cohorts with 8.35 tracts per area-cohort

Area-Cohort Hazard Rates and Heterogeneity

- Area-cohorts into three categories:
  - area-cohorts with 10 or fewer tracts
  - area-cohorts with 11 to 20 tracts
  - with more than 20 tracts.
- For each size category, next table reports a 6×6 matrix whose (i, j) element is the number of tracts
  - first drilled in lease year j
  - in area-cohorts abandoned in lease year i, or its track lest drill was year j.
- Hazard rate A: assumes that the risk set consists of all tracts not yet drilled.
- Hazard rate B: assumes that the risk set consists of all tracts not yet drilled in area-cohorts that are still active.

Area-Cohort Drilling Patterns

Panel A: Area-cohorts of size 1–10

Last year of drilling	Number of tracts drilled in lease year: Never	1	2	3	4	5	Total	Number of area-cohorts
Never	32						32	13
1	20	63					83	37
2	11	44	40				95	32
3	25	32	17	38			112	28
4	24	20	19	5	23		91	22
5	62	98	49	30	19	95	353	69
Total	174	257	125	73	42	95	766	201
Hazard rate A		0.33	0.25	0.19	0.14	0.35		
Hazard rate B		0.35	0.27	0.22	0.19	0.61		

Panel B: Area-cohorts of size 11–20

Last year of drilling	Number of tracts drilled in lease year: Never	1	2	3	4	5	Total	Number of area-cohorts
Never	14						14	1
1	0	0					0	0
2	2	19	3				24	2
3	5	18	5	5			33	2
4	17	11	7	4	4		43	3
5	81	142	84	36	43	94	480	31
Total	119	190	99	45	47	94	594	39
Hazard rate A		0.32	0.25	0.15	0.18	0.44		
Hazard rate B		0.33	0.25	0.16	0.20	0.54		

Area-Cohort Drilling Patterns

Panel C: Area-cohorts of size > 20

Last year of drilling	Number of tracts drilled in lease year: Never	1	2	3	4	5	Total	Number of area-cohorts
Never	22						22	1
1	0	0					0	0
2	29	23	17				69	3
3	9	19	15	3			46	2
4	6	21	2	1	2		32	1
5	168	223	105	66	64	100	726	23
Total	234	286	139	70	66	100	895	30
Hazard rate A		0.32	0.23	0.15	0.17	0.30		
Hazard rate B		0.33	0.24	0.17	0.19	0.37		

Area-Cohort Hazard Rates and Heterogeneity

- The empirical hazard functions are all U-shaped
- Drilling in most area-cohort of size greater than 10 did not end until year 5
- Because panel A (low number of tracks in an area-cohort) firms may be able to coordinate drilling plans in area-cohorts where the number of tracts is quite small
- 15 area-cohorts, mostly containing a small number of tracts, abandoned without any drilling.
- Probably purchased for their option value or concludes from low biding that are worthless

Area-Cohort Hazard Rates and Heterogeneity

- BID: mean of the logarithm of the high bid
- HIT: number of explored tracts where there was subsequent production
- REV: mean of the logarithm of discounted revenues on productive tracts (5% discount rate)
- BIDDIFI: difference between BID on tracts that were drilled, and the average level of BID on tracts in the risk set that were sold in the same year.
- BIDDIF2: difference between BID on tracts that were drilled, and the average level of BID on tracts in the risk set that belong to the same area-cohort

Track Characteristics by Year of Initial Drilling

	Year after acquisition				
Variable	1	2	3	4	5
Risk set:					
Number	2,255	1,522	1,159	971	816
BID	14.46	13.91	13.64	13.52	13.46
	(1.62)	(1.48)	(1.43)	(1.44)	(1.46)
Number of bids	3.76	2.87	2.51	2.41	2.30
	(3.26)	(2.47)	(2.15)	(2.13)	(2.06)
Tracts drilled:					
Number	733	363	188	155	214
(fraction)	(0.325)	(0.239)	(0.162)	(0.160)	(0.262)
BID	15.62	14.77	14.26	13.82	13.50
	(1.26)	(1.31)	(1.18)	(1.31)	(1.18)
BIDDIF1	0.757	0.687	0.553	0.464	0.148
	(0.041)	(0.059)	(0.079)	(0.096)	(0.075)
BIDDIF2	0.616	0.502	0.389	0.381	0.058
	(0.037)	(0.054)	(0.066)	(0.082)	(0.060)
Number of bids	5.62	4.00	3.06	2.99	2.33
	(3.86)	(3.03)	(2.22)	(2.37)	(1.75)
HIT	403	163	87	70	82
(fraction)	(0.550)	(0.449)	(0.463)	(0.452)	(0.383)
REV	16.29	15.52	15.55	15.52	15.22
	(1.54)	(1.64)	(1.71)	(1.96)	(1.55)

Area-Cohort Hazard Rates and Heterogeneity

- Tracts that were a priori judged to be more productive, as indicated by BID, were more likely to be drilled.
  - means of BIDDIFl and BIDDIF2 are significantly positive throughout.
- Hit rates, and deposit sizes conditional on a hit, fall over the lease term, and the decreases are largest after the first year and in the final year of the lease.
- Average bids on drilled tracts fall more than hit rates or average revenues
  - Ex post tract profits are increasing for the set of tracts that are drilled.
- Quality of tracts drilled in the last year of the lease is significantly lower (based on hit rate and REV)

Results

- Support the hypothesis of noncooperative behavior.
- Substantial delay and duplication in an area-cohort, with many firms waiting until the last period to drill.
- Heterogeneity across and within area-cohorts can explain the decreasing hazard function,
  - not increasing portion of hazard rate near end of lease term.
- Some tracts, prior expectation of profits is sufficiently high that they are drilled immediately.
- Area-cohorts with higher bid are drilled more rapidly
  - within an area-cohort, tracts with high bids tend to be drilled earlier than those with low bids.

Variables

- local drilling post-sale experience for the relevant area-cohort
  - DRPOST: total number of tracts explored since the sale date (=0 if no drilling)
  - HITPOST: logarithm of (1 plus) the number of drilled tracts that were productive
  - REVPOST: mean of the logarithm of discounted revenues on productive tracts (=0 if no hits)
- Information revealed at the sale date
  - BID: logarithm of the winning bid
  - ONEBID: (competition) a dummy variable that equals one if the winning bid was the only bid submitted
  - MLT: "money left on the table":logarithm of the ratio of the highest to the second highest bid. (if one bid, the announced reserve price is employed)

Variables

- area-cohort pre-drilling information
  - NRISKSET: logarithm of the number of tracts in the area-cohort not yet drilled in each year of the lease
  - AREABID: average value of BID for the tracts in the area-cohort risk set.
- HERF: a Herfindahl index of the dispersion of lease holdings among solo bidders in an area-cohort.
  - According to the strategic model, areacohorts with higher values of HERF should experience less delay.
- ACRE: logarithm of tract acreage
  - Some blocks that are ex ante believed to be more valuable are split into two tracts for the wildcat auction
  - exacerbate war of attrition problems
  - tracts are known to be productive, with smaller acreage are drilled right away

Variables

- Yearly dummy variables,
- REOFFER: dummy variable =1 if tract is being re-offered. (158 tracks)
  - the government previously rejected the high bid, (107)
  - or if a previous leaseholder relinquished the lease without drilling. (51)
- Other variables
  - number of submitted bids on a tract
  - winning bid is submitted by a consortium of firms
  - fraction of leases in an area-cohort that were acquired by joint bids.

Determinants of Drilling Activity

- The dependent variable is a dummy variable equaling 1 if exploratory drilling began in that year.
- In logarithms., estimates in probit
- BID is initially large and significant, but negative & insignificant by the final year.
- leaseholders do not respond to other bidding (MLT)
- Maybe this information anticipated by the winner when it submitted its bid, and hence included in BID.

Determinants of Drilling Activity

Variable	Year after acquisition 1	2	3	4	5
NRISKSET	−0.062	−0.212	−0.290	−0.201	−0.316
	(0.050)	(0.064)	(0.077)	(0.081)	(0.078)
AREABID	−0.148	−0.025	0.022	−0.134	0.087
	(0.061)	(0.076)	(0.095)	(0.100)	(0.097)
BID	0.564	0.437	0.155	0.236	−0.026
	(0.037)	(0.047)	(0.060)	(0.066)	(0.068)
MLT	−0.035	−0.103	−0.064	0.057	−0.113
	(0.039)	(0.060)	(0.077)	(0.092)	(0.106)
ONEBID	−0.142	0.230	−0.798	0.141	−0.576
	(0.211)	(0.209)	(0.241)	(0.248)	(0.241)
ONEBID × MLT	0.064	−0.049	0.264	−0.157	0.248
	(0.073)	(0.089)	(0.109)	(0.126)	(0.132)
HERF	0.067	−0.075	−0.286	−0.234	−0.224
	(0.206)	(0.231)	(0.290)	(0.296)	(0.294)
ACRE	−0.095	0.041	0.597	0.649	0.267
	(0.107)	(0.125)	(0.208)	(0.210)	(0.161)
REOFFER	0.126	0.046	0.466	−0.379	0.505
	(0.146)	(0.177)	(0.203)	(0.308)	(0.251)
DRPOST		0.236	−0.077	0.108	0.139
		(0.121)	(0.134)	(0.130)	(0.131)
HITPOST		0.188	0.292	−0.069	−0.023
		(0.102)	(0.121)	(0.125)	(0.128)
REVPOST		−0.017	0.007	0.029	0.042
		(0.009)	(0.012)	(0.013)	(0.014)
Sample size	2,255	1,522	1,159	971	816
Log-likelihood	−1,032.5	−692.0	−446.1	−377.3	−404.6

Determinants of Drilling Activity

- Firms are less likely to drill if AREABID is high. (other leaseholders are more likely to drill early)
- Final year, firms more likely to drill if AREABID is high
- In the first year of the lease term, the coefficient of HERF is positive, but not significant.
- The coefficient of DRPOST is positive and significant in the first year.
- Initially more drilling in areas with substantial post-sale activity

Determinants of Drilling Activity

- The sum of the two coefficients DRPOST & HITPOST is positive, so increase the likelihood of drilling.
- But DRPOST is larger than HITPOST, may be significant unobservable heterogeneity correlated with DRPOST
- For example, HITPOST only observed with a lag
- The above results should be viewed as suggestive. (simple functional forms and loose information set)
- May be actual production is unobservable by firms at the time.
- May explain why REVPOST no effect on drilling decisions, except in the last two years of the lease

Determinants of Drilling Outcomes

Variable	Year after acquisition 1	2	3	4	5
NRISKSET	−0.282	−1.181	−0.355	0.338	−0.420
	(0.410)	(0.794)	(0.890)	(0.997)	(0.855)
AREABID	0.481	−0.132	1.187	−0.245	1.914
	(0.502)	(0.898)	(1.111)	(1.056)	(0.943)
BID	2.337	0.832	0.815	0.374	−0.848
	(0.320)	(0.540)	(0.668)	(0.720)	(0.698)
MLT	−0.897	0.395	−0.126	1.561	2.960
	(0.314)	(0.635)	(0.792)	(0.897)	(1.021)
ONEBID	−3.434	−2.746	0.575	0.190	−0.345
	(2.693)	(2.967)	(2.715)	(2.963)	(2.245)
ONEBID × MLT	1.497	−0.205	−0.489	−0.483	−2.298
	(0.758)	(1.139)	(1.095)	(1.366)	(1.176)
HERF	−2.023	−4.359	−2.004	−0.638	2.141
	(1.919)	(2.728)	(3.482)	(3.154)	(2.693)
ACRE	−2.188	−2.861	−4.016	3.256	−1.201
	(0.981)	(1.215)	(2.779)	(3.209)	(1.659)
REOFFER	3.170	−0.482	−0.020	6.400	0.741
	(1.320)	(1.886)	(1.758)	(3.395)	(2.133)
DRPOST		6.892	1.368	2.136	2.367
		(1.500)	(1.476)	(1.397)	(1.339)
HITPOST		−4.766	−2.072	−4.816	−1.839
		(1.366)	(1.483)	(1.478)	(1.571)
REVPOST		0.006	0.201	0.125	0.076
		(0.104)	(0.129)	(0.138)	(0.146)
Sample size	733	363	188	155	214
Log-likelihood	−1,575	−669	−330	−266	−321

Determinants of Drilling Outcomes

- A Tobit regression, dependent variable is REV (logarithm of discounted revenues if the tract had positive revenues, or zero if it was unproductive)
- Whether the determinants of drilling activity are correlated with drilling outcomes? how accurate ex ante information is?
- There is an obvious sample selection problem in that we observe outcomes only on tracts that are viewed most favorably and hence drilled.
- AREABID coefficient is not significant initially, is positive and significant in the last year
- indicating that the firms rationally incorporated this information in their end-of-lease drilling decisions.

Determinants of Drilling Outcomes

- MLT is negative & significant in the first year
- firms may not correctly update their beliefs about the value of their leases after the sale.
- DRPOST coefficient is positive and often significant
- HITPOST coefficient is negative and often significant, contrary to expectation
- indicate that firms may not be processing information optimally
- or may observe HITPOST with a longer lag than DRPOST

Determinants of Drilling Outcomes

- REVPOST coefficient is positive
- Variables not explain drilling outcomes very well because of uncertainty that firms encounter in their drilling decisions

Other Papers on Drilling

- The Effect of Uncertainty on Investment: Evidence from Texas Oil Drilling by Kellogg (AER, 2014)
- Recent paper proposes clear sign of learning.
- Real options theory views investments with sunk cost as options in that, at any point in time, a firm may choose to either invest immediately or delay
- Insight: option to delay has value with positive future even NPV¡0
- Firms should delay irreversible investments until a significant gap develops between the investments' expected benefits and costs.
- No empirically well-known test for option theory

Other Papers on Drilling

- As uncertainty increases in theory
  - the incentive to delay should grow stronger
  - gap between the expected benefit and cost necessary to trigger investment should widen.
- Paper: a descriptive analysis show increases in the expected volatility of the future price associated with decreases in drilling activity
- To test, construct and estimate a dynamic, econometric model of firms' optimal drilling timing (Rust's (1987) nested fixed point approach)
- But allows the volatility of the process governing future oil prices to vary over time.

Kellogg 2014

- Paper finds that the response of drilling investment to changes in uncertainty is broadly consistent with optimal decision-making.
- That is, when the expected volatility of the future price of oil increases, drilling activity decreases by a magnitude that aligns with that predicted by the real options model.
- variation in oil price volatility can reduce the value of a drilling prospect by more than 25%

Learning: Source of Productivity

- Kellogg, Ryan. "Learning by drilling: Interfirm learning and relationship persistence in the Texas oil patch." The Quarterly Journal of Economics (2011)
- Relationship-specific learning drives productivity + shape contracting decisions
- Accumulation of experience with one producer improves productivity twice a rig that frequently changes contracting partners.
- $\Rightarrow$ strong incentive to maintain relationships
- Relationship-specific learning stems from personal interactions b/w personnel, rather than firm-specific technical knowledge.

Model

- Max productivity or min. inverse drilling rate: $y = \phi g(\Omega)\nu$
- ϕ baseline drilling rate, Ω efficiency, ν field or well specific
- Optimum is Ω^* in which minimized at $g(\Omega)$
- Experience (E, learning by doing) shift ω close to Ω^*
- If $log(h(E))$ is a learning process, then $log(g(\Omega)) = log(h(E)) + log(\eta)$ with combined specification:

$$log(y_{fprt}) = log(h(E)) + \gamma_f + \delta_p + \phi_r + \theta X_{fprt} + \varepsilon_{fprt}$$

- p: producer, r: rig, f: field, t: date drilling completed

$$log(h(E)) = \beta_1 log(E_{ft}) + \beta_2 log(E_{pt}) + \beta_3 log(E_{fpt}) + \beta_4 log(\widehat{E_{rt}}) + \beta_5 log(\widehat{E_{prt}})$$

- E_j overall experience of industry at j, $\widehat{E_{prt}}$ experience r & p work together

Drilling Technology

Learning-by-Doing Estimates. Dependent Variable Is $\log \frac{\text{Drilling Time}}{\text{Well Depth}}$

Log of experience with:	(1) Reference case model	(2) No producer–rig fixed effects	(3) Splines	(4) Field variables only	(5) Field variables with rig FE	(6) Field variables with forgetting	(7) All variables, with forgetting
Same field	−0.009	−0.010	spline	−0.001	−0.010	−0.002	−0.011
(E_{ft})	(0.012)	(0.007)		(0.008)	(0.007)	(0.008)	(0.007)
Same producer	0.003	−0.015	spline	−0.015	−0.024**	−0.016	−0.016
(E_{pt})	(0.020)	(0.011)		(0.011)	(0.011)	(0.011)	(0.011)
Same field, same producer	−0.023***	−0.022***	spline	−0.038***	−0.032***	−0.048***	−0.040***
(E_{fpt})	(0.007)	(0.006)		(0.006)	(0.005)	(0.007)	(0.011)
Same rig	−0.014	−0.019***	spline	–	–	–	−0.020***
($\hat{E}_{rt}$)	(0.009)	(0.006)		–	–	–	(0.006)
Same producer, same rig	−0.019***	−0.019***	−0.018***	–	–	–	−0.016***
($\hat{E}_{prt}$)	(0.007)	(0.004)	(0.006)	–	–	–	(0.004)
Forgetting parameter δ on	–	–	–	–	–	−0.928**	−2.495
E_{fpt}	–	–	–	–	–	(0.443)	(1.718)
Producer fixed effects	Y	Y	Y	Y	Y	Y	Y
Rig fixed effects	Y	Y	Y	N	Y	N	Y
Producer X rig fixed effects	Y	N	Y	N	N	N	N