ECO 392M: Computational Economics I
Fall 2010, University of Texas
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Problem Set #8- Due 11/10/10

You are to compute an approximate equilibrium of an Aiyagari (1994) paper with aggregate uncertainty using the techniques in Krusell and Smith (1998). As discussed in class, there is a unit measure of agents, the time period is one quarter, preferences are given by

$$\sum_{\tau=0}^{\infty} \beta^t \ln(c_t)$$

where $\beta = 0.99$.The production technology is given by

$$y_t = z_t k_t^{\alpha} l_t^{1-\alpha}$$

where $\alpha = 0.36$,and aggregate technology shocks $z_t \in \{z_g = 1.01, z_b = 0.99\}$ are drawn from a markov process to be described more fully below. Capital depreciates at rate $\delta = 0.025$.Agents have 1 unit of time and face idiosyncratic employment opportunities $\varepsilon_t \in \{0, 1\}$ where $\varepsilon_t = 1$ means the agent is employed an receives wage $w_t \overline{e}$ (where $\overline{e} = 0.3271$ denotes labor efficiency per unit of time worked) and $\varepsilon_t = 0$ means he is unemployed. The probability of transition from state (z, ε) to (z', ε'), denoted $\pi_{zz'\varepsilon\varepsilon'}$ must statisfy certain conditions:

$$\pi_{zz'00} + \pi_{zz'01} = \pi_{zz'10} + \pi_{zz'11} = \pi_{zz'}$$

and

$$u_z \frac{\pi_{zz'00}}{\pi_{zz'}} + (1 - u_z) \frac{\pi_{zz'10}}{\pi_{zz'}} = u_{z'}$$

where u_z denotes the fraction of those unemployed in state z with $u_g = 4\%$ and $u_b = 10\%$. The other restrictions on $\pi_{zz'\varepsilon\varepsilon'}$ necessary to pin down the transition matrix are that: the average duration of good and bad times is 8 quarters; the average duration of unemployment spells is 1.5 quarters in good times and 2.5 quarters in bad times; and

$$\frac{\pi_{gb00}}{\pi_{gb}} = 1.25 \cdot \frac{\pi_{bb00}}{\pi_{bb}} \text{ and } \frac{\pi_{bg00}}{\pi_{bg}} 0.75 \cdot \frac{\pi_{gg00}}{\pi_{gg}}.$$

See my website for the file transmatrix.m which actually computes the transition matrix for you. Capital is the only asset to self insure fluctuations; households rent their capital $k_t \in [0, \infty)$ to firms and receive rate of return r_t. Without loss of generality, we can consider one firm which hires L_t units of labor efficiency units (so that $L_t = \overline{e}(1 - u_t)$) and rents capital K so that wages and rental rates are given by their marginal products:

$$\begin{aligned} w_t &\equiv w(K_t, L_t, z_t) = (1-\alpha) z_t \left(\frac{K_t}{L_t} \right)^{\alpha} \quad (1) \\ r_t &\equiv r(K_t, L_t, z_t) = \alpha z_t \left(\frac{K_t}{L_t} \right)^{\alpha - 1} \end{aligned}$$

As in Krusell and Smith, approximate the true distribution Γ_t over (k_t, ε_t) in state z_t by I moments and let the law of motion for the moment be $m' = h_I(m, z, z')$.

Like the previous models without a labor leisure decison and a unit measure of agents, L is trivially given in any state z. Specifically, $L_g = 1 - u_g = 0.96$ and $L_b = 1 - u_b = 0.9$.

Algorithm

1. Let $I = 1$ (which means only average capital holdings matter).

2. Conjecture a log linear functional form for h_1;Specifically let
$$\log K' = \begin{cases} a_0 + a_1 \log K & \text{if } z = z_g \\ b_0 + b_1 \log K & \text{if } z = z_b \end{cases} \tag{2}$$
As an initial guess, one could simply start with $a_0 = b_0 = 0.1$ and $a_1 = b_1 = 0.95$

3. Given h_I, solve the conumers problem. For the above example
$$v(k, \varepsilon; K, z) = \max_{c,k'} u(c) + \beta E_t \left[v(k', \varepsilon'; K', z')\right]$$
s.t.
$$c + k' = r(K, L, z)k + w(K, L, z)\varepsilon + (1 - \delta)k$$
as well as (1) and (2). Let k and K lie in $[0, 15)$ and use bilinear interpolation over these two dimensions of the state vector (the other 4 states are discrete so simply index 4 different value functions by $(g, 0), (b, 0), (g, 1), (b, 1)$). For bilinear interpolation see Numerical Recipes, Section 3.6.

4. Use the decision rules generated in step 3 and the transition function $\pi_{zz'\varepsilon\varepsilon'}$ to simulate the behavior of N households starting from an initial condition $K^{ss} = 5.7163$, where $N = 5000$, for $T = 11,000$ periods, discarding the first 1000 periods (to deal with initial condition dependence).[1] This generates a huge $N \times \widetilde{T}$ matrix where each row is an agent's k_{t+1} choice in state z_t. Call it V. Keep track of the $\widetilde{T} \times 1$ vector of good and bad shocks. Call it Z.

5. Use the simulated data in step 4 to (re-)estimate a set of parameters for the functional form conjectured in step 2. That is, average over all agents in each period (i.e. down a column of V). The resulting $\widetilde{T} \times 1$ vector of aggregate capital holdings is K..Run the (auto)regression in (2) using the information in Z to know which branch to run. Obtain a measure of "goodness of fit" (e.g. R^2 of regression in (2)).

6. If the new parameter vector (a_0', a_1', b_0', b_1') is sufficiently close to the original parameter vector (a_0, a_1, b_0, b_1) and the "goodness of fit" (e.g. R^2 of regression in (2)), is sufficiently high, stop. If the parameter values have converged, but the goodness of fit is not high enough, increase I in step 1 or try a different functional form in step 2.

[1] The initial capital stock “guess” is just the average capital stock in a version of the model without aggregate uncertainty (i.e. Aiyagari).