

Problem Set #11 - Due 12/18/07

You are to estimate a subset of deep parameters of the structural model in Hopenhayn and Rogerson (1993) via the simulated method of moments. You first need to generate an equilibrium where entry and exit occur. Particularly, the firms having the lowest shock will exit and others will stay. You can use the Matlab files on my website that compute decision rules for problem set #10 in order to generate model moments or if you did the problem set, you can use your own.

This exercise will help us find unreported parameters that Hopenhayn and Rogerson use in their computation by matching data moments they report in Table 1. The parameters to be estimated are c_f , A and parameters of the distribution of shocks $F(s'|s)$, where we assume that F has the following form:

$$F(s'|s) = \begin{bmatrix} \pi_{11} & 1 - \pi_{11} & 0 & 0 & 0 \\ \frac{1 - \pi_{22}}{2} & \pi_{22} & \frac{1 - \pi_{22}}{2} & 0 & 0 \\ \frac{1 - \pi_{22}}{2} & \frac{1 - \pi_{33} - \frac{1 - \pi_{22}}{2}}{2} & \pi_{33} & \frac{1 - \pi_{33} - \frac{1 - \pi_{22}}{2}}{2} & 0 \\ \frac{1 - \pi_{22}}{2} & 0 & \frac{1 - \pi_{44} - \frac{1 - \pi_{22}}{2}}{2} & \pi_{44} & \frac{1 - \pi_{44} - \frac{1 - \pi_{22}}{2}}{2} \\ \frac{1 - \pi_{22}}{2} & 0 & 0 & 1 - \pi_{55} - \frac{1 - \pi_{22}}{2} & \pi_{55} \end{bmatrix}$$

Then, we have 5 probability parameters (the diagonals), c_f and A , a total of 7 parameters to be estimated. So, we need at least 7 moments to have an exactly identified model. The data moments reported in the paper are the serial correlation in log employment ($\log n$), the variance in growth rates (we call σ_g), mean employment (we call μ_n) and exit rate (we call x') in Table 1A, and the share of firms $q_{0-19}, q_{20-99}, q_{100-499}, q_{500+}$ reported in Table 1B. Note that we can use only three of the four shares in SMM due to collinearity. Call the data moment vector $M_T^d = (\log n, \sigma_g, \mu_n, x', q_{0-19}, q_{20-99}, q_{100-499})$. Call the parameter vector $\psi = (c_f, A, \pi_{11}, \pi_{22}, \pi_{33}, \pi_{44}, \pi_{55})$. Call the simulated model moments $M_{TN}^m(\psi) = (\log n(\psi), \sigma_g(\psi), \mu_n(\psi), x'(\psi), q_{0-19}(\psi), q_{20-99}(\psi), q_{100-499}(\psi))$.

Here are the steps of the algorithm:

1. Write a subroutine that generates an economy using a set of initial parameter values. We are interested in an equilibrium where only the lowest shock will lead to exit. Similarly, among the possible entrants, only the lowest shock receivers will decide not to enter. Call this subroutine "generator". Your "generator" will find μ , M , sales, profits and values $W(s; p)$. You will also find the c_e that ensures $p = 1$.

- Write a function (name it "simulator") that creates large panels of incumbent and entrant firms. Take $N = 1000$ for each set of firms. Create 50 or more panels of $T = 5$ period economies. By using a random number generator, and using the outcome of "generator", attain firms' shocks and derive their exit/stay or enter/do not enter decisions. You also need to adjust the panel in a way that the measure of entrants equals the measure of those who exit to ensure stationarity. Your "simulator" should calculate the model moments for each set of panels and will produce their mean as its output.
- Write a small function that calculates

$$J^{TN} = [M_T^d - M_{TN}^m(\psi)]' \widehat{W}_T [M_T^d - M_{TN}^m(\psi)].$$

by using the moments simulated in "simulator".

- The idea of SMM is to choose ψ in such a way as to bring the simulated moments $M_{TN}^m(\psi)$ constructed in "simulator" as close as possible to the data moments in step (1). That is, we solve

$$\widehat{\psi}_{TN} = \arg \min_{\psi} J^{TN}$$

In order to do that, take a set of initial parameter values (a good set is $(c_f = 2, A = 0.0111, \pi_{11} = 0.9713, \pi_{22} = 0.9664, \pi_{33} = 0.893, \pi_{44} = 0.8238, \pi_{55} = 0.3648)$), use Matlab's built-in function 'fminsearch' to minimize J^{TN} . Use $\widehat{W}_T = I$ initially. Also, take the entrants' probability distribution, ν , equal to the invariant distribution of F .

- What you will get is a set of consistent estimators for the parameters. You need to find efficient estimators by using the efficient weighting matrix. To calculate it, use the moments from "simulator" and get the variance covariance matrix, S . Your optimal weighting matrix will be $\widehat{W} = S^{-1}$.
- Redo the estimation by using the optimal weighting matrix.