Computational Economics I
Fall 2018, Sharif University of Technology
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Problem Set #7 - Due 0/0/18

This problem set is designed to have you solve a simplified version of the general equilibrium model of firm dynamics in Hopenhayn and Rogerson (1993, JPE, hereafter H-R). The next problem set will apply simulated method of moments to this model in order to estimate the underlying parameters of their model given certain moments in the data.

Environment. Consider the following model of firm dynamics. There is a unit measure of identical households and a continuum of firms (mass not necessarily 1) which produce a homogeneous final product that sells at price p_t (the numeraire is units of labor). Any given firm's production function is given by $q_t = s_t n_t^\theta$ where $s_t \in \mathbb{R}_+$ is a productivity shock which follows a first order Markov process, iid across firms with conditional distribution $F(s' = s_{t+1}|s = s_t)$. Since the data on the size distribution of firms in Table 2 of H-R lists firm size (number of workers employed) in 4 bins (1-19, 20-99, 100-499, 500+), we will include a very low productivity state (which should induce exit) and consider a 5 state markov process which is persistent. Each period that the firm stays in the market, it bears a fixed cost c_f (denominated in units of output). The timing of incumbent firm's decisions is:

1. enter period t in state s_{t-1}.
2. decide whether to exit. If the firm exits, it avoids c_f but receives profits of zero in all future periods.
3. If firm doesn't exit, it pays costs c_f and receives this period's shock s_t from $F(s_t|s_{t-1})$
4. firm chooses labor demand $n_t = N^d(s_t; p_t)$.

The timing for a potential entrant is:

1. decide whether to pay fixed cost c_e and subsequently receive this period's shock s_t from $\nu(s_t)$ (which is iid across entrants).
2. same as 4 above.

Household preferences are given by

$$\sum_{t=0}^{\infty} \beta^t \left[\ln(C_t) - AN_t\right].$$

Profits are distributed equally among all households.

Equilibrium. Since the only uncertainty is idiosyncratic, we will focus on a stationary equilibrium where $p_t = p$. There are two decisions of an incumbent firm: optimal employment $n = N^d(s_t;p)$ and optimal exit next period $x' = X(s_t;p) \in \{0,1\}$ with convention that exit equals 1. The dynamic programming problem is:

$$W(s;p) = \max_{n\geq 0, x'\in\{0,1\}} \left\{ psn^\theta - n - pc_f + \beta(1-x')\int_{s'} W(s';p)dF(ds'|s) \right\} \quad (1)$$

Conditional upon incurring the cost c_e, an entrant solves

$$W^e(s;p) = \max_{n\geq 0, x'\in\{0,1\}} \left\{ psn^\theta - n + \beta(1-x')\int_{s'} W(s';p)dF(ds'|s) \right\}$$

and free entry requires

$$\int W^e(s;p)\nu(ds) \leq pc_e$$

with equality if the mass of new entrants in period t, denoted M, is strictly positive.

The distribution of firms at the beginning of stage 4 of period t is denoted $\mu(s;p)$. For any set $S_0 \in S$, the law of motion for the distribution of firms is:

$$\begin{aligned} \mu'(S_0) &= \int_{s'\in S_0} \left\{ \int_{s\in S} \left[1 - X(s;p)\right] dF(s'|s)d\mu(s) \right\} ds' \\ &+ \int_{s'\in S_0} \left\{ \int_{s\in S} \left[1 - X(s;p)\right] dF(s'|s)M\nu(ds) \right\} ds'. \end{aligned} \quad (2)$$

Defining the operator T^*, (2) can be written as[1]

$$\mu' = T^*(\mu, M; p). \quad (3)$$

In a steady state equilibrium where interest rates satisfy $\beta(1+r) = 1$ and all households own the same diversified portfolio of firms, the household's problem simplifies to a static labor/leisure choice since there is no desire to save when there is no uncertainty at the individual household or aggregate level:

$$\begin{aligned} &\max_{C,N^s} u(C) - AN^s \\ s.t. pC &\leq N^s + \Pi \end{aligned} \quad (4)$$

where

$$\begin{aligned} \Pi(\mu, M; p) &= \int \left[ps\left(N^d(s;p)\right)^\theta - N^d(s;p) - pc_f \right] d\mu(s) \\ &+ M\int \left[ps\left(N^d(s;p)\right)^\theta - N^d(s;p) - pc_e \right] d\nu(s). \end{aligned} \quad (5)$$

[1] The operator T^* is linearly homogeneous in μ and M jointly.

The solution to (4) implies a decision rule $N^s\left[p, \Pi(\mu, M; p)\right]$.

A **stationary competitive equilibrium** is a list $\{p^*, \mu^*, M^*\}$ such that: (i) the labor market clears $L^d(\mu^*, M^*; p^*) = N^s\left[p^*, \Pi(\mu^*, M^*; p^*)\right]$ where $L^d(\mu, M; p) = \int N^d(s;p)d\mu(s) + M\int N^d(s;p)d\nu(s)$ (ii) there is an invariant distribution over firms $\mu^* = T(\mu^*, M^*; p^*)$; and (iii) $\int W^e(s;p^*)\nu(ds) \leq p^* c_e$ with equality if $M^* > 0$.

Algorithm. Basically, 2 "Do Loops"

1. Iterate over p_i until the entry condition is satified at p^* :

   (a) For each p_i, calculate $W_i(s;p_i)$ and $W_i^e(s;p_i)$.

   (b) Let $EC(p_i) \equiv \left[\int W^e(s;p_i)\nu(ds)\right]/p_i - c_e$. See the FigureEntryProblemSet10F09 at the end of this problem set. If $EC(p_i) > 0$, then set $p_{i+1} < p_i$, otherwise set $p_{i+1} > p_i$. When $EC(p_{i+1}) \approx 0$, $p_{i+1} = p^*$.

2. Iterate over (μ_i, M_i) until the labor market clearing condition is satisfied at (μ^*, M^*).

   (a) Letting $M_0 = 1$, use the T^*operator in (2) to find a fixed point $\mu_0^{ss}(M_0 = 1)$.

   (b) Let $LMC(\mu_i, M_i) = L^d(\mu_i^{ss}(M_i), M_i; p^*) - N^s\left[p^*, \Pi(\mu_i^{ss}(M_i), M_i; p^*)\right]$. See FigureLMCProblemSet10F09. If $LMC(\mu_i, M_i) > 0$, then set $M_{i+1} < M_i$, otherwise set $M_{i+1} > M_i$. When $LMC(\mu_{i+1}, M_{i+1}) \approx 0$, then $(\mu_{i+1}, M_{i+1}) = (\mu^*, M^*)$.

Actually, H-R prove linear homogeneity of (2) in (μ, M), so you needn't iterate again in 2a, but since the calculation is quick, it's possibly more intuitive as explained above

Calibration

Two parameters can be set independent of the firm distribution data: $\{\beta = 0.8, \theta = 0.64\}$. The distribution of shocks $\{F, \nu\}$, costs $\{c_f, c_e\}$, and the employment to population ratio (which pins down A) need to be set to match the firm distribution data. For this assignment, let $s \in \{3.98e^{-4}, 3.58, 6.82, 12.18, 18.79\}$. This grid of shocks gives employment levels of $\{1.3e^{-9}, 10, 60, 300, 1000\}$ which except for 0, are in the bins of firm size in Table 1.B. Let

$$F(s'|s) = \begin{bmatrix} 0.6598 & 0.2600 & 0.0416 & 0.0331 & 0.0055 \\ 0.1997 & 0.7201 & 0.0420 & 0.0326 & 0.0056 \\ 0.2000 & 0.2000 & 0.5555 & 0.0344 & 0.0101 \\ 0.2000 & 0.2000 & 0.2502 & 0.3397 & 0.0101 \\ 0.2000 & 0.2000 & 0.2500 & 0.3400 & 0.0100 \end{bmatrix}$$

This transition matrix gives an invariant distribution, which we take to be the entrant distribution $\nu(s)$,given by:

$$v(s) = \{0.37, 0.4631, 0.1102, 0.0504, 0.0063\}.$$

The way we calibrated $F(s'|s)$ was to change parameters until the invariant distribution matched the exit rate 37% in Table 1 and then take $(1-0.37)*$Table 1 bin to arrive at $v(s), s \in \{s_2, s_3, s_4, s_5\}$. For example, for the 500+ bin, the share of Total Firms is 0.01,so $v(s_5) = (1-0.37)*0.01 = 0.0063$. Let $A = 1/200$, $c_f = 10$, and $c_e = 15$.