ECO 392M: Computational Economics I
Fall 2009, University of Texas
Instructor: Dean Corbae

Problem Set #11- Due 11/30/09

This problem set is based on a simple MA(1) example in Section 3 of Michaelides and Ng (2000, Journal of Econometrics) used to assess the properties of a Simulated Methods of Moments (SMM) estimator, as well as indirect inference and efficient method of moments estimators. A simple statement of the three estimators is given in Section 2 of that paper. As stated on p. 237, "With the SMM, the practitioner only needs to specify the empirical moments and is the easiest to implement".

Suppose the true data generating process for a series $\{y_t\}_{t=1}^{T}$ is given by the following $MA(1)$ model:

$$y_t = e_t - \theta e_{t-1} \tag{1}$$

where $\theta = 0.5$, $e_t \sim N(0,1)$, and $e_0 = 0$. The econometrician knows that the structural model is an $MA(1)$ and is interested in estimating the parameter θ using SMM. Given a weighting matrix Ω_T for a sample of length T, the SMM estimate $\widehat{\theta}_{\Omega_T}$ is asymptotically normally distributed; that is by the law of large numbers $\sqrt{T}(\widehat{\theta}_{\Omega_T} - \theta) \sim N(0, W)$ as $T \to \infty$. Let Y_t and $\widetilde{Y}_t(\theta)$ denote the observations at time t of the actual and simulated variables. Let T be the sample size of the observed series and H the number of simulated series of size T. The SMM estimate $\widehat{\theta}_{\Omega_T}$ is obtained from:

$$\widehat{\theta}_{\Omega_T} = \arg\min_{\theta} D'\Omega_T D, \tag{2}$$

where D is a vector with the distance between data moments and model moments and Ω is a positive semi-definite weighting matrix. More specifically,

$$D = \left(\frac{1}{T} \sum_{t=1}^{T} m(Y_t) - \frac{1}{TH} \sum_{t=1}^{TH} m(\widetilde{Y}_t(\theta)) \right).$$

In the last expression $\sum_{t=1}^{T} m(Y_t)$ is the vector of empirical moments based on the true data Y_t and $\sum_{t=1}^{TH} m(\widetilde{Y}_t)$ is based on the simulated data. For example, when $m(Y_t) = Y_t$, we simply match the sample average.

To obtain a consistent estimate of $\widehat{\theta}$ we can use $\Omega_T = I$. However, to find the efficient estimator, we need the optimal weighting matrix Ω_T^*. The asymptotic optimal weighting matrix is given by:

$$\Omega^* = \lim_{T \to \infty} Var\left(\frac{1}{\sqrt{T}} \sum_{t=1}^{T} m(Y_t) \right)^{-1}. \tag{3}$$

Once we know Ω^* we can also find the variance covariance matrix W_H^* of the estimate $\widehat{\theta}$ (which in this simple one dimensional MA(1) case is just the variance of the estimate):

$$W_H^*(\Omega^*) = \left(1 + \frac{1}{H}\right)\left(E\left[\frac{\partial m(\widetilde{Y}(\theta))'}{\partial \theta}\right]\Omega^* E\left[\frac{\partial m(\widetilde{Y}(\theta))'}{\partial \theta}\right]\right)^{-1}. \tag{4}$$

Expressions (3) and (4) correspond to the asymptotic optimal weighting matrix and variance covariance matrix. However, when implementing SMM we will need estimates of Ω^* and W_H^* given we will be working with finite samples. One possible estimate of Ω^*, $\widehat{\Omega}_T$ can be obtained directly from the data (thus independent from $\hat{\theta}$) using the estimator proposed by Newey and West (1987). More specifically,

$$\widehat{\Omega}_T = \Gamma_{0,T} + \sum_{v=1}^{q}\left(1 - \left[\frac{v}{q+1}\right]\right)\left(\Gamma_{v,T} + \Gamma'_{v,T}\right)$$

with

$$\Gamma_{v,T} = \frac{1}{T}\sum_{t=v+1}^{T} m(Y_t)m(Y_{t-v})'$$

where q is the number of non-zero autocovariances. In later problem sets we will address what to do when you don't have the actual data Y_t to construct the optimal weighting matrix.

Let the objective function be denoted $F_T(\theta) \equiv D'\Omega_T D$. The estimate of $W_H^*(\Omega^*)$, $\widehat{W_{TH}}(\widehat{\Omega}_T)$ can be obtained using $\widehat{\Omega}_T$. Moreover, $\frac{\partial m(\widetilde{Y}(\theta))}{\partial \theta}$ can be obtained numerically. The moments we will use for SMM are the mean, the variance and three autocovariances (i.e. $q = 3$); that is

$$m(Y_t) = \left\{\ Y_t,\ \ (Y_t - \overline{Y})^2,\ \ (Y_t - \overline{Y})(Y_{t-1} - \overline{Y}),\ \ (Y_t - \overline{Y})(Y_{t-2} - \overline{Y}), (Y_t - \overline{Y})(Y_{t-3} - \overline{Y})\ \right\}$$

1. Simulate a series of "true" data of length $T = 200$ using (1).

2. Set $H = 10$ and simulate H vectors of length T random variables e_t from $N(0, 1)$. Store these vectors. You will use the same vector of random variables throughout the entire exercise.

3. Set $\Omega = I$ and plot $F_T(\theta) \equiv D'ID$ for values of $\theta \in [0.1, 0.8]$. Obtain an estimate of θ by using $\Omega = I$ in (2). Report $\widehat{\theta}_I$.

4. Set $q = 4$. Obtain an estimate of Ω^*. Using $\widehat{\Omega}_T$, plot $F_T(\theta) \equiv D'\widehat{\Omega}_T D$ for values of $\theta \in [0.1, 0.8]$. Obtain an estimate of $\theta_{\widehat{\Omega}_T}$ by using $\widehat{\Omega}_T$ in (2). Report $\widehat{\theta}_{\widehat{\Omega}_T}$.

5. Obtain numerically $\left.\frac{\partial m(\widetilde{Y}(\theta))}{\partial \theta}\right|_{\theta=\widehat{\theta}_{\widehat{\Omega}_T}}$. On my website is a matlab function named mom_smm.m which is used to compute the numerical derivative. This function calculates the value of

the simulated moments at a given parameter value, $m(\widetilde{Y}(\theta))$, and it should be similar to the one used to find $\widehat{\theta}$ in (2). Then, the derivative can be computed as

$$\frac{\partial m(\widetilde{Y}(\theta))}{\partial \theta}\Big|_{\theta=\widehat{\theta}} \approx \frac{m(\widetilde{Y}(\widehat{\theta}+h)) - m(\widetilde{Y}(\widehat{\theta}))}{h}$$

where h is a small number. Report the values of $\frac{\partial m(\widetilde{Y}(\theta))}{\partial \theta}\Big|_{\theta=\widehat{\theta}_{\widehat{\Omega}_T}}$.

6. Obtain the standard error of $\widehat{\theta}_{\widehat{\Omega}_T}$; that is compute $\widehat{W_{TH}}(\widehat{\Omega}_T)$.

7. Given the fact that we have an overidentified model, we can use these extra moments to evaluate the model. Under the null that the model is true, a global specification test (see equation (5) on p. 236 of Michaelides and Ng) can be obtained

$$TH \times D'\widehat{\Omega}_T D \to \chi^2$$

with $q - p = 3 - 1 = 2$ degrees of freedom (where the number of parameters is given by p). Compute this statistic. Is the model rejected?