

Problem Set #7- Due 10/14/09

You are to compute the decision problem of an agent in a nonstochastic growth model facing different prices using different interpolation methods. There is a unit measure of agents. Let lower case letters denote individual holdings and uppercase letters denote aggregates. The preferences of any agent are given by

$$\sum_{\tau=0}^{\infty} \beta^{\tau} \ln(c_{\tau})$$

where $\beta = 0.99$. The production technology is given by

$$Y_t = K_t^{\alpha} L_t^{1-\alpha}$$

where $\alpha = 0.36$ and capital depreciates fully each period. Agents have 1 unit of time. Agents are endowed with initial capital k_0 and can rent their capital $k_t \in [0, \infty)$ to firms and receive rate of return r_t . Without loss of generality, we can consider one firm which hires all workers so that $L_t = 1$ and rents $K_t = K$ units of capital so that wages w_t and rental rates r_t are given by their marginal products:

$$\begin{aligned} w(K) &= (1 - \alpha)K^{-\alpha} \\ r(K) &= \alpha K^{\alpha-1} \end{aligned} \tag{1}$$

Consumers solve

$$v(k; K) = \max_{c, k'} u(c) + \beta v(k'; K)$$

s.t.

$$c + k' = r(K)k + w(K)$$

as well as (1).

1. Solve for a closed form solution of the value function and decision rules using the method of undetermined coefficients when $k = K$.
2. Let $K \in [0.15, 0.25]$ and $k \in [0, 0.5]$. Solve for the value function and decision rules using
 - a. Bilinear interpolation.
 - b. Cubic splines.
 - c. Cubic splines in the k dimension and linear interpolation in the K dimension.
3. Compare your results from the closed form solution to the computational solution when $k = K$.