General Physics I

chapter 5

Sharif University of Technology
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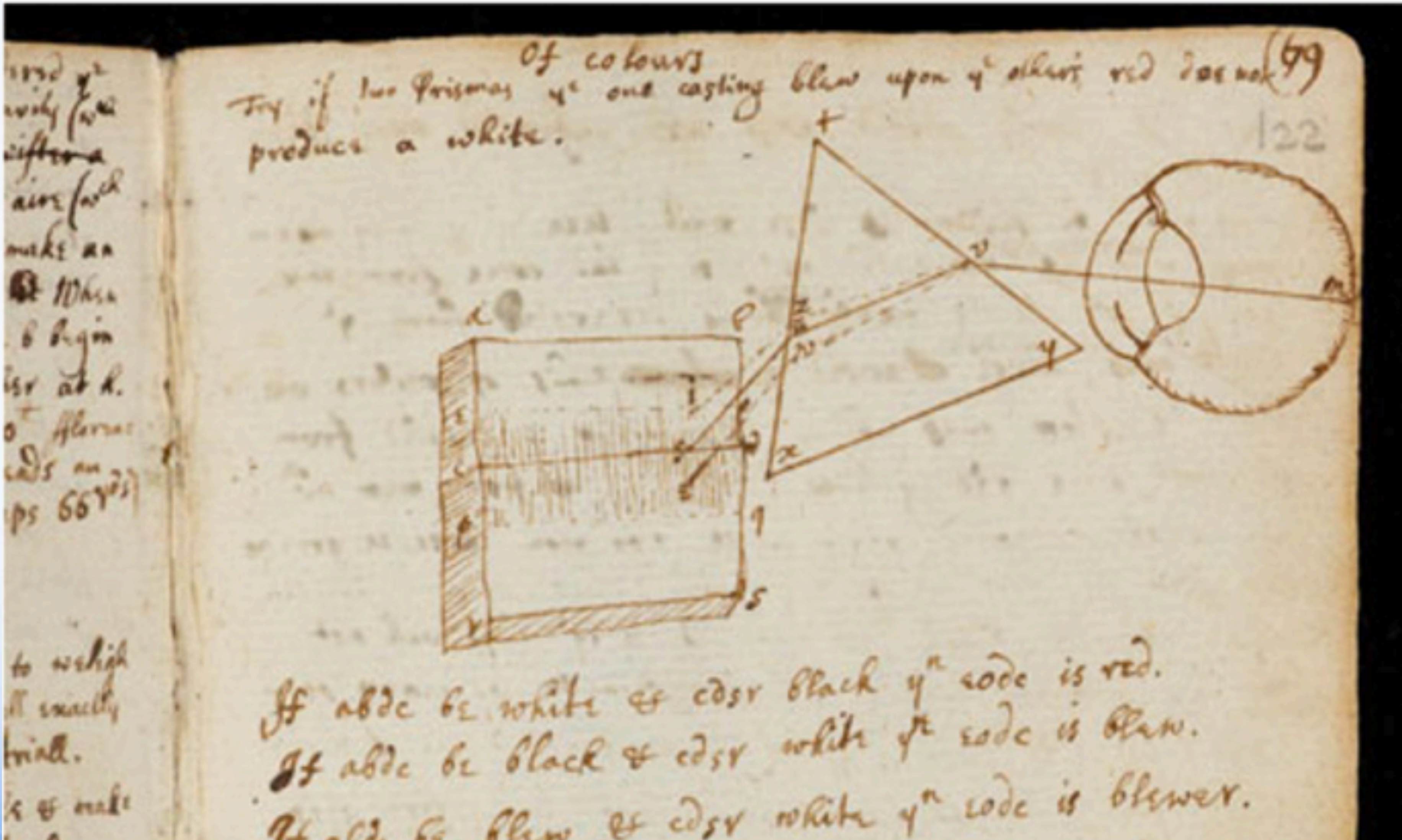
Chapter 5
Force and Motion-I

Bridgeman-Giraudon/Art Resource, NY

Isaac Newton

English physicist and mathematician (1642–1727)

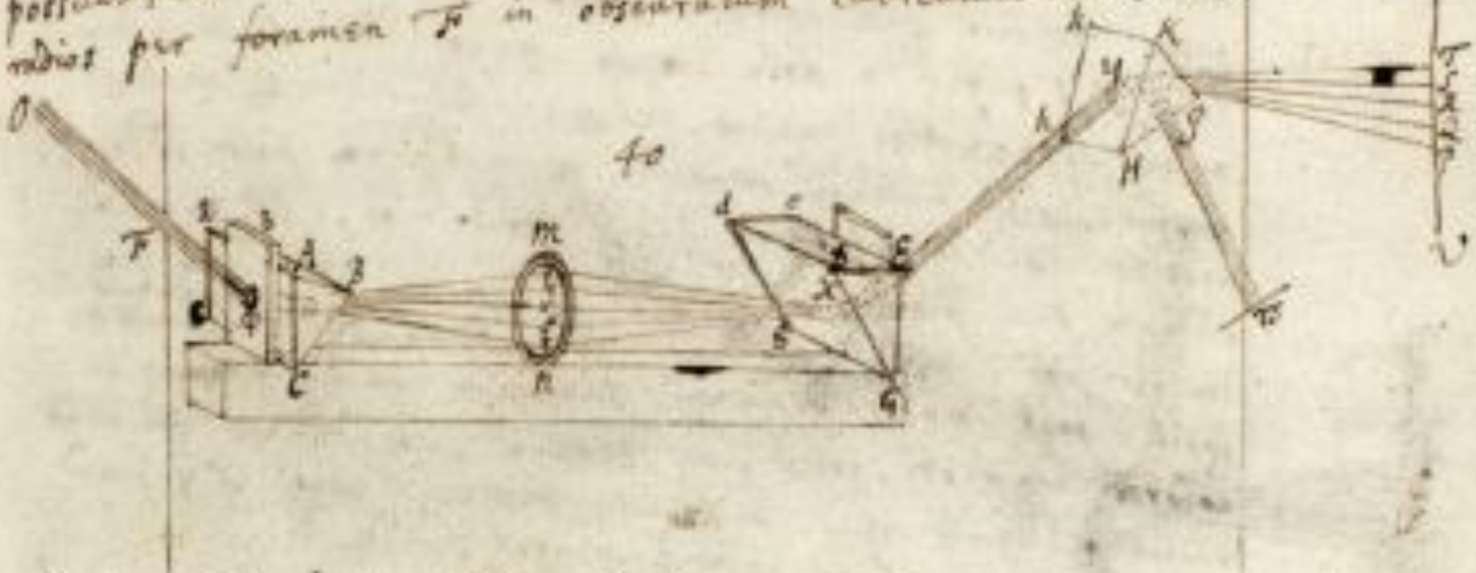
Isaac Newton was one of the most brilliant scientists in history. Before the age of 30, he formulated the basic concepts and laws of mechanics, discovered the law of universal gravitation, and invented the mathematical methods of calculus. As a consequence of his theories, Newton was able to explain the motions of the planets, the ebb and flow of the tides, and many special features of the motions of the Moon and the Earth. He also interpreted many fundamental observations concerning the nature of light. His contributions to physical theories dominated scientific thought for two centuries and remain important today.



Original manuscript by Sir Isaac Newton, one of several by the English physicist to be made available on the web. Photograph: Cambridge University Library/PA

Caeterum [illegible]
ex inventâ tandem veritate, suâ sponte corruent. Tamen ut
fecero, si proferam experimentum tandem, quo omnia quae
de genesi colorum hactenus explicui, non modò probari
possunt, sed etiam videri. Quamobrem sit ABCc prisma quod
radios per foramen F in obscuratum cubiculum transmissos

[illegible] de coloribus [illegible]
dita dilucidissimè pro-
bentur.



refringat versus lentem MN ut colores quos efficit in p, q, r, s, t
per lentem deinde trajiciantur ad X et ibidem commisceantur
in albedinem, ~~[illegible]~~ sicut in praecedentibus ostendi. Deinde
aliud prisma DEGb priori parallelum ad locum X, ubi
albedo redintegrata est, statuatur, quod lucem versus Y
refringat. Hujus autem prismatis verticalis angulus Gb sit
aequalis angulo verticali Cc prismatis anterioris, aut eo

Num. 54

Chapter 5
Force and Motion-I

5.01-- Identify that a force is a vector quantity and thus has both magnitude and direction and also components.

5.02-- Given two or more forces acting on the same particle, add the forces as vectors to get the net force.

5.03-- Identify Newton's first and second laws of motion.

5.04-- Identify inertial reference frames.

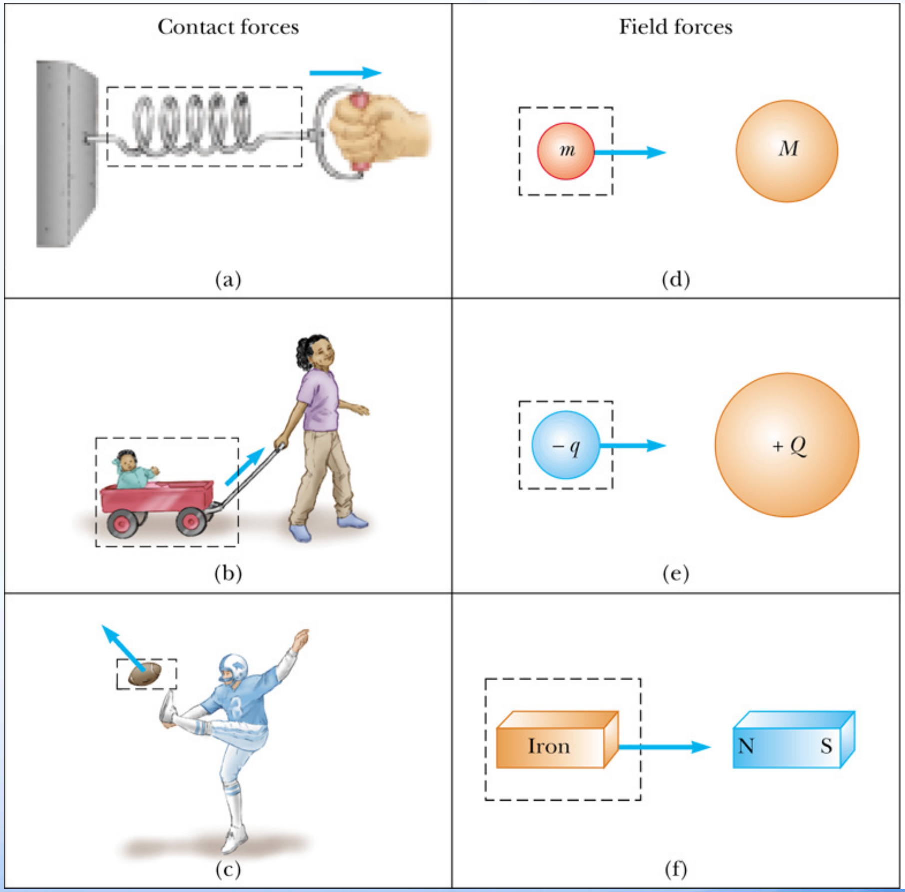
5.05-- Sketch a free-body diagram for an object, showing the object as a particle and drawing the forces acting on it as vectors with their tails anchored on the particle.

5.06-- Apply the relationship (Newton's second law) between the net force on an object, the mass of the object, and the acceleration produced by the net force.

5.07-- Identify that only external forces on an object can cause the object to accelerate.

Forces

- Usually think of a force as a push or pull
- Vector quantity
- May be contact or field force



Fundamental Forces

- Types
  - Strong nuclear force
  - Electromagnetic force
  - Weak nuclear force
  - Gravity
- Characteristics
  - All field forces
  - Listed in order of decreasing strength

Newton’s First Law (1)

- If no force acts on a body, the body’s velocity cannot change; that is, the body cannot accelerate.

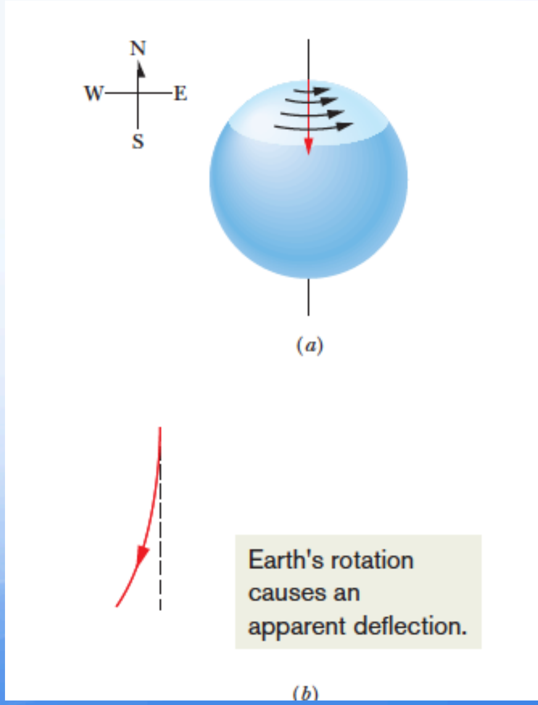
Newton’s First Law (2)

- Newton’s First Law: If no net force acts on a body ($\sum \vec{F} = 0$), the body’s velocity cannot change; that is, the body cannot accelerate.

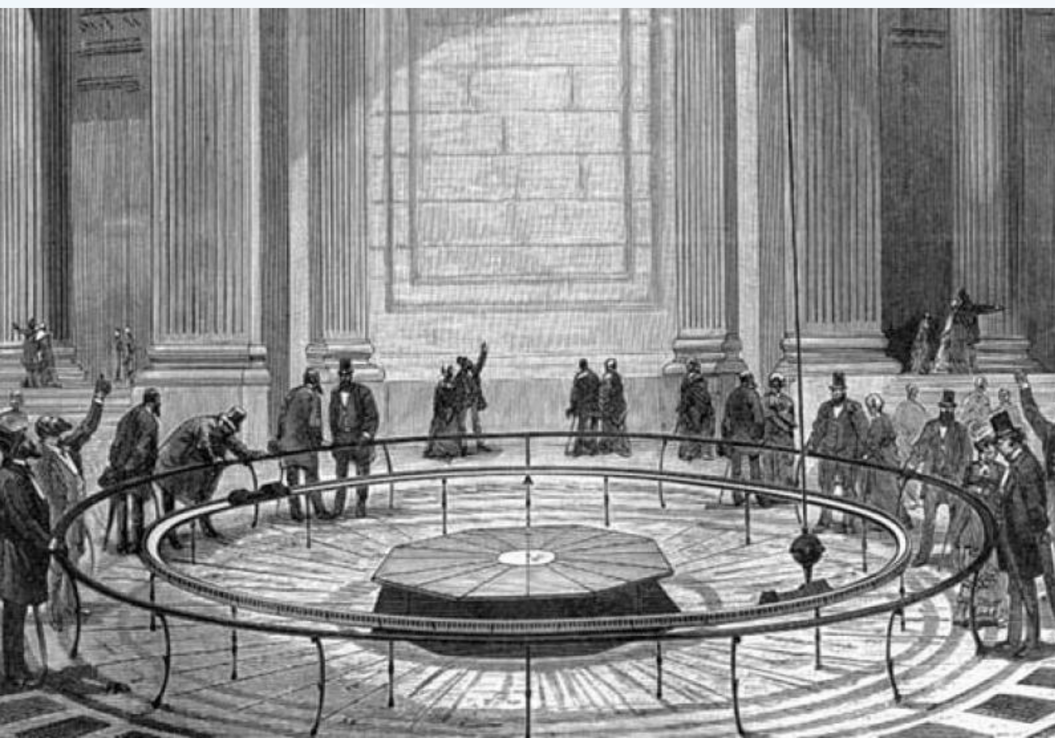
Inertial Reference Frames

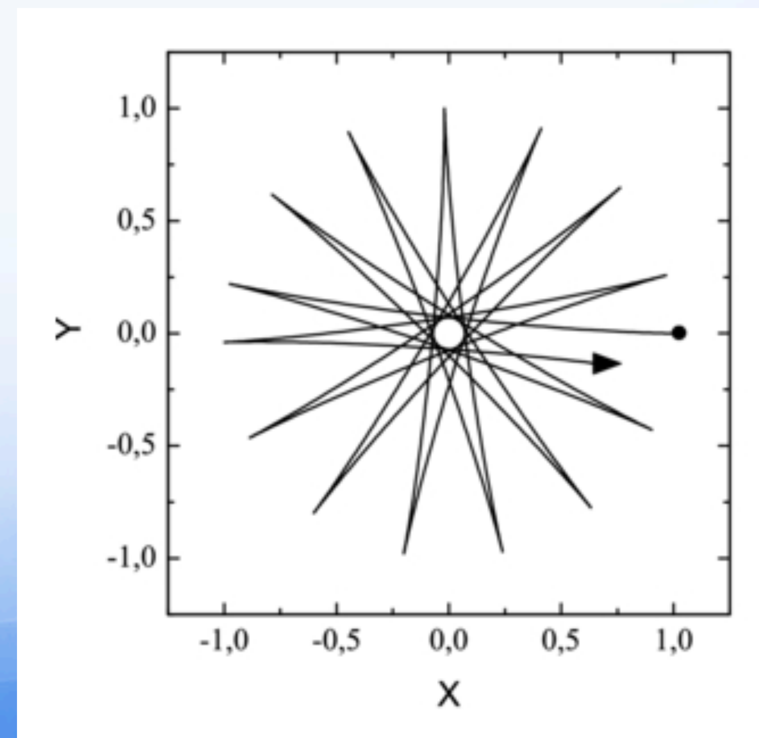
- An inertial reference frame is one in which Newton's laws hold.

- Noninertial frame



Foucault's pendulum





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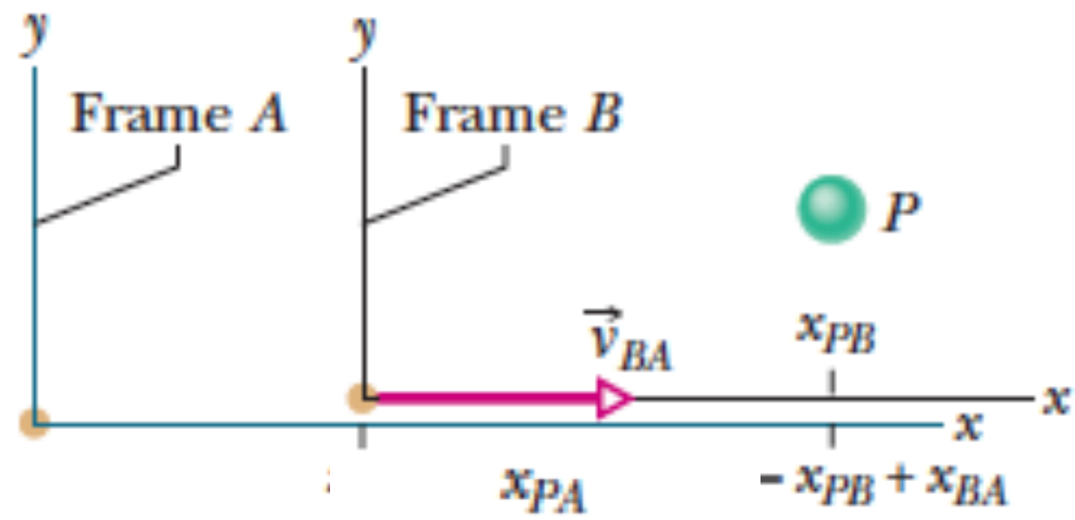


$$x_{PA} = x_{PB} + x_{BA}.$$

$$\frac{d}{dt}(x_{PA}) = \frac{d}{dt}(x_{PB}) + \frac{d}{dt}(x_{BA}).$$

$$\frac{d}{dt}(v_{PA}) = \frac{d}{dt}(v_{PB}) + \frac{d}{dt}(v_{BA}).$$

$$v_{PA} = v_{PB} + v_{BA}.$$

$$\boxed{a_{PA} = a_{PB}.}$$

Newton’s First Law, cont.

- External force
  - any force that results from the interaction between the object and its environment
- Alternative statement of Newton’s 1st Law
  - When there are no external forces acting on an object, the acceleration of the object is zero.
  - An inertial reference frame is one in which Newton's laws hold.

Inertia and Mass

- Inertia is the tendency of an object to continue in its original motion
- Mass is a measure of the inertia, i.e resistance of an object to changes in its motion due to a force
- Recall: mass is a scalar quantity

Units of mass	
SI	kilograms (kg)
CGS	grams (g)

$$\frac{m_X}{m_0} = \frac{a_0}{a_X},$$

Inertia and Mass:

Runaway train

Newton's Second Law

- The net force on a body is equal to the product of the body's mass and its acceleration.

$$\sum \vec{F} = m\vec{a}$$

  - **F** and **a** are both vectors
- Can also be applied three-dimensionally

Newton's Second Law

- Note: $\sum \vec{F}$ represents the vector sum of all external forces acting on the object.
- We can always write it in terms of components:

$$\sum \vec{F} = m\vec{a} : \quad \begin{cases} \sum F_x = ma_x \\ \sum F_y = ma_y \\ \sum F_z = ma_z \end{cases}$$

Units of Force

- SI unit of force is a Newton (N)

$$1\,N \equiv 1\frac{kg\,m}{s^2}$$

Units of force	
SI	Newton ($N = kg\ m/\ s^2$)
CGS	Dyne ($dyne = g\ cm/s^2$)
US Customary	Pound ($lb = slug\ ft/s^2$)

- 1 N = 10^5 dyne = 0.225 lb, 1 Slug = 14.5939 Kg

Example

EXAMPLE 4–2 ESTIMATE Force to accelerate a fast car. Estimate the net force needed to accelerate (*a*) a 1000-kg car at $\frac{1}{2}g$; (*b*) a 200-gram apple at the same rate.

APPROACH We use Newton's second law to find the net force needed for each object; we are given the mass and the acceleration. This is an estimate (the $\frac{1}{2}$ is not said to be precise) so we round off to one significant figure.

SOLUTION (*a*) The car's acceleration is $a = \frac{1}{2}g = \frac{1}{2}(9.8\ \mathrm{m/s^2}) \approx 5\ \mathrm{m/s^2}$. We use Newton's second law to get the net force needed to achieve this acceleration:

$$\Sigma F = ma \approx (1000\ \mathrm{kg})(5\ \mathrm{m/s^2}) = 5000\ \mathrm{N}.$$

(If you are used to British units, to get an idea of what a 5000-N force is, you can divide by 4.45 N/lb and get a force of about 1000 lb.)

(*b*) For the apple, $m = 200\ \mathrm{g} = 0.2\ \mathrm{kg}$, so

$$\Sigma F = ma \approx (0.2\ \mathrm{kg})(5\ \mathrm{m/s^2}) = 1\ \mathrm{N}.$$

Example

EXAMPLE 4–3 **Force to stop a car.** What average net force is required to bring a 1500-kg car to rest from a speed of 100 km/h within a distance of 55 m?

Example

EXAMPLE 4–3 Force to stop a car. What average net force is required to bring a 1500-kg car to rest from a speed of 100 km/h within a distance of 55 m?

APPROACH We use Newton's second law, $\Sigma F = ma$, to determine the force, but first we need to calculate the acceleration a. We assume the acceleration is constant so that we can use the kinematic equations, Eqs. 2–11, to calculate it.

FIGURE 4–6 Example 4–3.

SOLUTION We assume the motion is along the $+x$ axis (Fig. 4–6). We are given the initial velocity $v_0 = 100\,\mathrm{km/h} = 27.8\,\mathrm{m/s}$ (Section 1–6), the final velocity $v = 0$, and the distance traveled $x - x_0 = 55\,\mathrm{m}$. From Eq. 2–11c, we have

$$v^2 = v_0^2 + 2a(x - x_0),$$

so

$$a = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{0 - (27.8\,\mathrm{m/s})^2}{2(55\,\mathrm{m})} = -7.0\,\mathrm{m/s^2}.$$

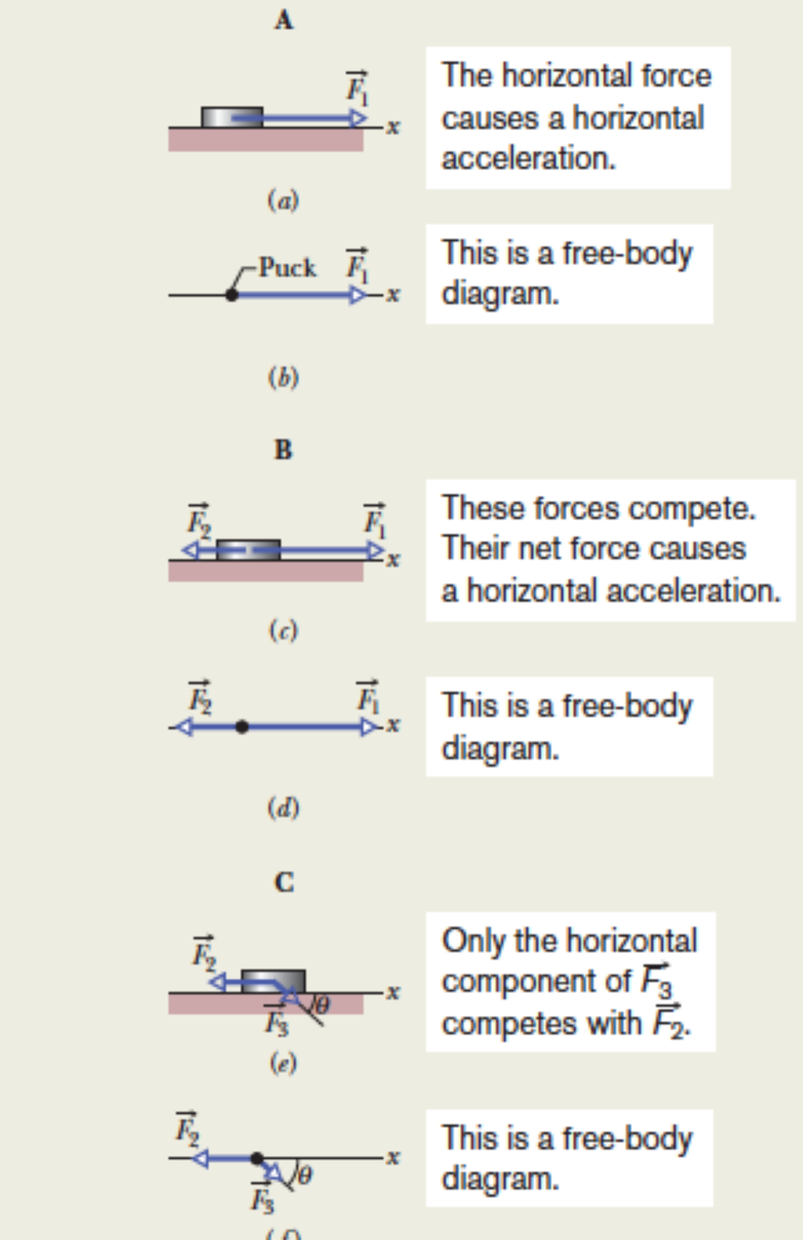
The net force required is then

$$\Sigma F = ma = (1500\,\mathrm{kg})(-7.0\,\mathrm{m/s^2}) = -1.1 \times 10^4\,\mathrm{N},$$

or 11,000 N. The force must be exerted in the direction *opposite* to the initial velocity, which is what the negative sign means.

Example

Here are examples of how to use Newton's second law for a puck when one or two forces act on it. Parts A, B, and C of Fig. 5-3 show three situations in which one or two forces act on a puck that moves over frictionless ice along an x axis, in one-dimensional motion. The puck's mass is $m = 0.20$ kg. Forces $\vec{F}_1$ and $\vec{F}_2$ are directed along the axis and have magnitudes $F_1 = 4.0$ N and $F_2 = 2.0$ N. Force $\vec{F}_3$ is directed at angle $\theta = 30°$ and has magnitude $F_3 = 1.0$ N. In each situation, what is the acceleration of the puck?



Example

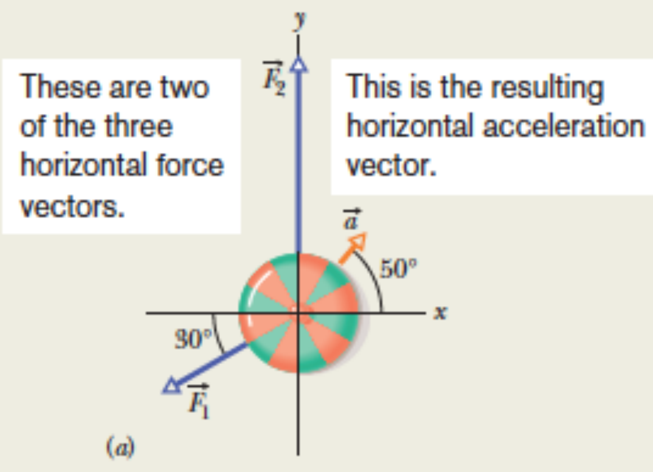


Figure 5-4 (*a*) An overhead view of two of three horizontal forces that act on a co tin, resulting in acceleration $\vec{a}$. $\vec{F}_3$ is not shown. (*b*) An arrangement of vectors *m* and $-\vec{F}_2$ to find force $\vec{F}_3$.

Here we find a missing force by using the acceleration. In the overhead view of Fig. 5-4*a*, a 2.0 kg cookie tin is accelerated at 3.0 m/s^2 in the direction shown by $\vec{a}$, over a frictionless horizontal surface. The acceleration is caused by three horizontal forces, only two of which are shown: $\vec{F}_1$ of magnitude 10 N and $\vec{F}_2$ of magnitude 20 N. What is the third force $\vec{F}_3$ in unit-vector notation and in magnitude-angle notation?

Example

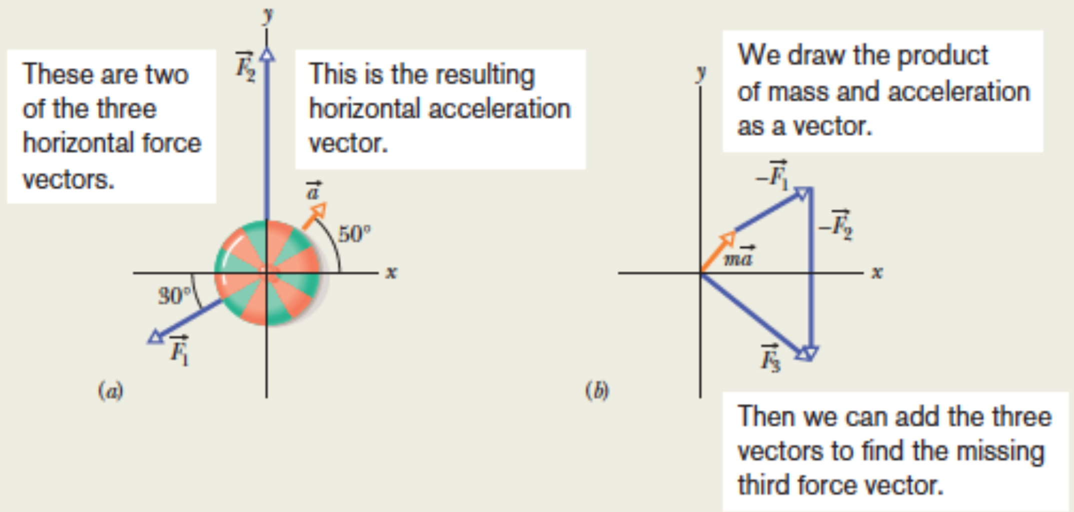


Figure 5-4 (*a*) An overhead view of two of three horizontal forces that act on a cookie tin, resulting in acceleration $\vec{a}$. $\vec{F}_3$ is not shown. (*b*) An arrangement of vectors $m\vec{a}$, $-\vec{F}_1$, and $-\vec{F}_2$ to find force $\vec{F}_3$.

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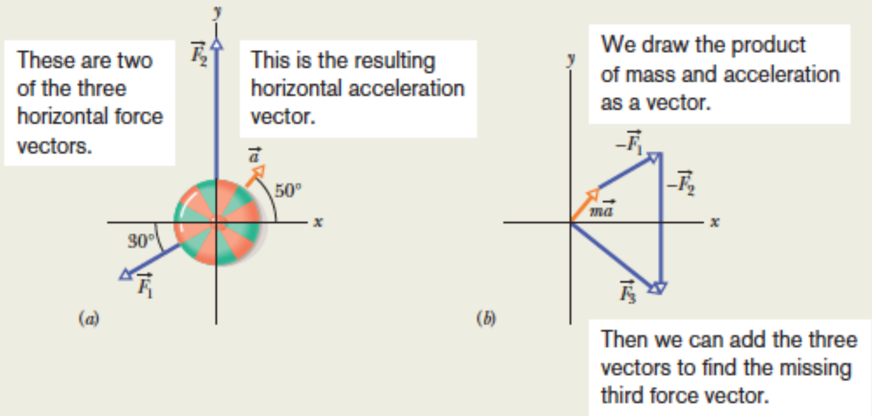


Figure 5-4 (*a*) An overhead view of two of three horizontal forces that act on a cookie tin, resulting in acceleration $\vec{a}$. $\vec{F}_3$ is not shown. (*b*) An arrangement of vectors $m\vec{a}$, $-\vec{F}_1$, and $-\vec{F}_2$ to find force $\vec{F}_3$.

The net force $\vec{F}_{net}$ on the tin is the sum of the three forces and is related to the acceleration $\vec{a}$ via Newton's second law ($\vec{F}_{net} = m\vec{a}$). Thus,

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = m\vec{a}, \tag{5-6}$$

which gives us

$$\vec{F}_3 = m\vec{a} - \vec{F}_1 - \vec{F}_2. \tag{5-7}$$

***x* components:** Along the *x* axis we have

$$F_{3,x} = ma_x - F_{1,x} - F_{2,x}$$
$$= m(a\cos 50°) - F_1\cos(-150°) - F_2\cos 90°.$$

Then, substituting known data, we find

$$F_{3,x} = (2.0\text{ kg})(3.0\text{ m/s}^2)\cos 50° - (10\text{ N})\cos(-150°)$$
$$- (20\text{ N})\cos 90°$$
$$= 12.5\text{ N}.$$

***y* components:** Similarly, along the *y* axis we find

$$F_{3,y} = ma_y - F_{1,y} - F_{2,y}$$
$$= m(a\sin 50°) - F_1\sin(-150°) - F_2\sin 90°$$
$$= (2.0\text{ kg})(3.0\text{ m/s}^2)\sin 50° - (10\text{ N})\sin(-150°)$$
$$- (20\text{ N})\sin 90°$$
$$= -10.4\text{ N}.$$

Vector: In unit-vector notation, we can write

$$\vec{F}_3 = F_{3,x}\hat{\imath} + F_{3,y}\hat{\jmath} = (12.5\text{ N})\hat{\imath} - (10.4\text{ N})\hat{\jmath}$$
$$\approx (13\text{ N})\hat{\imath} - (10\text{ N})\hat{\jmath}. \qquad \text{(Answer)}$$

We can now use a vector-capable calculator to get the magnitude and the angle of $\vec{F}_3$. We can also use Eq. 3-6 to obtain the magnitude and the angle (from the positive direction of the *x* axis) as

$$F_3 = \sqrt{F_{3,x}^2 + F_{3,y}^2} = 16\text{ N}$$

and

$$\theta = \tan^{-1}\frac{F_{3,y}}{F_{3,x}} = -40°. \qquad \text{(Answer)}$$

5-2 Some Particular Forces

- Gravitational Force

✓ Mutual force of attraction between any two objects

✓ Expressed by Newton's Law of Universal Gravitation:

$$F_g = G\frac{m_1 m_2}{r^2}$$

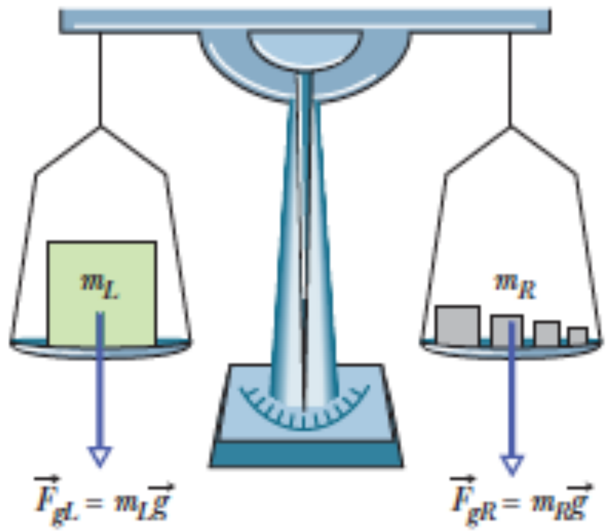
Weight

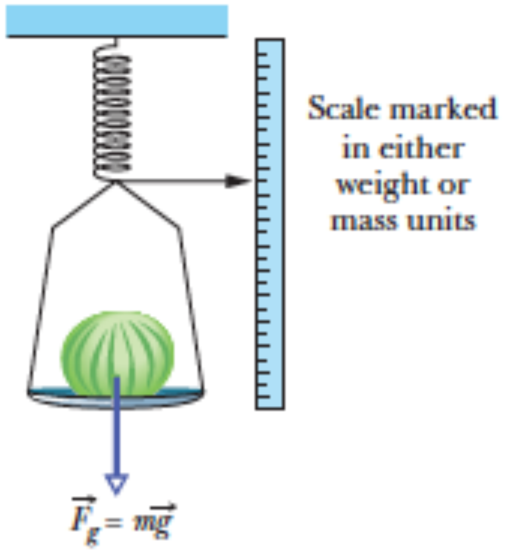
- The magnitude of the gravitational force acting on an object of mass *m* near the Earth's surface is called the weight *w* of the object

  ✓ $w = m g$ is a special case of Newton's Second Law

- g can also be found from the Law of Universal Gravitation

Weight





$$F_{\text{net},y} = ma_y.$$

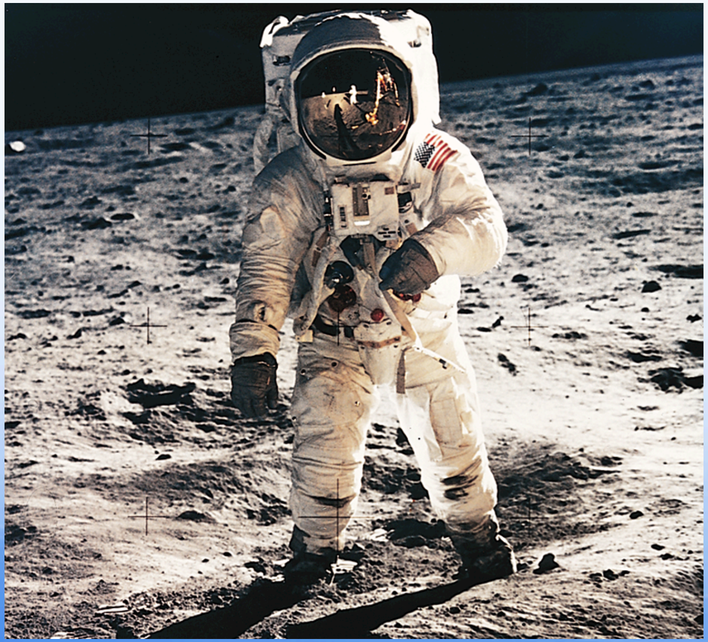
In our situation, this becomes

$$W - F_g = m(0)$$

or $\quad W = F_g \quad$ (weight, with ground as inertial frame).

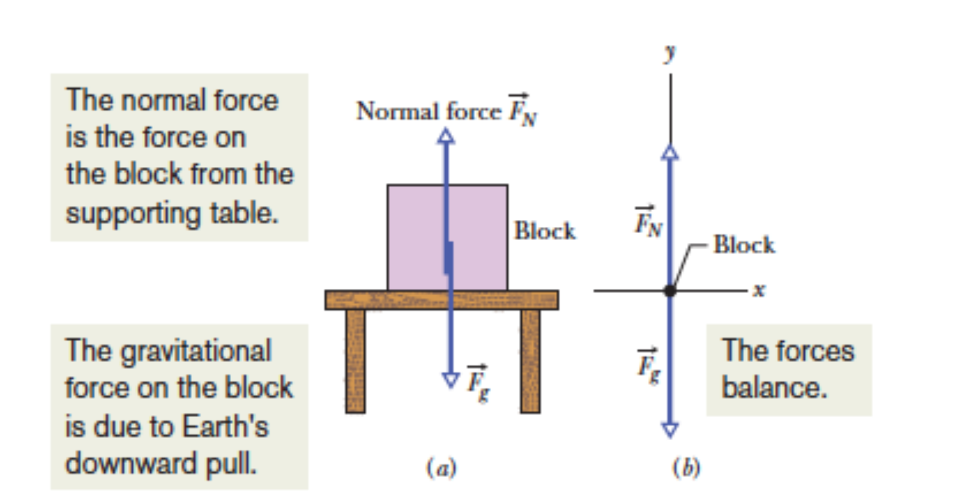
More about weight

- Weight is **not** an inherent property of an object
  - ✓ mass **is** an inherent property
- Weight depends upon location

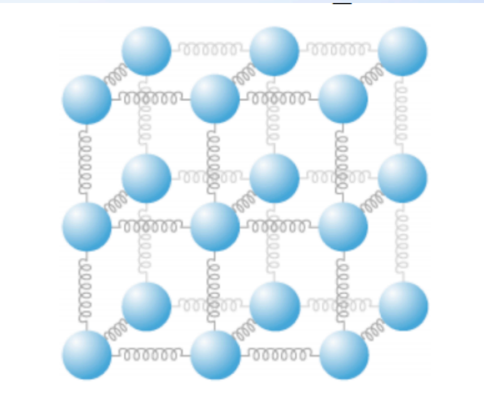


The Normal Force

- When a body presses against a surface, the surface (even a seemingly rigid one) deforms and pushes on the body with a normal force $\vec{F}_N$ that is perpendicular to the surface.



$$F_N = mg.$$



Friction (see next chapter)

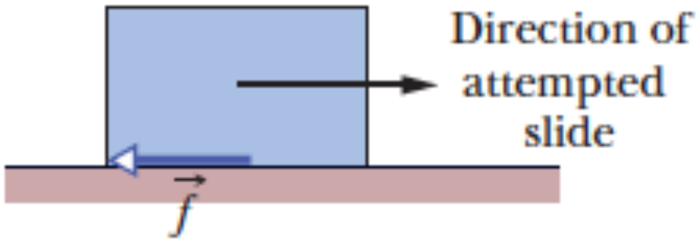
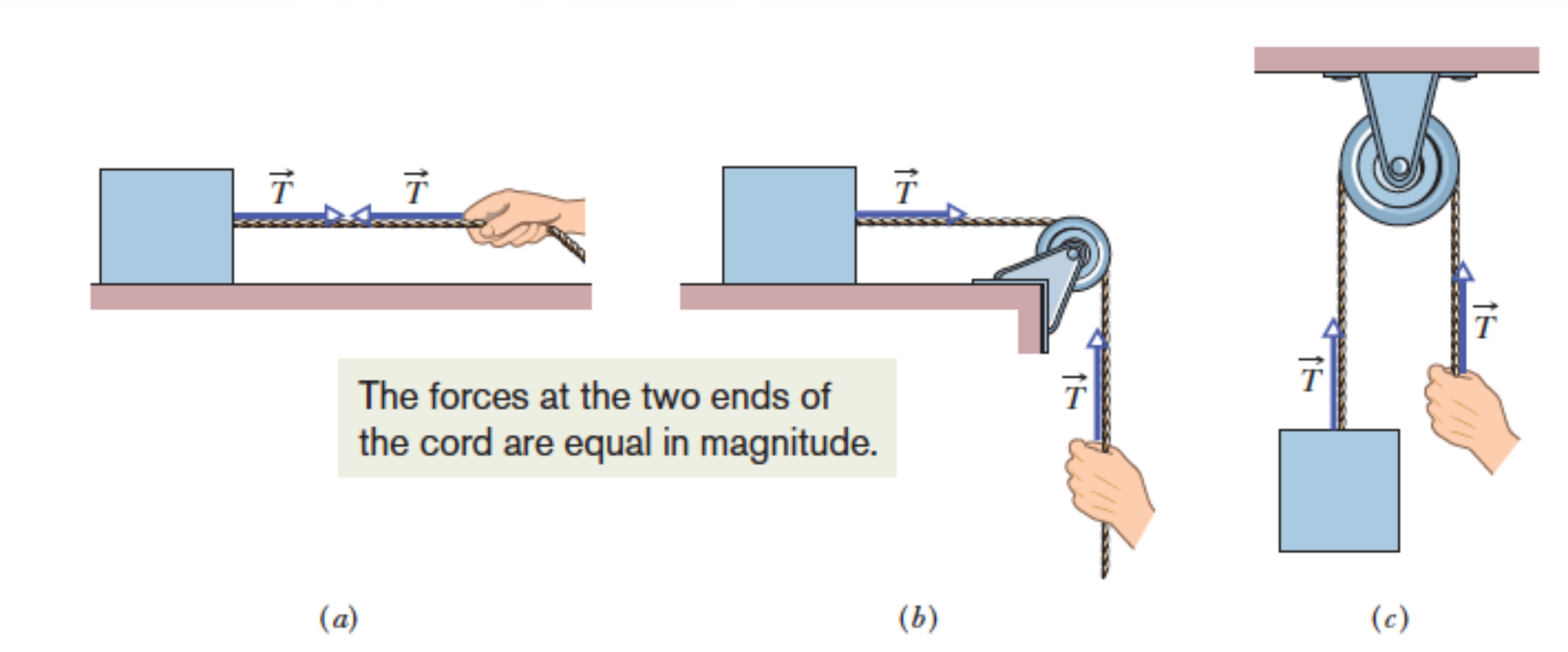


Figure 5-8 A frictional force $\vec{f}$ opposes the attempted slide of a body over a surface.

Tension



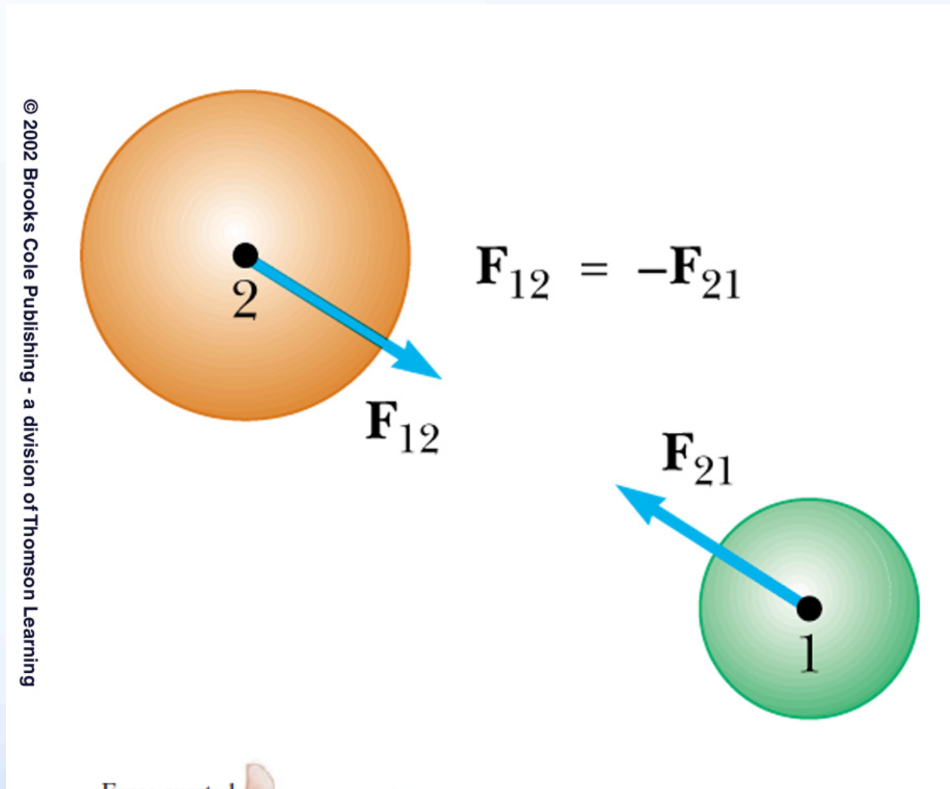
$\vec{T}$
$\vec{T}$
$\vec{T}$
$\vec{T}$
$\vec{T}$
$\vec{T}$
The forces at the two ends of the cord are equal in magnitude.
(a)
(b)
(c)

Newton’s Third Law

- When two bodies interact, the forces on the bodies from each other are always equal in magnitude and opposite in direction.

Example: Newton’s Third Law

- Consider collision of two spheres
- F_{12} may be called the *action* force and F_{21} the *reaction* force
  - Actually, either force can be the action or the reaction force
- The action and reaction forces act on **different** objects



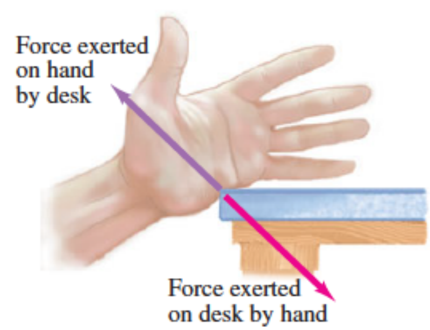
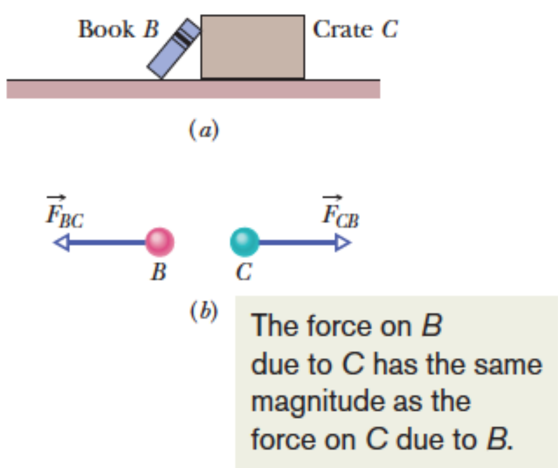


FIGURE 4–8 If your hand pushes against the edge of a desk (the force vector is shown in red), the desk pushes back against your hand (this force vector is shown in a different color, violet, to remind us that this force acts on a different object).

Example



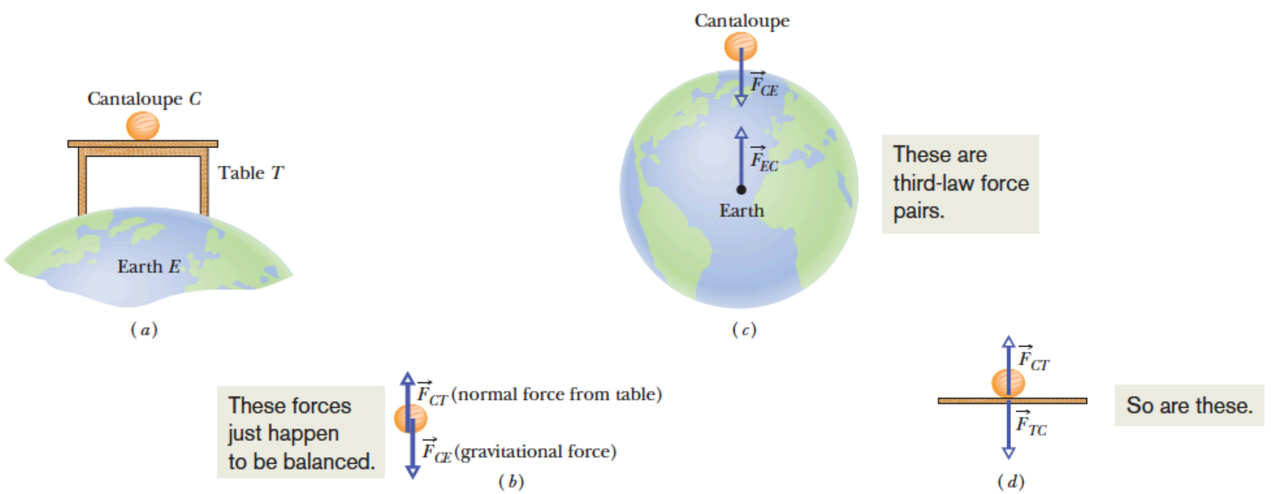
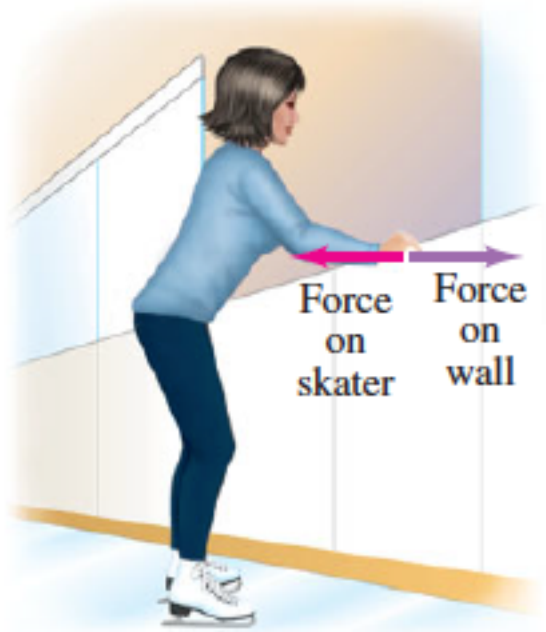


Figure 5-11 (a) A cantaloupe lies on a table that stands on Earth. (b) The forces *on the cantaloupe* are $\vec{F}_{CT}$ and $\vec{F}_{CE}$. (c) The third-law force pair for the cantaloupe–Earth interaction. (d) The third-law force pair for the cantaloupe–table interaction.

Example

FIGURE 4–9 An example of Newton's third law: when an ice skater pushes against the wall, the wall pushes back and this force causes her to accelerate away.

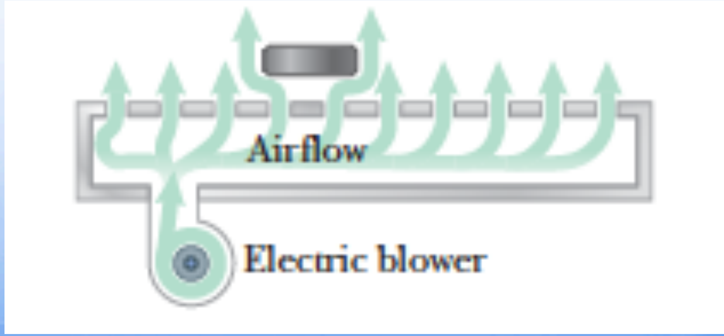


Applying Newton's Laws

- Assumptions
  - ✓ Objects behave as particles
    - ✓ can ignore rotational motion (for now)
  - ✓ Masses of strings or ropes are negligible
  - ✓ Interested only in the forces acting on the object

Free Body Diagram

- Must identify all the forces acting on the object of interest
- Choose an appropriate coordinate system
- If the free body diagram is incorrect, the solution will likely be incorrect



Example

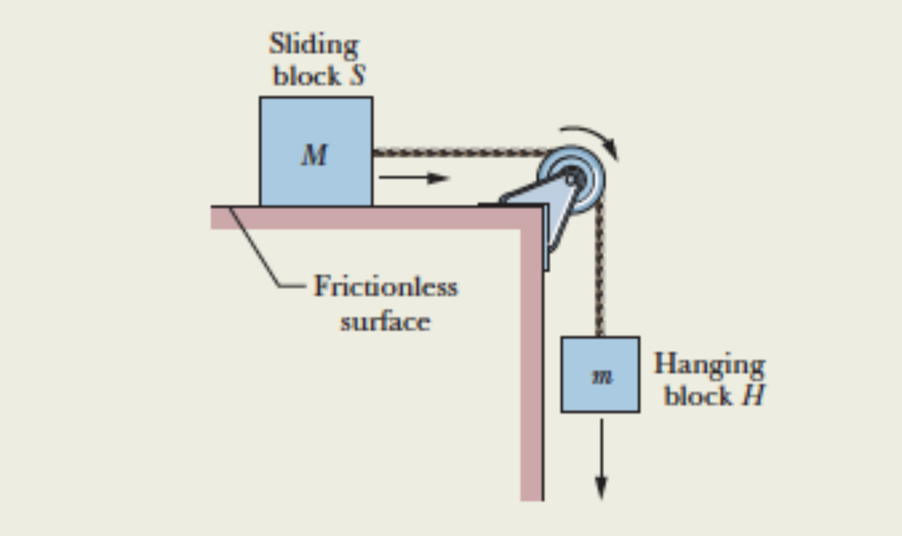
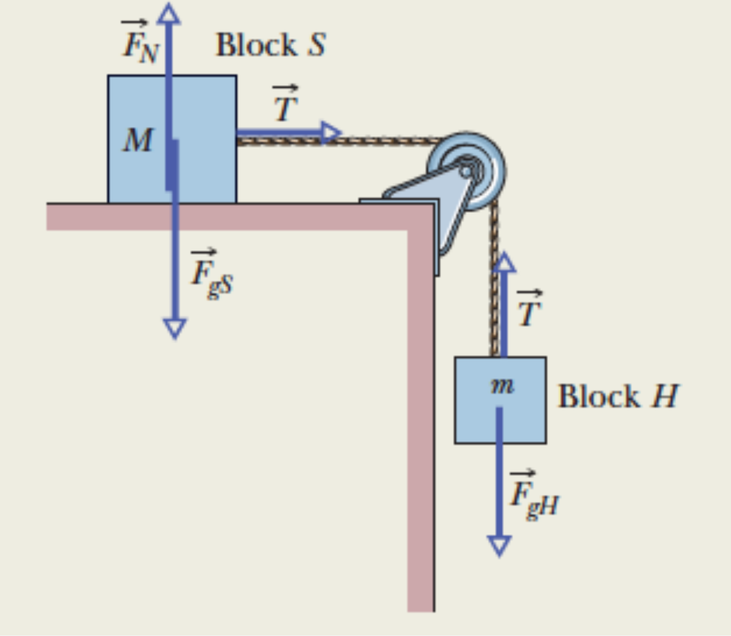


Figure 5-12 shows a block S (the *sliding block*) with mass $M = 3.3$ kg. The block is free to move along a horizontal frictionless surface and connected, by a cord that wraps over a frictionless pulley, to a second block H (the *hanging block*), with mass $m = 2.1$ kg. The cord and pulley have negligible masses compared to the blocks (they are "massless"). The hanging block H falls as the sliding block S accelerates to the right. Find (a) the acceleration of block S, (b) the acceleration of block H, and (c) the tension in the cord.



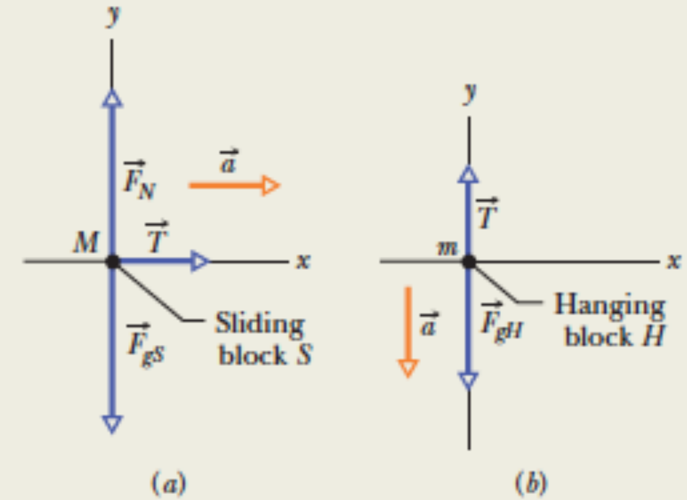


Figure 5-14 (*a*) A free-body diagram for block *S* of Fig. 5-12. (*b*) A free-body diagram for block *H* of Fig. 5-12.

$$\vec{F}_{\text{net}} = M\vec{a}$$

$$F_{\text{net},x} = Ma_x \qquad F_{\text{net},y} = Ma_y \qquad F_{\text{net},z} = Ma_z$$

S x $F_N - F_{gS} = 0$ or $F_N = F_{gS}$.

y $T = Ma.$

H y $T - F_{gH} = ma_y.$

$$T - mg = -ma.$$

$$a = \frac{m}{M+m}g.$$

$$T = \frac{Mm}{M+m}g.$$

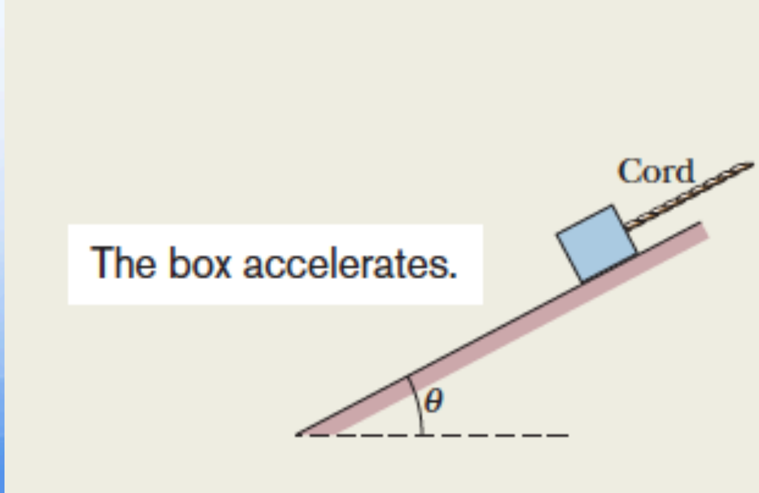
Putting in the numbers gives, for these two quantities,

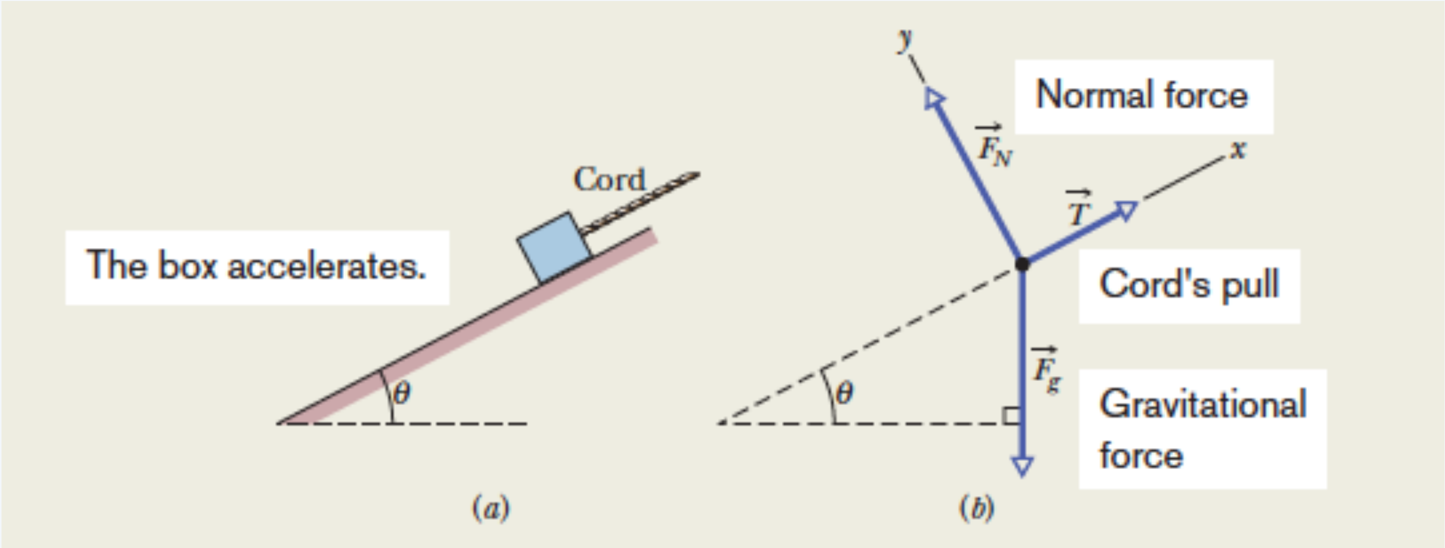
$$a = \frac{m}{M+m}g = \frac{2.1\text{ kg}}{3.3\text{ kg} + 2.1\text{ kg}}(9.8\text{ m/s}^2)$$
$$= 3.8\text{ m/s}^2 \qquad \text{(Answer)}$$

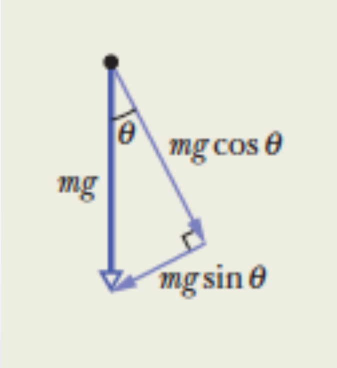
and $$T = \frac{Mm}{M+m}g = \frac{(3.3\text{ kg})(2.1\text{ kg})}{3.3\text{ kg} + 2.1\text{ kg}}(9.8\text{ m/s}^2)$$
$$= 13\text{ N}. \qquad \text{(Answer)}$$

Example

Figure 5-15 (*a*) A box is pulled up a plane by a cord. (*b*) The three forces acting on the box: the cord's force $\vec{T}$, the gravitational force $\vec{F}_g$, and the normal force $\vec{F}_N$. (*c*)–(*i*) Finding the force components along the plane and perpendicular to it. **In *WileyPLUS*, this figure is available as an animation with voiceover.**







$$F_{\text{net},x} = ma_x.$$

$$T - mg\sin\theta = ma. \tag{5-24}$$

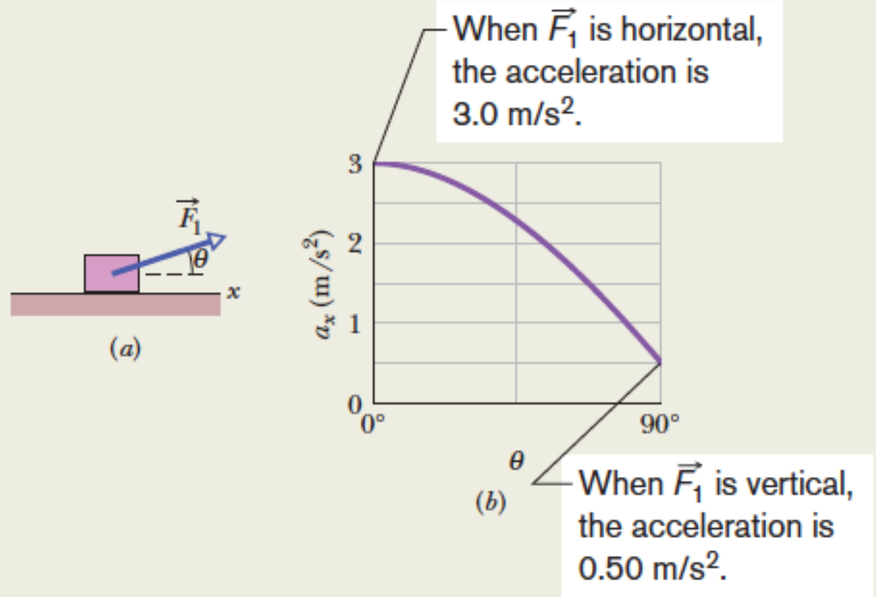
Substituting data and solving for a, we find

$$a = 0.100\ \text{m/s}^2. \qquad \text{(Answer)}$$

Example

Sample Problem 5.05 Reading a force graph

Here is an example of where you must dig information out of a graph, not just read off a number. In Fig. 5-16*a*, two forces are applied to a 4.00 kg block on a frictionless floor, but only force $\vec{F}_1$ is indicated. That force has a fixed magnitude but can be applied at an adjustable angle θ to the positive direction of the x axis. Force $\vec{F}_2$ is horizontal and fixed in both magnitude and angle. Figure 5-16*b* gives the horizontal acceleration a_x of the block for any given value of θ from 0° to 90°. What is the value of a_x for $\theta = 180°$?



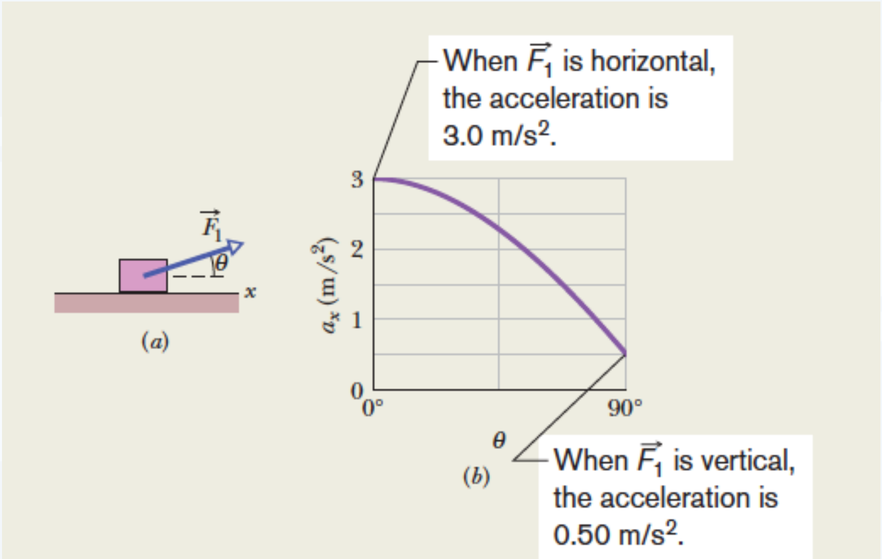


Figure 5-16 (*a*) One of the two forces applied to a block is shown. Its angle θ can be varied. (*b*) The block's acceleration component a_x versus θ.

(1) The horizontal acceleration a_x depends on the net horizontal force $F_{\text{net},x}$, as given by Newton's second law. (2) The net horizontal force is the sum of the horizontal components of forces $\vec{F}_1$ and $\vec{F}_2$.

Calculations: The x component of $\vec{F}_2$ is F_2 because the vector is horizontal. The x component of $\vec{F}_1$ is $F_1 \cos\theta$. Using these expressions and a mass m of 4.00 kg, we can write Newton's second law ($\vec{F}_{\text{net}} = m\vec{a}$) for motion along the x axis as

$$F_1 \cos\theta + F_2 = 4.00a_x. \qquad (5\text{-}25)$$

From this equation we see that when angle $\theta = 90°$, $F_1 \cos\theta$ is zero and $F_2 = 4.00a_x$. From the graph we see that the corresponding acceleration is $0.50\ \text{m/s}^2$. Thus, $F_2 = 2.00$ N and $\vec{F}_2$ must be in the positive direction of the x axis.

From Eq. 5-25, we find that when $\theta = 0°$,

$$F_1 \cos 0° + 2.00 = 4.00a_x. \qquad (5\text{-}26)$$

From the graph we see that the corresponding acceleration is $3.0\ \text{m/s}^2$. From Eq. 5-26, we then find that $F_1 = 10$ N.

Substituting $F_1 = 10$ N, $F_2 = 2.00$ N, and $\theta = 180°$ into Eq. 5-25 leads to

$$a_x = -2.00\ \text{m/s}^2. \qquad \text{(Answer)}$$

Example

$$F_N - F_g = ma$$

or

$$F_N = F_g + ma. \tag{5-27}$$

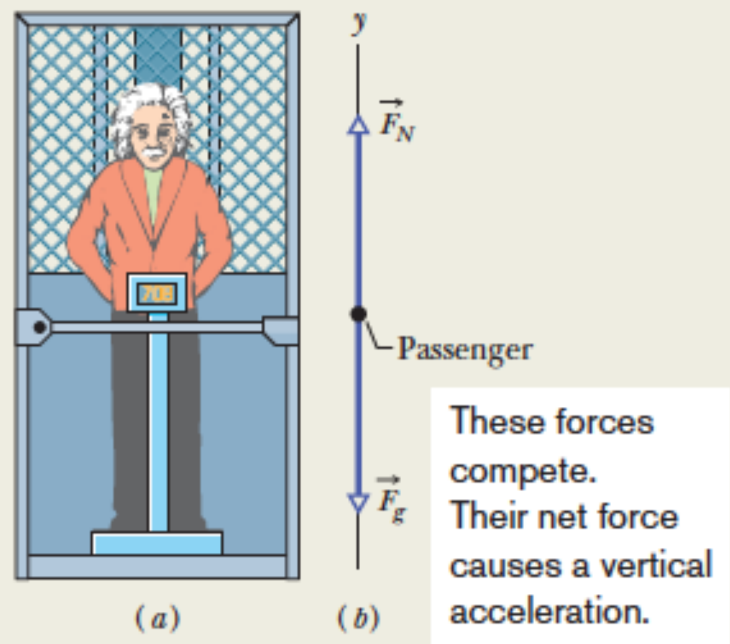
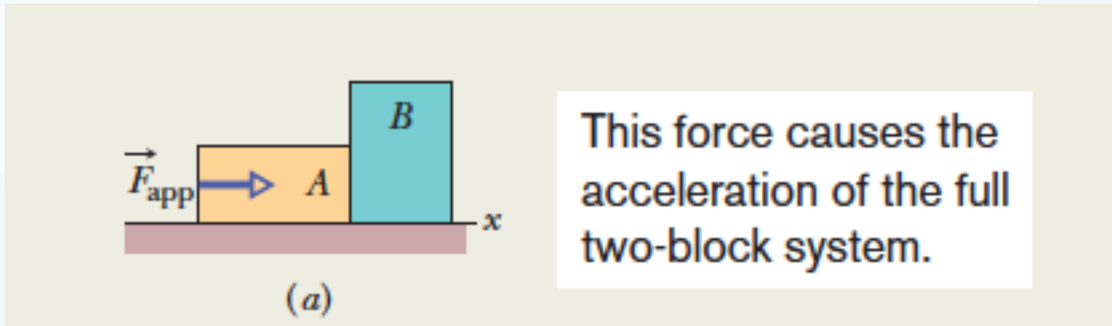


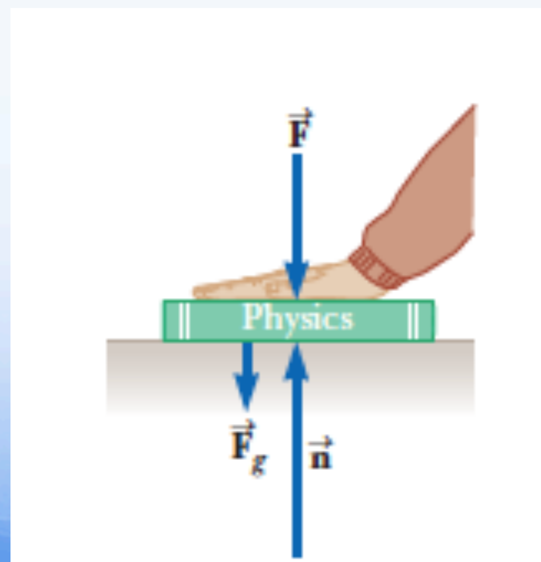
Figure 5-17 (*a*) A passenger stands on a platform scale that indicates either his weight or his apparent weight. (*b*) The free-body diagram for the passenger, showing the normal force $\vec{F}_N$ on him from the scale and the gravitational force $\vec{F}_g$.

$$F_N = m(g + a) \quad \text{(Answer)}$$

(a) Find a general solution for the scale reading, whatever the vertical motion of the cab.

Example

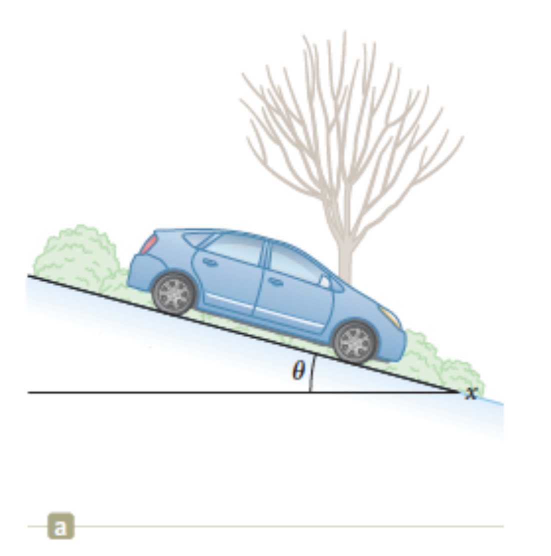

$\vec{F}_{app}$
A
B
x
(a)
This force causes the acceleration of the full two-block system.


$\vec{F}$
Physics
$\vec{F}_g$
$\vec{n}$

Example

A car of mass m is on an icy driveway inclined at an angle θ as in Figure 5.11a.

(A) Find the acceleration of the car, assuming the driveway is frictionless.

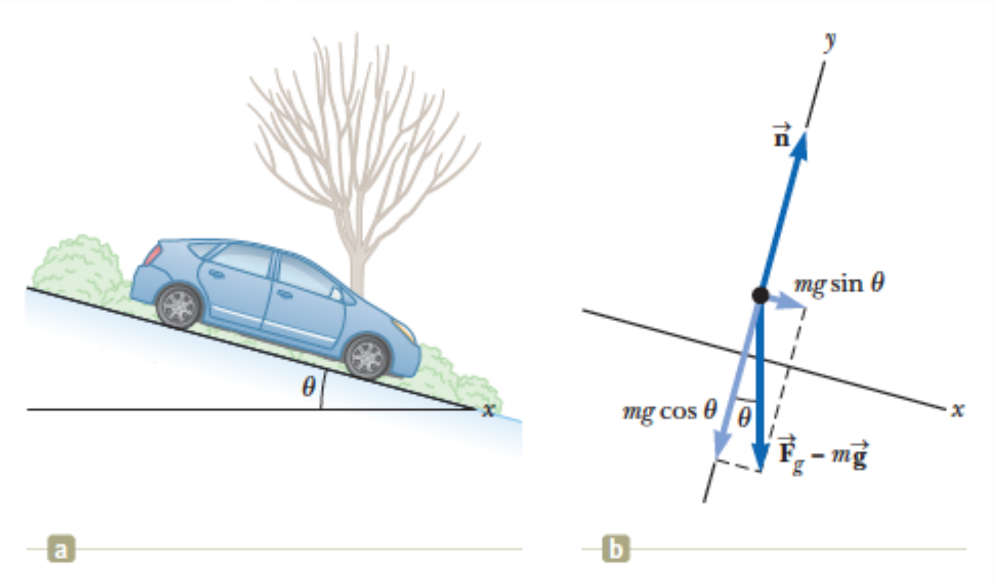


(B) Suppose the car is released from rest at the top of the incline and the distance from the car's front bumper to the bottom of the incline is d. How long does it take the front bumper to reach the bottom of the hill, and what is the car's speed as it arrives there?

$$(1)\ \sum F_x = mg\sin\theta = ma_x$$

$$(2)\ \sum F_y = n - mg\cos\theta = 0$$

$$(3)\ a_x = \boxed{g\sin\theta}$$



Analyze Defining the initial position of the front bumper as $x_i = 0$ and its final position as $x_f = d$, and recognizing that $v_{xi} = 0$, apply Equation 2.16, $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$:

$$d = \tfrac{1}{2}a_x t^2$$

Solve for t:

$$(4)\ t = \sqrt{\frac{2d}{a_x}} = \boxed{\sqrt{\frac{2d}{g\sin\theta}}}$$

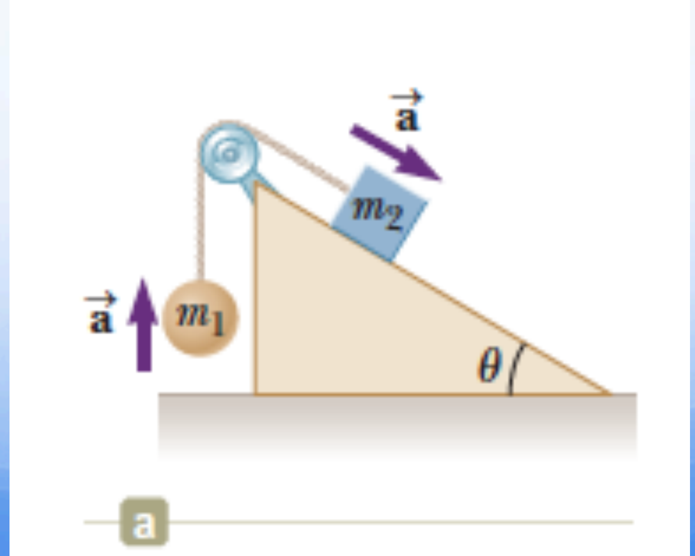
Use Equation 2.17, with $v_{xi} = 0$, to find the final velocity of the car:

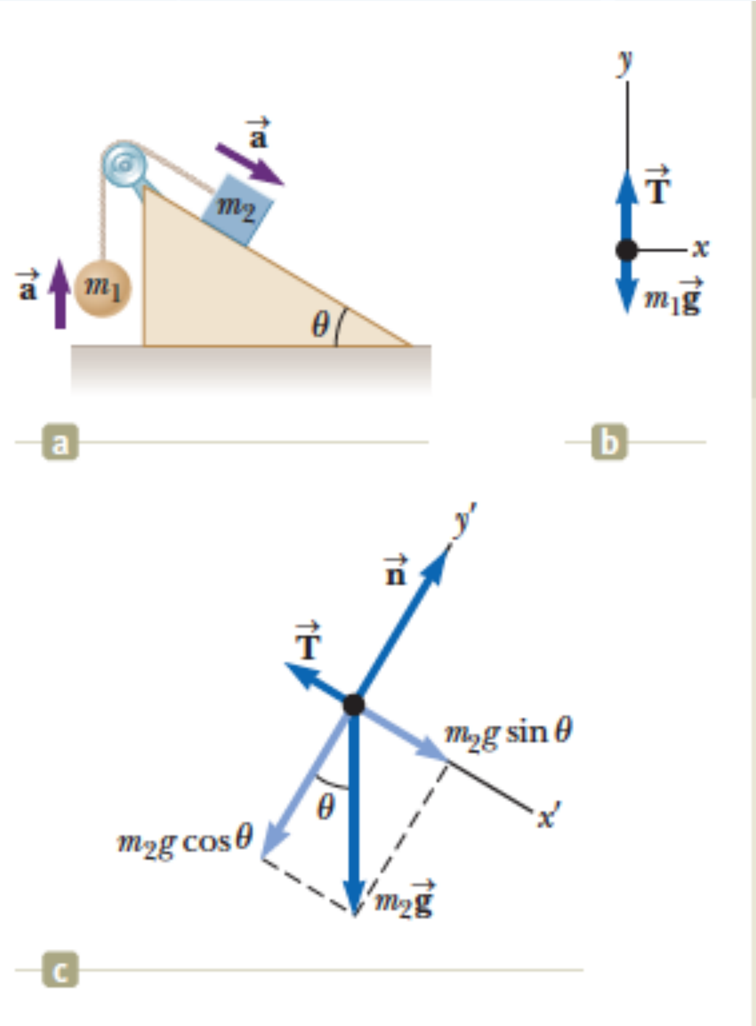
$$v_{xf}^2 = 2a_x d$$

$$(5)\ v_{xf} = \sqrt{2a_x d} = \boxed{\sqrt{2gd\sin\theta}}$$

Example

A ball of mass m_1 and a block of mass m_2 are attached by a lightweight cord that passes over a frictionless pulley of negligible mass as in Figure 5.15a. The block lies on a frictionless incline of angle θ. Find the magnitude of the acceleration of the two objects and the tension in the cord.





$$(1)\ \sum F_x = 0$$

$$(2)\ \sum F_y = T - m_1 g = m_1 a_y = m_1 a$$

$$(3)\ \sum F_{x'} = m_2 g \sin\theta - T = m_2 a_{x'} = m_2 a$$

$$(4)\ \sum F_{y'} = n - m_2 g \cos\theta = 0$$

$$(5)\ T = m_1(g + a)$$

$$m_2 g \sin\theta - m_1(g + a) = m_2 a$$

$$(6)\ a = \left(\frac{m_2 \sin\theta - m_1}{m_1 + m_2}\right) g$$

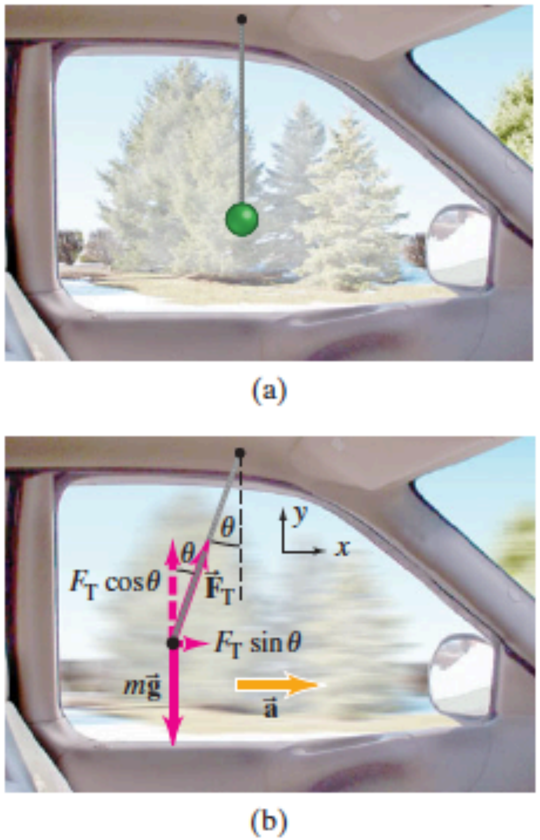
$$(7)\ T = \left(\frac{m_1 m_2(\sin\theta + 1)}{m_1 + m_2}\right) g$$

Atwood Machine!

Example

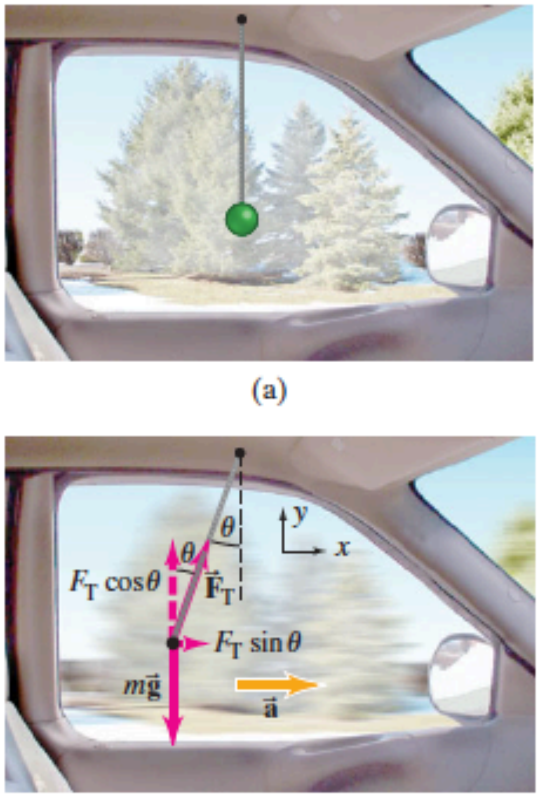
PHYSICS APPLIED
Accelerometer

FIGURE 4–25 Example 4–15.



PHYSICS APPLIED
Accelerometer

FIGURE 4–25 Example 4–15.



(a)

(b)

EXAMPLE 4–15 **Accelerometer.** A small mass m hangs from a thin string and can swing like a pendulum. You attach it above the window of your car as shown in Fig. 4–25a. When the car is at rest, the string hangs vertically. What angle θ does the string make (*a*) when the car accelerates at a constant $a = 1.20\ \mathrm{m/s^2}$, and (*b*) when the car moves at constant velocity, $v = 90\ \mathrm{km/h}$?

APPROACH The free-body diagram of Fig. 4–25b shows the pendulum at some angle θ relative to the vertical, and the forces on it: $m\vec{g}$ downward, and the tension $\vec{F}_T$ in the cord (including its components). These forces do not add up to zero if $\theta \neq 0$; and since we have an acceleration a, we expect $\theta \neq 0$.

SOLUTION (*a*) The acceleration $a = 1.20\ \mathrm{m/s^2}$ is horizontal ($= a_x$), and the only horizontal force is the x component of $\vec{F}_T$, $F_T \sin\theta$ (Fig. 4–25b). Then from Newton's second law,

$$ma = F_T \sin\theta.$$

The vertical component of Newton's second law gives, since $a_y = 0$,

$$0 = F_T \cos\theta - mg.$$

So

$$mg = F_T \cos\theta.$$

Dividing these two equations, we obtain

$$\tan\theta = \frac{F_T \sin\theta}{F_T \cos\theta} = \frac{ma}{mg} = \frac{a}{g}$$

or

$$\tan\theta = \frac{1.20\ \mathrm{m/s^2}}{9.80\ \mathrm{m/s^2}} = 0.122,$$

so

$$\theta = 7.0°.$$

(*b*) The velocity is constant, so $a = 0$ and $\tan\theta = 0$. Hence the pendulum hangs vertically ($\theta = 0°$).

Example

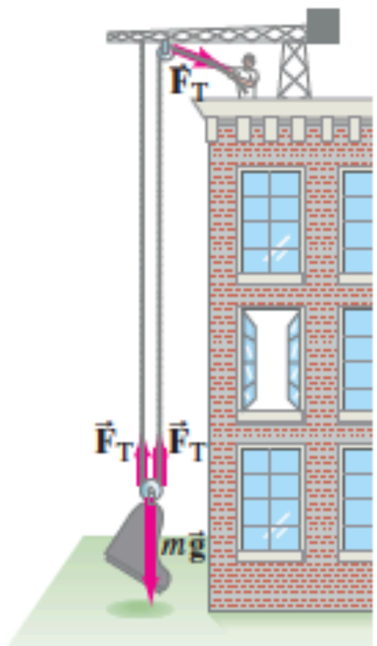


FIGURE 4–24 Example 4–14.

CONCEPTUAL EXAMPLE 4–14 **The advantage of a pulley.** A mover is trying to lift a piano (slowly) up to a second-story apartment (Fig. 4–24). He is using a rope looped over two pulleys as shown. What force must he exert on the rope to slowly lift the piano's 1600-N weight?

RESPONSE The magnitude of the tension force F_T within the rope is the same at any point along the rope if we assume we can ignore its mass. First notice the forces acting on the lower pulley at the piano. The weight of the piano $(= mg)$ pulls down on the pulley. The tension in the rope, looped through this pulley, pulls up *twice*, once on each side of the pulley. Let us apply Newton's second law to the pulley–piano combination (of mass m), choosing the upward direction as positive:

$$2F_T - mg = ma.$$

To move the piano with constant speed (set $a = 0$ in this equation) thus requires a tension in the rope, and hence a pull on the rope, of $F_T = mg/2$. The piano mover can exert a force equal to half the piano's weight.

NOTE We say the pulley has given a **mechanical advantage** of 2, since without the pulley the mover would have to exert twice the force.

Example

EXAMPLE 4–13 **Elevator and counterweight (Atwood machine).** A system of two objects suspended over a pulley by a flexible cable, as shown in Fig. 4–23a, is sometimes referred to as an *Atwood machine*. Consider the real-life application of an elevator (m_E) and its counterweight (m_C). To minimize the work done by the motor to raise and lower the elevator safely, m_E and m_C are made similar in mass. We leave the motor out of the system for this calculation, and assume that the cable's mass is negligible and that the mass of the pulley, as well as any friction, is small and ignorable. These assumptions ensure that the tension F_T in the cable has the same magnitude on both sides of the pulley. Let the mass of the counterweight be $m_C = 1000$ kg. Assume the mass of the empty elevator is 850 kg, and its mass when carrying four passengers is $m_E = 1150$ kg. For the latter case ($m_E = 1150$ kg), calculate (*a*) the acceleration of the elevator and (*b*) the tension in the cable.

APPROACH Again we have two objects, and we will need to apply Newton's second law to each of them separately. Each mass has two forces acting on it: gravity downward and the cable tension pulling upward, $\vec{F}_T$. Figures 4–23b and c show the free-body diagrams for the elevator (m_E) and for the counterweight (m_C). The elevator, being the heavier, will accelerate downward, whereas the counterweight will accelerate upward. The magnitudes of their accelerations will be equal (we assume the cable is massless and doesn't stretch). For the counterweight, $m_C g = (1000\text{ kg})(9.80\text{ m/s}^2) = 9800$ N, so F_T must be greater than 9800 N (in order that m_C will accelerate upward). For the elevator, $m_E g = (1150\text{ kg})(9.80\text{ m/s}^2) = 11{,}300$ N, which must have greater magnitude than F_T so that m_E accelerates downward. Thus our calculation must give F_T between 9800 N and 11,300 N.

SOLUTION (*a*) To find F_T as well as the acceleration a, we apply Newton's second law, $\Sigma F = ma$, to each object. We take upward as the positive y direction for both objects. With this choice of axes, $a_C = a$ because m_C accelerates upward, and $a_E = -a$ because m_E accelerates downward. Thus

$$F_T - m_E g = m_E a_E = -m_E a$$
$$F_T - m_C g = m_C a_C = +m_C a.$$

We can subtract the first equation from the second to get

$$(m_E - m_C)g = (m_E + m_C)a,$$

where a is now the only unknown. We solve this for a:

$$a = \frac{m_E - m_C}{m_E + m_C}g = \frac{1150\text{ kg} - 1000\text{ kg}}{1150\text{ kg} + 1000\text{ kg}}g = 0.070g = 0.68\text{ m/s}^2.$$

The elevator (m_E) accelerates downward (and the counterweight m_C upward) at $a = 0.070g = 0.68\text{ m/s}^2$.

(*b*) The tension in the cable F_T can be obtained from either of the two $\Sigma F = ma$ equations at the start of our solution, setting $a = 0.070g = 0.68\text{ m/s}^2$:

$$F_T = m_E g - m_E a = m_E(g - a)$$
$$= 1150\text{ kg}\,(9.80\text{ m/s}^2 - 0.68\text{ m/s}^2) = 10{,}500\text{ N},$$

or

$$F_T = m_C g + m_C a = m_C(g + a)$$
$$= 1000\text{ kg}\,(9.80\text{ m/s}^2 + 0.68\text{ m/s}^2) = 10{,}500\text{ N},$$

which are consistent. As predicted, our result lies between 9800 N and 11,300 N.

PHYSICS APPLIED
Elevator (as Atwood machine)

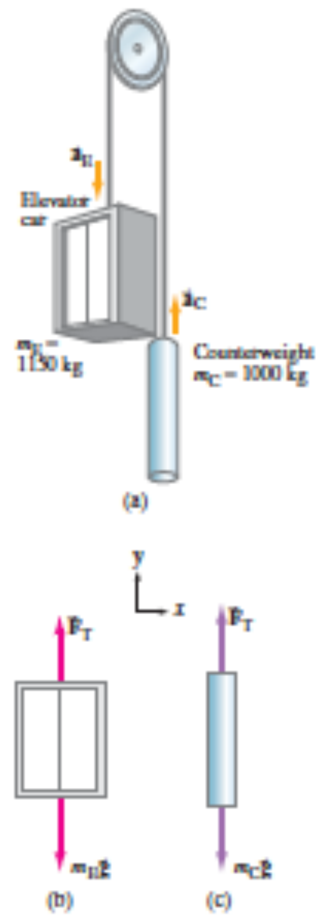


FIGURE 4–23 Example 4–13. (a) Atwood machine in the form of an elevator–counterweight system. (b) and (c) Free-body diagrams for the two objects.

Example

EXAMPLE 4–9 **Adding force vectors.** Calculate the sum of the two forces exerted on the boat by workers A and B in Fig. 4–19a.

APPROACH We add force vectors like any other vectors as described in Chapter 3. The first step is to choose an xy coordinate system (see Fig. 4–19a), and then resolve vectors into their components.

SOLUTION The two force vectors are shown resolved into components in Fig. 4–19b. We add the forces using the method of components. The components of $\vec{\mathbf{F}}_A$ are

$$F_{Ax} = F_A \cos 45.0° = (40.0\text{ N})(0.707) = 28.3\text{ N},$$
$$F_{Ay} = F_A \sin 45.0° = (40.0\text{ N})(0.707) = 28.3\text{ N}.$$

The components of $\vec{\mathbf{F}}_B$ are

$$F_{Bx} = +F_B \cos 37.0° = +(30.0\text{ N})(0.799) = +24.0\text{ N},$$
$$F_{By} = -F_B \sin 37.0° = -(30.0\text{ N})(0.602) = -18.1\text{ N}.$$

F_{By} is negative because it points along the negative y axis. The components of the resultant force are (see Fig. 4–19c)

$$F_{Rx} = F_{Ax} + F_{Bx} = 28.3\text{ N} + 24.0\text{ N} = 52.3\text{ N},$$
$$F_{Ry} = F_{Ay} + F_{By} = 28.3\text{ N} - 18.1\text{ N} = 10.2\text{ N}.$$

To find the magnitude of the resultant force, we use the Pythagorean theorem,

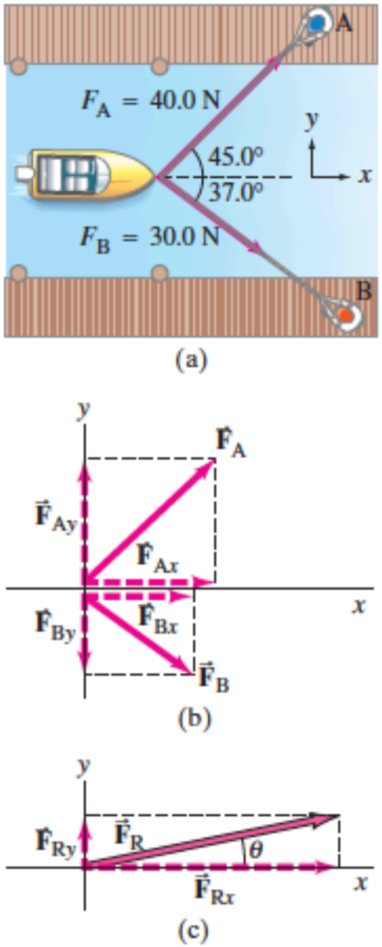
$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{(52.3)^2 + (10.2)^2}\text{ N} = 53.3\text{ N}.$$

The only remaining question is the angle θ that the net force $\vec{\mathbf{F}}_R$ makes with the x axis. We use:

$$\tan\theta = \frac{F_{Ry}}{F_{Rx}} = \frac{10.2\text{ N}}{52.3\text{ N}} = 0.195,$$

and $\tan^{-1}(0.195) = 11.0°$. The net force on the boat has magnitude 53.3 N and acts at an 11.0° angle to the x axis.

FIGURE 4–19 Example 4–9: Two force vectors act on a boat.



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SOLUTION The two force vectors are shown resolved into components in Fig. 4–19b. We add the forces using the method of components. The components of $\vec{\mathbf{F}}_A$ are

$$F_{Ax} = F_A \cos 45.0^\circ = (40.0\,\mathrm{N})(0.707) = 28.3\,\mathrm{N},$$
$$F_{Ay} = F_A \sin 45.0^\circ = (40.0\,\mathrm{N})(0.707) = 28.3\,\mathrm{N}.$$

The components of $\vec{\mathbf{F}}_B$ are

$$F_{Bx} = +F_B \cos 37.0^\circ = +(30.0\,\mathrm{N})(0.799) = +24.0\,\mathrm{N},$$
$$F_{By} = -F_B \sin 37.0^\circ = -(30.0\,\mathrm{N})(0.602) = -18.1\,\mathrm{N}.$$

F_{By} is negative because it points along the negative y axis. The components of the resultant force are (see Fig. 4–19c)

$$F_{Rx} = F_{Ax} + F_{Bx} = 28.3\,\mathrm{N} + 24.0\,\mathrm{N} = 52.3\,\mathrm{N},$$
$$F_{Ry} = F_{Ay} + F_{By} = 28.3\,\mathrm{N} - 18.1\,\mathrm{N} = 10.2\,\mathrm{N}.$$

To find the magnitude of the resultant force, we use the Pythagorean theorem,

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{(52.3)^2 + (10.2)^2}\,\mathrm{N} = 53.3\,\mathrm{N}.$$

The only remaining question is the angle θ that the net force $\vec{\mathbf{F}}_R$ makes with the x axis. We use:

$$\tan\theta = \frac{F_{Ry}}{F_{Rx}} = \frac{10.2\,\mathrm{N}}{52.3\,\mathrm{N}} = 0.195,$$

and $\tan^{-1}(0.195) = 11.0^\circ$. The net force on the boat has magnitude 53.3 N and acts at an 11.0° angle to the x axis.

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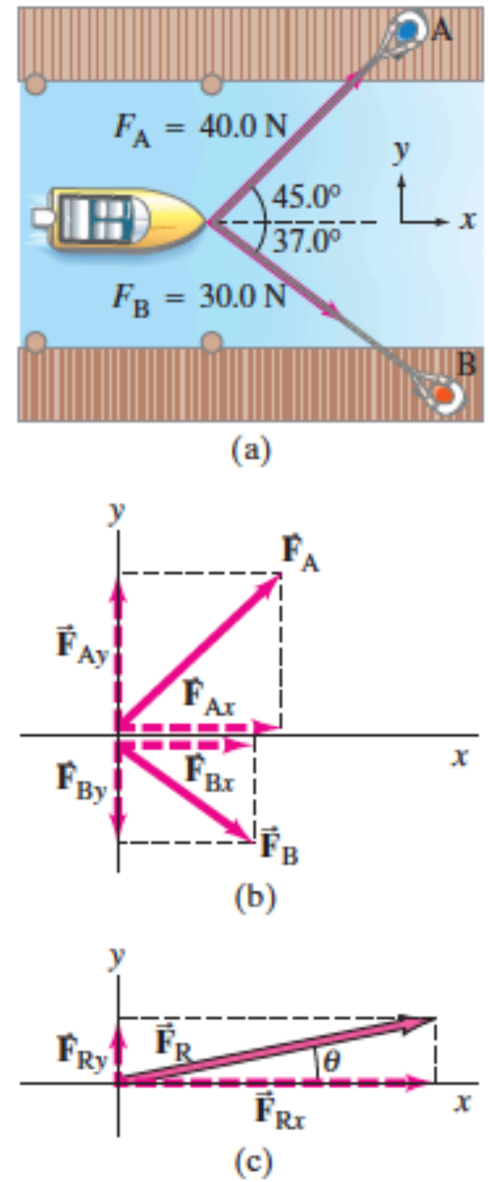


Example

Example 5.4 A Traffic Light at Rest

A traffic light weighing 122 N hangs from a cable tied to two other cables fastened to a support as in Figure 5.10a. The upper cables make angles of 37.0° and 53.0° with the horizontal. These upper cables are not as strong as the vertical cable and will break if the tension in them exceeds 100 N. Does the traffic light remain hanging in this situation, or will one of the cables break?

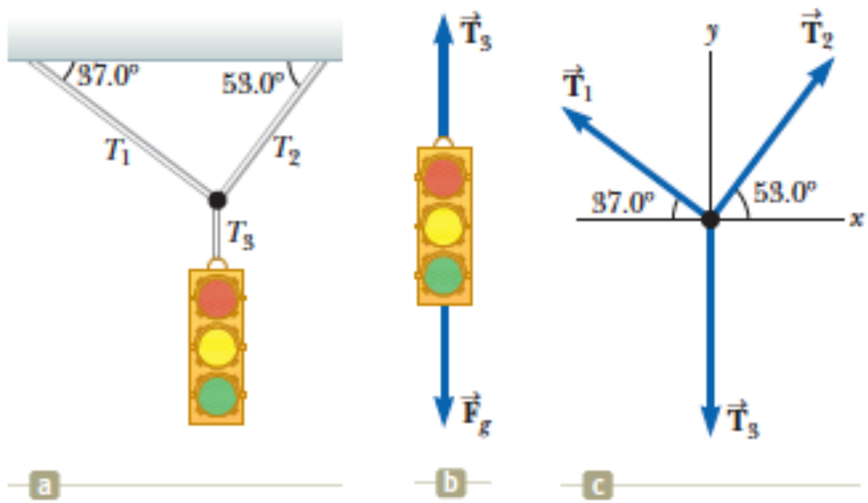


Figure 5.10 (Example 5.4) (a) A traffic light suspended by cables. (b) The forces acting on the traffic light. (c) The free-body diagram for the knot where the three cables are joined.

SOLUTION

Conceptualize Inspect the drawing in Figure 5.10a. Let us assume the cables do not break and nothing is moving.

Categorize If nothing is moving, no part of the system is accelerating. We can now model the light as a particle in equilibrium on which the net force is zero. Similarly, the net force on the knot (Fig. 5.10c) is zero.

Analyze We construct a diagram of the forces acting on the traffic light, shown in Figure 5.10b, and a free-body diagram for the knot that holds the three cables together, shown in Figure 5.10c. This knot is a convenient object to choose because all the forces of interest act along lines passing through the knot.

Apply Equation 5.8 for the traffic light in the y direction:

$$\sum F_y = 0 \rightarrow T_3 - F_g = 0$$

$$T_3 = F_g = 122 \text{ N}$$

5.4 cont.

Choose the coordinate axes as shown in Figure 5.10c and resolve the forces acting on the knot into their components:

Force	x Component	y Component
$\vec{\mathbf{T}}_1$	$-T_1 \cos 37.0°$	$T_1 \sin 37.0°$
$\vec{\mathbf{T}}_2$	$T_2 \cos 53.0°$	$T_2 \sin 53.0°$
$\vec{\mathbf{T}}_3$	0	-122 N

Apply the particle in equilibrium model to the knot:

$$(1)\ \sum F_x = -T_1 \cos 37.0° + T_2 \cos 53.0° = 0$$

$$(2)\ \sum F_y = T_1 \sin 37.0° + T_2 \sin 53.0° + (-122\ \text{N}) = 0$$

Equation (1) shows that the horizontal components of $\vec{\mathbf{T}}_1$ and $\vec{\mathbf{T}}_2$ must be equal in magnitude, and Equation (2) shows that the sum of the vertical components of $\vec{\mathbf{T}}_1$ and $\vec{\mathbf{T}}_2$ must balance the downward force $\vec{\mathbf{T}}_3$, which is equal in magnitude to the weight of the light.

Solve Equation (1) for T_2 in terms of T_1:

$$(3)\ T_2 = T_1\left(\frac{\cos 37.0°}{\cos 53.0°}\right) = 1.33T_1$$

Substitute this value for T_2 into Equation (2):

$$T_1 \sin 37.0° + (1.33T_1)(\sin 53.0°) - 122\ \text{N} = 0$$

$$T_1 = 73.4\ \text{N}$$

$$T_2 = 1.33T_1 = 97.4\ \text{N}$$

Both values are less than 100 N (just barely for T_2), so the cables will not break .

Finalize Let us finalize this problem by imagining a change in the system, as in the following What If?

WHAT IF? Suppose the two angles in Figure 5.10a are equal. What would be the relationship between T_1 and T_2?

Answer We can argue from the symmetry of the problem that the two tensions T_1 and T_2 would be equal to each other. Mathematically, if the equal angles are called θ, Equation (3) becomes

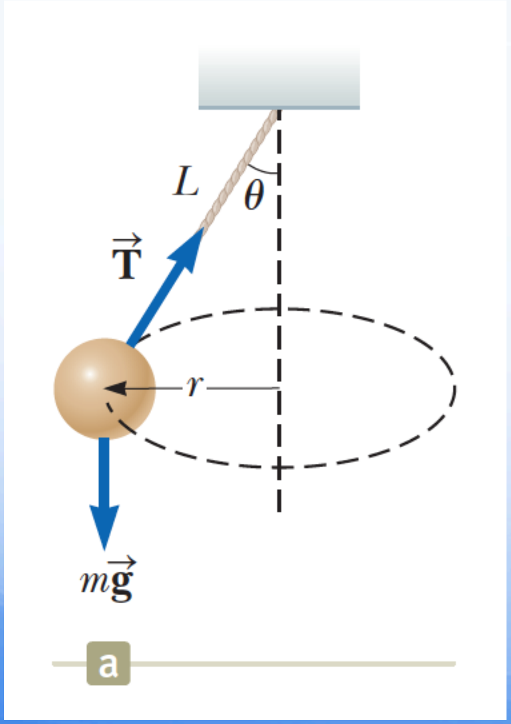
$$T_2 = T_1\left(\frac{\cos\theta}{\cos\theta}\right) = T_1$$

which also tells us that the tensions are equal. Without knowing the specific value of θ, we cannot find the values of T_1 and T_2. The tensions will be equal to each other, however, regardless of the value of θ.

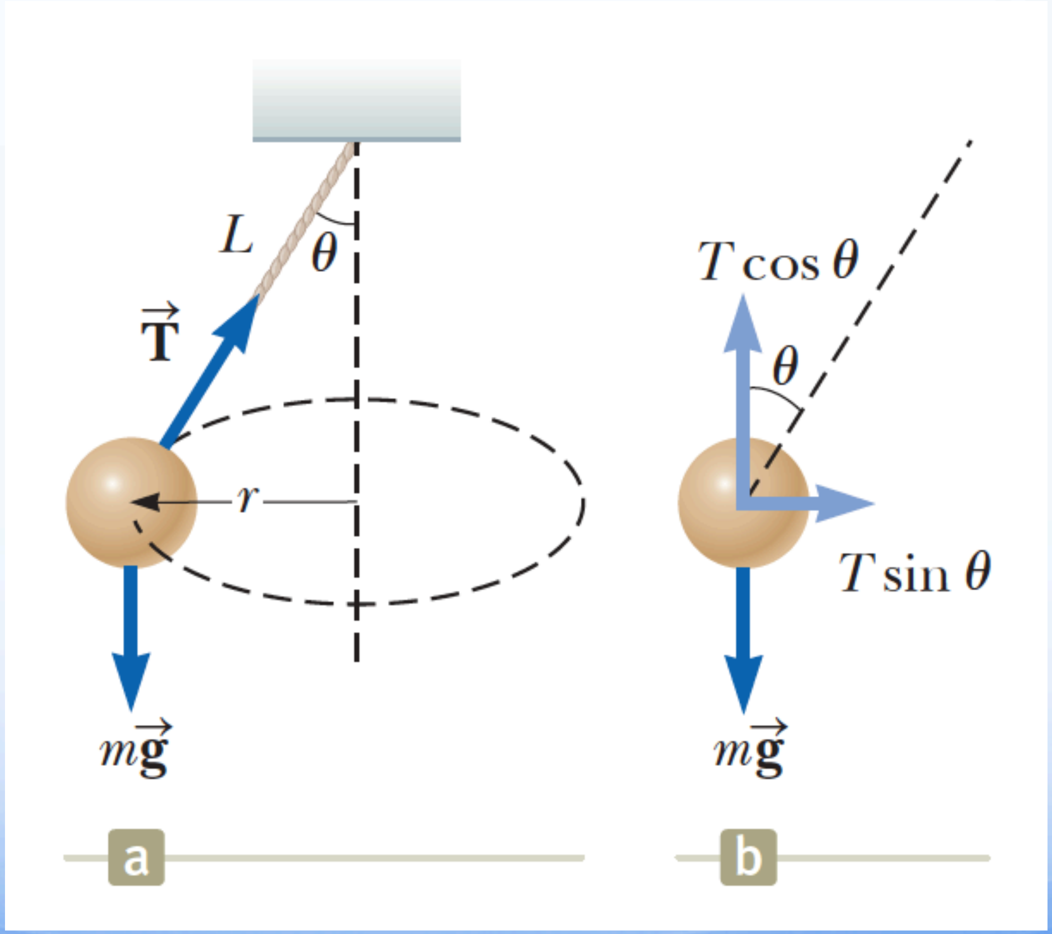
Example

Example 6.1 **The Conical Pendulum**

A small ball of mass m is suspended from a string of length L. The ball revolves with constant speed v in a horizontal circle of radius r as shown in Figure 6.3. (Because the string sweeps out the surface of a cone, the system is known as a *conical pendulum*.) Find an expression for v.



$$v = \sqrt{Lg\sin(\theta)\tan(\theta)}$$



L
θ
$\vec{\mathbf{T}}$
r
$m\vec{\mathbf{g}}$
a
$T\cos\theta$
θ
$T\sin\theta$
$m\vec{\mathbf{g}}$
b

SOLUTION

Conceptualize Imagine the motion of the ball in Figure 6.3a and convince yourself that the string sweeps out a cone and that the ball moves in a horizontal circle.

Categorize The ball in Figure 6.3 does not accelerate vertically. Therefore, we model it as a particle in equilibrium in the vertical direction. It experiences a centripetal acceleration in the horizontal direction, so it is modeled as a particle in uniform circular motion in this direction.

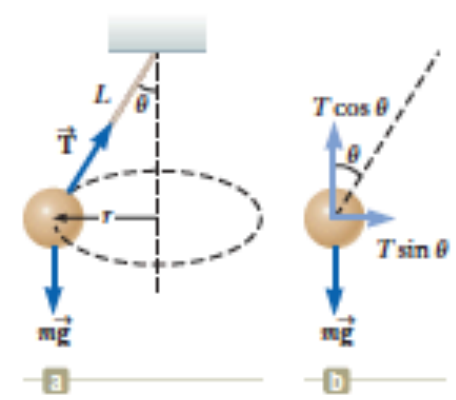


Figure 6.3 (Example 6.1) (a) A conical pendulum. The path of the ball is a horizontal circle. (b) The forces acting on the ball.

Analyze Let θ represent the angle between the string and the vertical. In the diagram of forces acting on the ball in Figure 6.3b, the force $\vec{\mathbf{T}}$ exerted by the string on the ball is resolved into a vertical component $T\cos\theta$ and a horizontal component $T\sin\theta$ acting toward the center of the circular path.

Apply the particle in equilibrium model in the vertical direction:

$$\sum F_y = T\cos\theta - mg = 0$$

$$(1)\quad T\cos\theta = mg$$

Use Equation 6.1 from the particle in uniform circular motion model in the horizontal direction:

$$(2)\quad \sum F_x = T\sin\theta = ma_c = \frac{mv^2}{r}$$

Divide Equation (2) by Equation (1) and use $\sin\theta/\cos\theta = \tan\theta$:

$$\tan\theta = \frac{v^2}{rg}$$

Solve for v:

$$v = \sqrt{rg\tan\theta}$$

Incorporate $r = L\sin\theta$ from the geometry in Figure 6.3a:

$$v = \boxed{\sqrt{Lg\sin\theta\tan\theta}}$$

Finalize Notice that the speed is independent of the mass of the ball. Consider what happens when θ goes to 90° so that the string is horizontal. Because the tangent of 90° is infinite, the speed v is infinite, which tells us the string cannot possibly be horizontal. If it were, there would be no vertical component of the force $\vec{\mathbf{T}}$ to balance the gravitational force on the ball. That is why we mentioned in regard to Figure 6.1 that the puck's weight in the figure is supported by a frictionless table.

Motion in Accelerated Frames

Example 6.7 Fictitious Forces in Linear Motion

A small sphere of mass m hangs by a cord from the ceiling of a boxcar that is accelerating to the right as shown in Figure 6.12. Both the inertial observer on the ground in Figure 6.12a and the noninertial observer on the train in Figure 6.12b agree that the cord makes an angle θ with respect to the vertical. The noninertial observer claims that a force, which we know to be fictitious, causes the observed deviation of the cord from the vertical. How is the magnitude of this force related to the boxcar's acceleration measured by the inertial observer in Figure 6.12a?

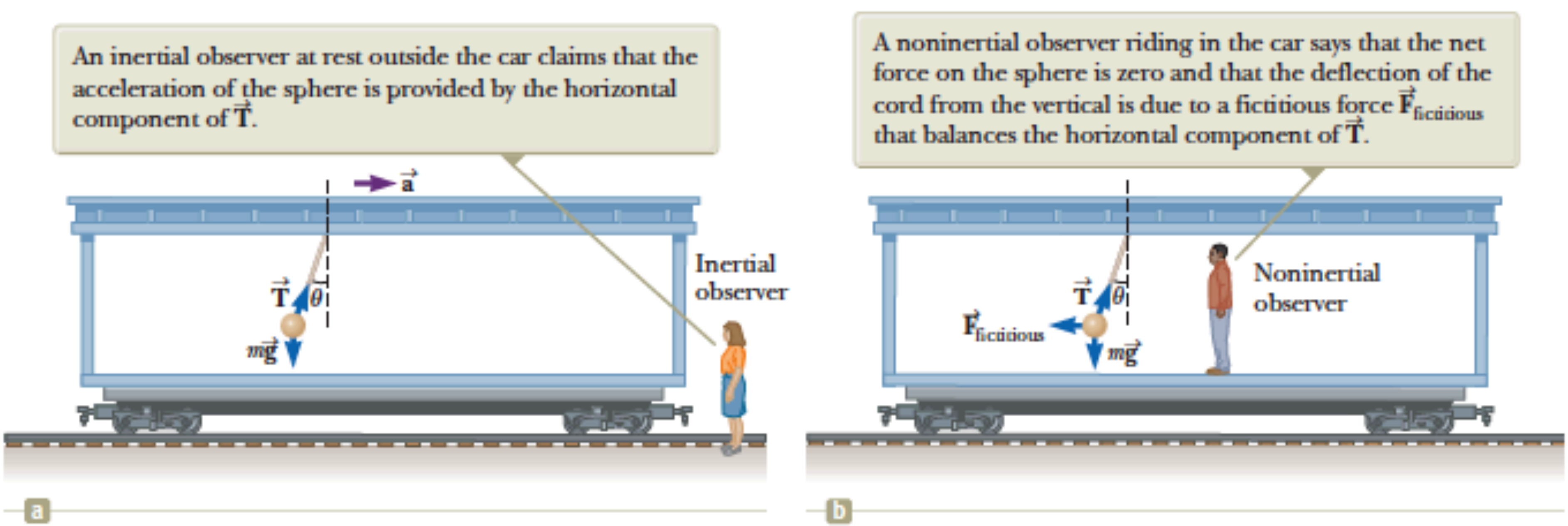


Figure 6.12 (Example 6.7) A small sphere suspended from the ceiling of a boxcar accelerating to the right is deflected as shown.

Fictitious Force: Derivation

$$x = v_0 t + \frac{1}{2} a t^2$$

Eq. of motion in fixed frame

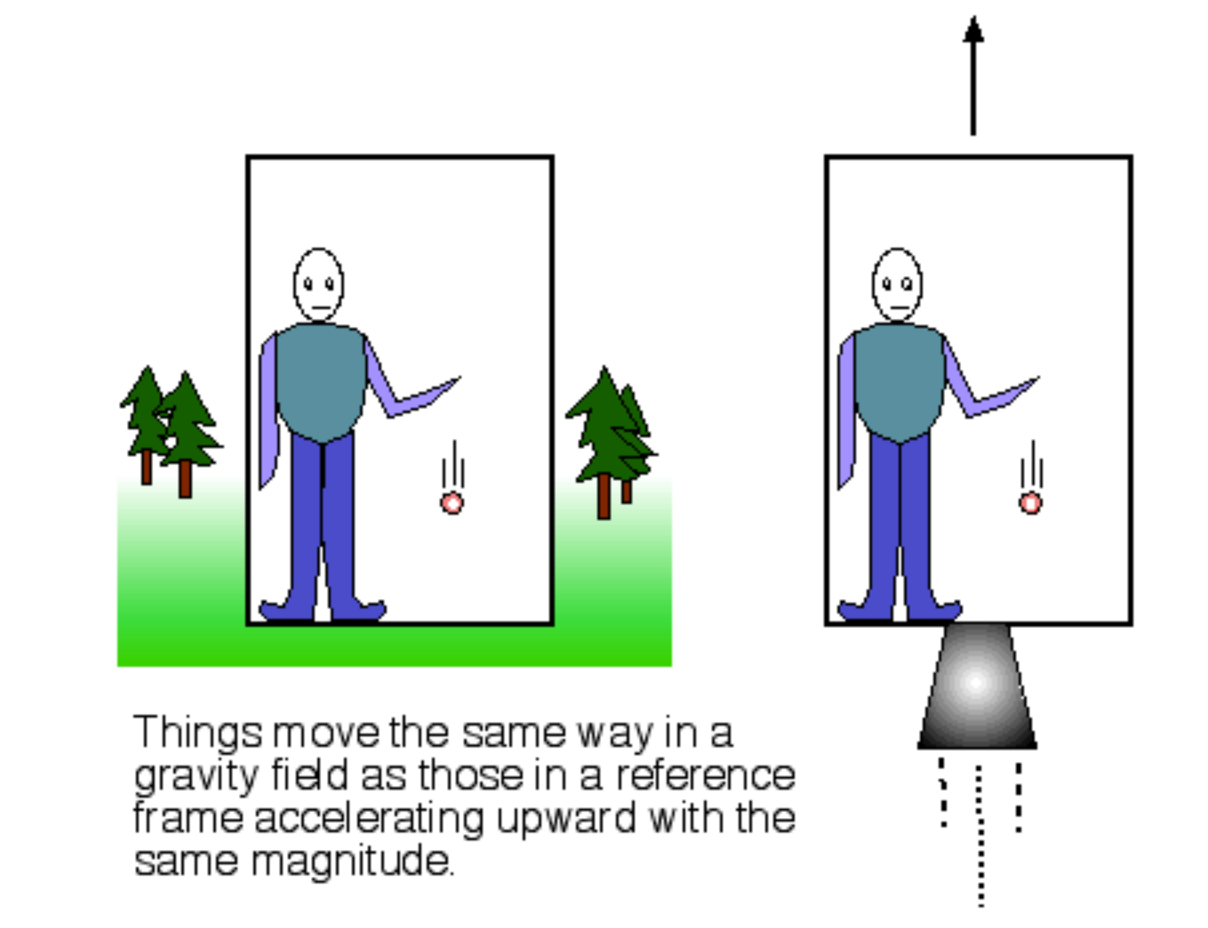
$$= v_0 t + \frac{1}{2} \frac{F}{m} t^2$$

$$x_0(t) = \frac{1}{2} a_f t^2$$

$$x - x_0(t) = v_0 t + \frac{1}{2} \frac{(F - m a_f)}{m} t^2$$

F-ma_f looks like force in new frame,
ma_f acts like fake gravitational force!

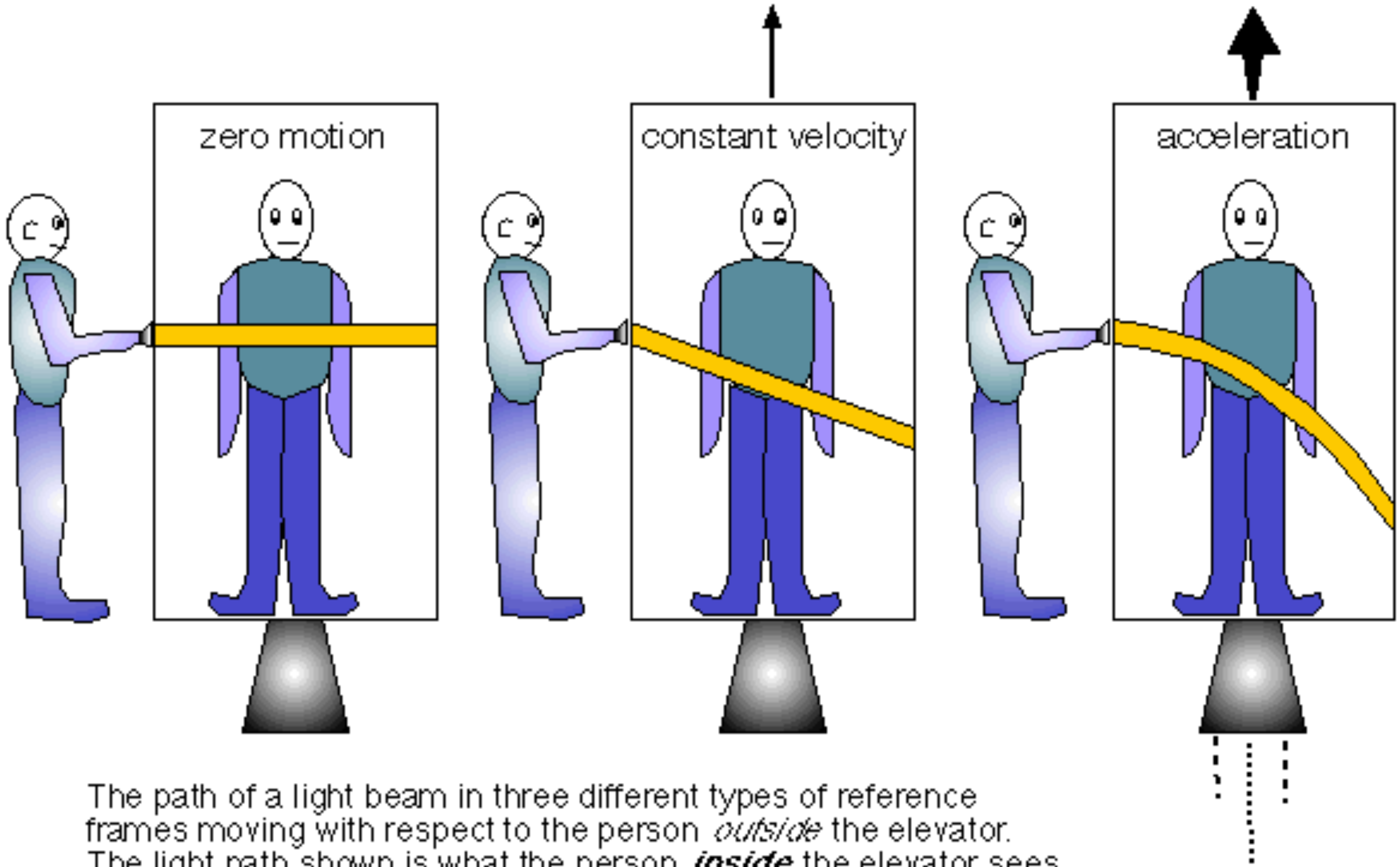
Einstein's Equivalence Principle



Example: Accelerating Reference Frames

- Equivalent to “Fictitious” gravitational force

$$g_{fictitious} = -a_{frame}$$



The path of a light beam in three different types of reference frames moving with respect to the person *outside* the elevator. The light path shown is what the person ***inside*** the elevator sees. Under large acceleration, the beam of light will curve downward. It should also do that in a region of strong gravity.

Review & Summary

Newtonian Mechanics The velocity of an object can change (the object can accelerate) when the object is acted on by one or more **forces** (pushes or pulls) from other objects. *Newtonian mechanics* relates accelerations and forces.

Force Forces are vector quantities. Their magnitudes are defined in terms of the acceleration they would give the standard kilogram. A force that accelerates that standard body by exactly 1 m/s^2 is defined to have a magnitude of 1 N. The direction of a force is the direction of the acceleration it causes. Forces are combined according to the rules of vector algebra. The **net force** on a body is the vector sum of all the forces acting on the body.

Newton's First Law If there is no net force on a body, the body remains at rest if it is initially at rest or moves in a straight line at constant speed if it is in motion.

Inertial Reference Frames Reference frames in which Newtonian mechanics holds are called *inertial reference frames* or *inertial frames*. Reference frames in which Newtonian mechanics does not hold are called *noninertial reference frames* or *noninertial frames*.

Mass The **mass** of a body is the characteristic of that body that relates the body's acceleration to the net force causing the acceleration. Masses are scalar quantities.

Newton's Second Law The net force $\vec{F}_{net}$ on a body with mass m is related to the body's acceleration $\vec{a}$ by

$$\vec{F}_{net} = m\vec{a}, \tag{5-1}$$

which may be written in the component versions

$$F_{net,x} = ma_x \quad F_{net,y} = ma_y \quad \text{and} \quad F_{net,z} = ma_z. \tag{5-2}$$

The second law indicates that in SI units

$$1\ \text{N} = 1\ \text{kg}\cdot\text{m/s}^2. \tag{5-3}$$

A **free-body diagram** is a stripped-down diagram in which only *one* body is considered. That body is represented by either a sketch or a dot. The external forces on the body are drawn, and a coordinate system is superimposed, oriented so as to simplify the solution.

Some Particular Forces A **gravitational force** $\vec{F}_g$ on a body is a pull by another body. In most situations in this book, the other body is Earth or some other astronomical body. For Earth, the force is directed down toward the ground, which is assumed to be an inertial frame. With that assumption, the magnitude of $\vec{F}_g$ is

$$F_g = mg, \tag{5-8}$$

where m is the body's mass and g is the magnitude of the free-fall acceleration.

The **weight** W of a body is the magnitude of the upward force needed to balance the gravitational force on the body. A body's weight is related to the body's mass by

$$W = mg. \tag{5-12}$$

A **normal force** $\vec{F}_N$ is the force on a body from a surface against which the body presses. The normal force is always perpendicular to the surface.

A **frictional force** $\vec{f}$ is the force on a body when the body slides or attempts to slide along a surface. The force is always parallel to the surface and directed so as to oppose the sliding. On a *frictionless surface*, the frictional force is negligible.

When a cord is under **tension**, each end of the cord pulls on a body. The pull is directed along the cord, away from the point of attachment to the body. For a *massless* cord (a cord with negligible mass), the pulls at both ends of the cord have the same magnitude T, even if the cord runs around a *massless, frictionless pulley* (a pulley with negligible mass and negligible friction on its axle to oppose its rotation).

Newton's Third Law If a force $\vec{F}_{BC}$ acts on body B due to body C, then there is a force $\vec{F}_{CB}$ on body C due to body B:

$$\vec{F}_{BC} = -\vec{F}_{CB}.$$

Example

•17 SSM WWW In Fig. 5-36, let the mass of the block be 8.5 kg and the angle θ be 30°. Find (a) the tension in the cord and (b) the normal force acting on the block. (c) If the cord is cut, find the magnitude of the resulting acceleration of the block.

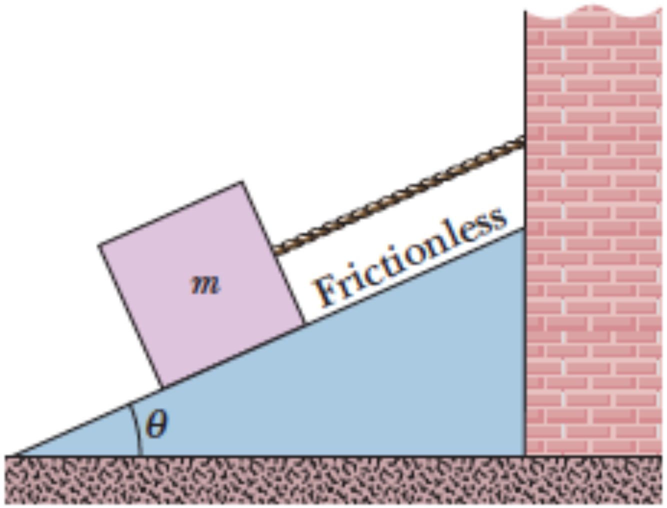


Figure 5-36 Problem 17.

Example

••42 GO In earlier days, horses pulled barges down canals in the manner shown in Fig. 5-42. Suppose the horse pulls on the rope with a force of 7900 N at an angle of $\theta = 18°$ to the direction of motion of the barge, which is headed straight along the positive direction of an x axis. The mass of the barge is 9500 kg, and the magnitude of its acceleration is $0.12\ m/s^2$. What are the (a) magnitude and (b) direction (relative to positive x) of the force on the barge from the water?

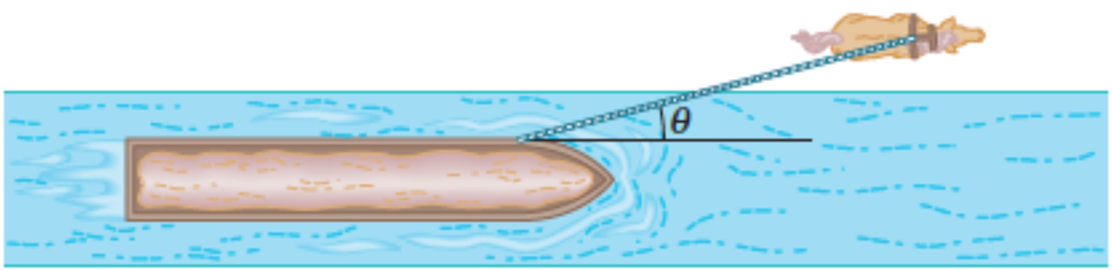


Figure 5-42 Problem 42.

Example

••43 SSM In Fig. 5-43, a chain consisting of five links, each of mass 0.100 kg, is lifted vertically with constant acceleration of magnitude $a = 2.50$ m/s^2. Find the magnitudes of (a) the force on link 1 from link 2, (b) the force on link 2 from link 3, (c) the force on link 3 from link 4, and (d) the force on link 4 from link 5. Then find the magnitudes of (e) the force $\vec{F}$ on the top link from the person lifting the chain and (f) the *net* force accelerating each link.

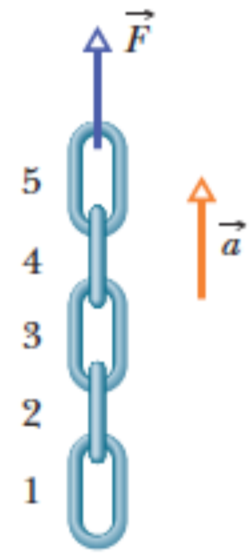


Figure 5-43 Problem 43.

Example

••59 SSM A 10 kg monkey climbs up a massless rope that runs over a frictionless tree limb and back down to a 15 kg package on the ground (Fig. 5-54). (a) What is the magnitude of the least acceleration the monkey must have if it is to lift the package off the ground? If, after the package has been lifted, the monkey stops its climb and holds onto the rope, what are the (b) magnitude and (c) direction of the monkey's acceleration and (d) the tension in the rope?

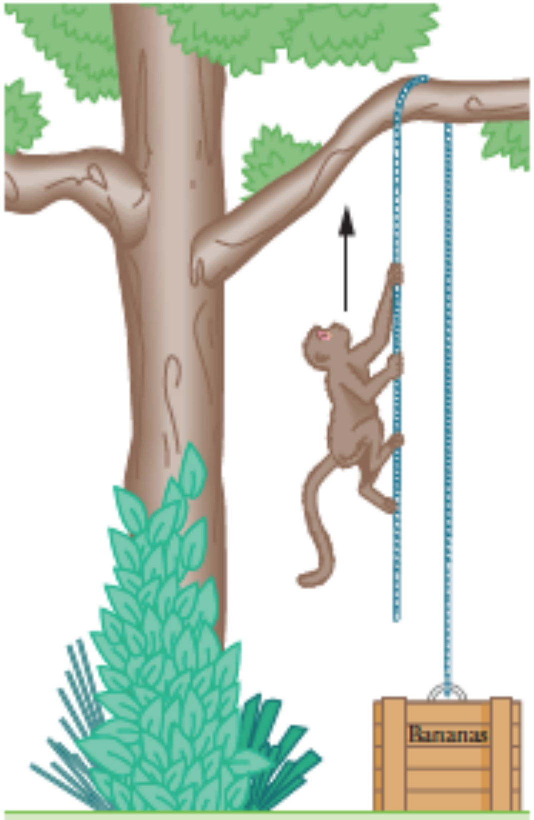
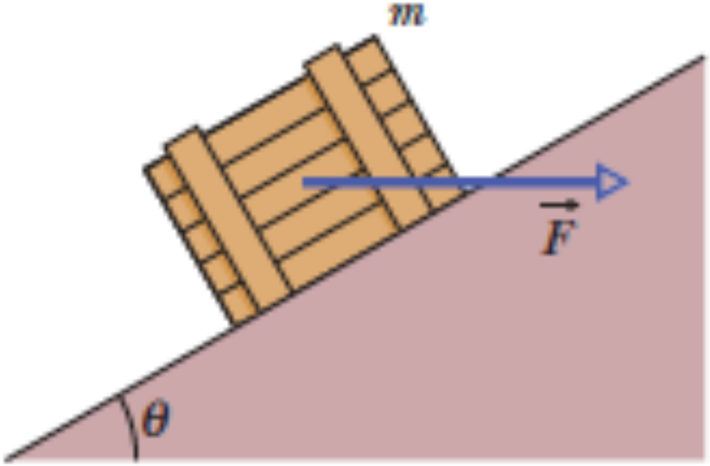


Figure 5-54 Problem 59.

Example

••34 In Fig. 5-40, a crate of mass $m = 100$ kg is pushed at constant speed up a frictionless ramp ($\theta = 30.0°$) by a horizontal force $\vec{F}$. What are the magnitudes of (a) $\vec{F}$ and (b) the force on the crate from the ramp?



Example

70 GO Figure 6-53 shows a *conical pendulum*, in which the bob (the small object at the lower end of the cord) moves in a horizontal circle at constant speed. (The cord sweeps out a cone as the bob rotates.) The bob has a mass of 0.040 kg, the string has length $L = 0.90$ m and negligible mass, and the bob follows a circular path of circumference 0.94 m. What are (a) the tension in the string and (b) the period of the motion?

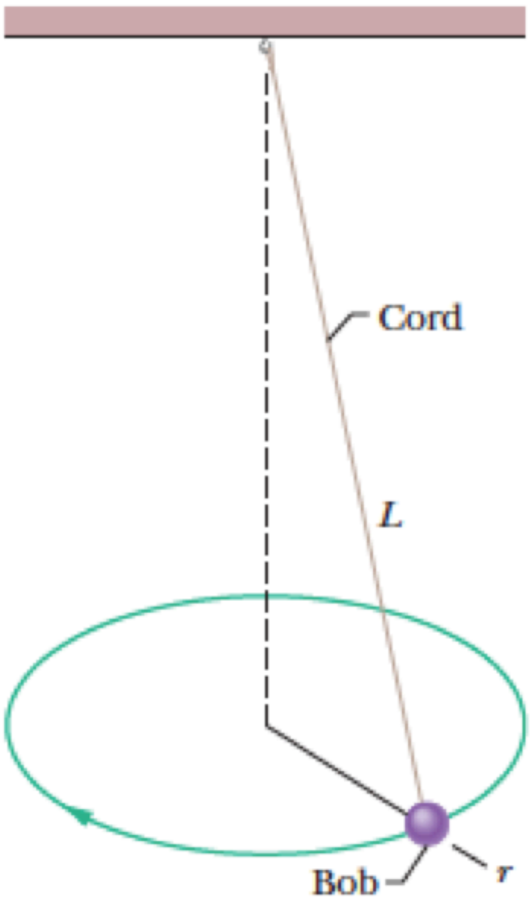


Figure 6-53 Problem 70.

Example

55. (III) A small block of mass m rests on the sloping side of a triangular block of mass M which itself rests on a horizontal table as shown in Fig. 4–47. Assuming all surfaces are frictionless, determine the magnitude of the force $\vec{\mathbf{F}}$ that must be applied to M so that m remains in a fixed position relative to M (that is, m doesn't move on the incline). [*Hint*: Take x and y axes horizontal and vertical.]

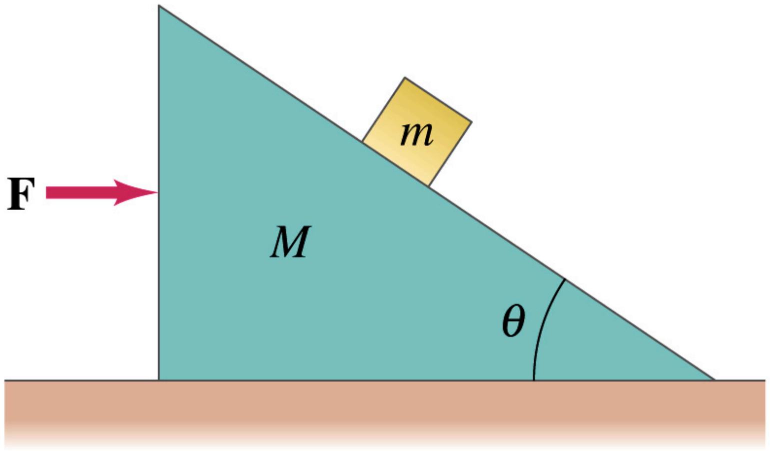


FIGURE 4–47 Problem 55.

$$F = (m + M) g \tan(\theta)$$

Example

67. **S** What horizontal force must be applied to a large block of mass M shown in Figure P5.67 so that the tan blocks remain stationary relative to M? Assume all surfaces and the pulley are frictionless. Notice that the force exerted by the string accelerates m_2.

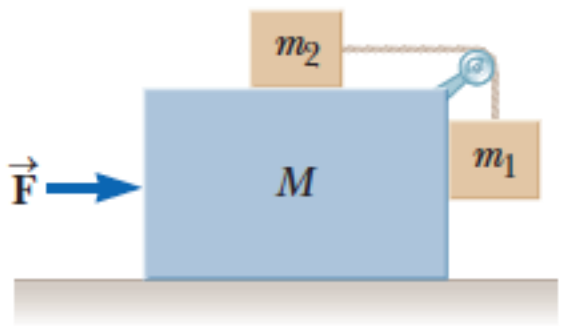
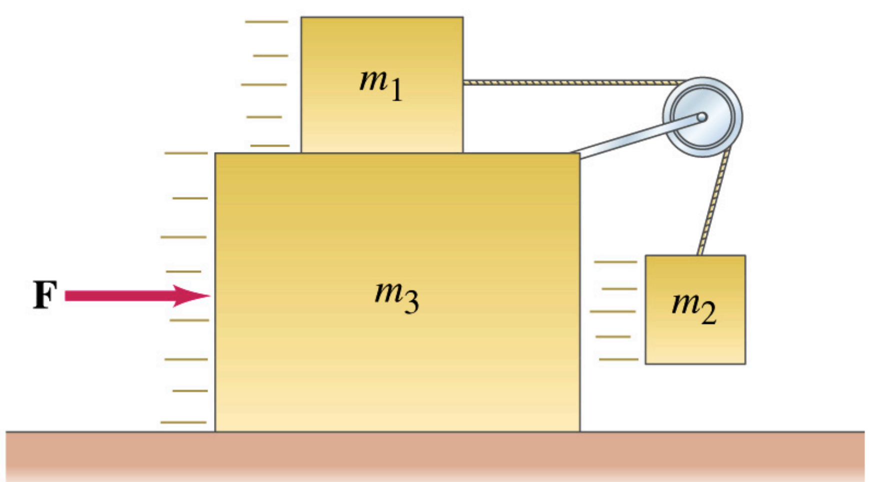


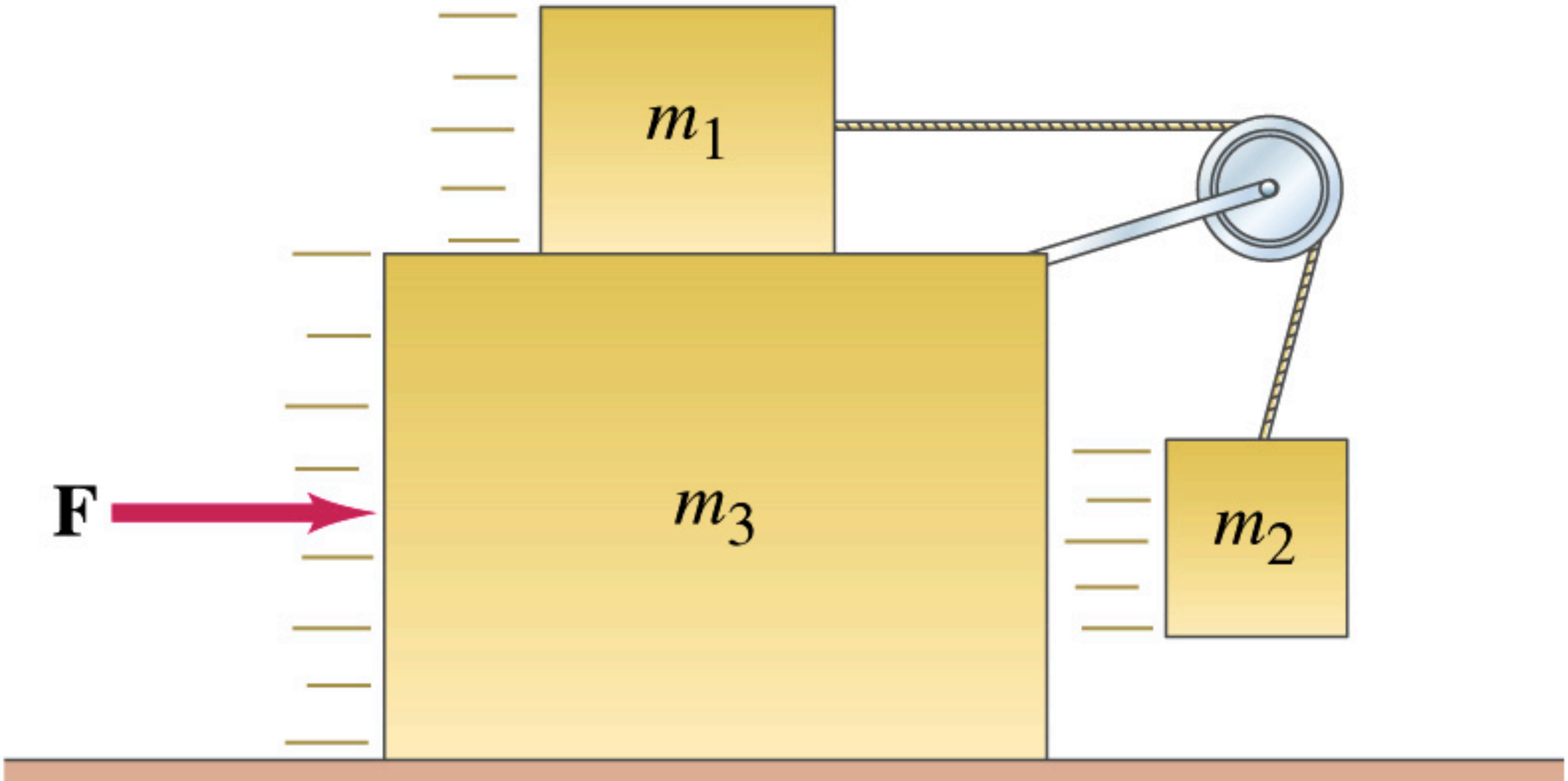
Figure P5.67

Example:

[redacted]

59. (III) Determine a formula for the magnitude of the force $\vec{\mathbf{F}}$ exerted on the large block (m_3) in Fig. 4–51 so that the mass m_1 does not move relative to m_3. Ignore all friction. Assume m_2 does not make contact with m_3.





$$\sin(\theta) = m_2/m_1$$

$$a = g\tan(\theta) = \frac{m_2 g}{(m_1^2 - m_2^2)^{1/2}}$$

$$F = \frac{(m_1 + m_2 + m_3)m_2 g}{(m_1^2 - m_2^2)^{1/2}}$$

Example

••57 GO A puck of mass $m = 1.50$ kg slides in a circle of radius $r = 20.0$ cm on a frictionless table while attached to a hanging cylinder of mass $M = 2.50$ kg by means of a cord that extends through a hole in the table (Fig. 6-43). What speed keeps the cylinder at rest?

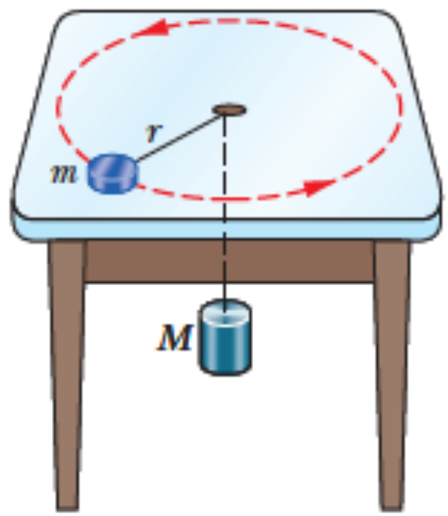
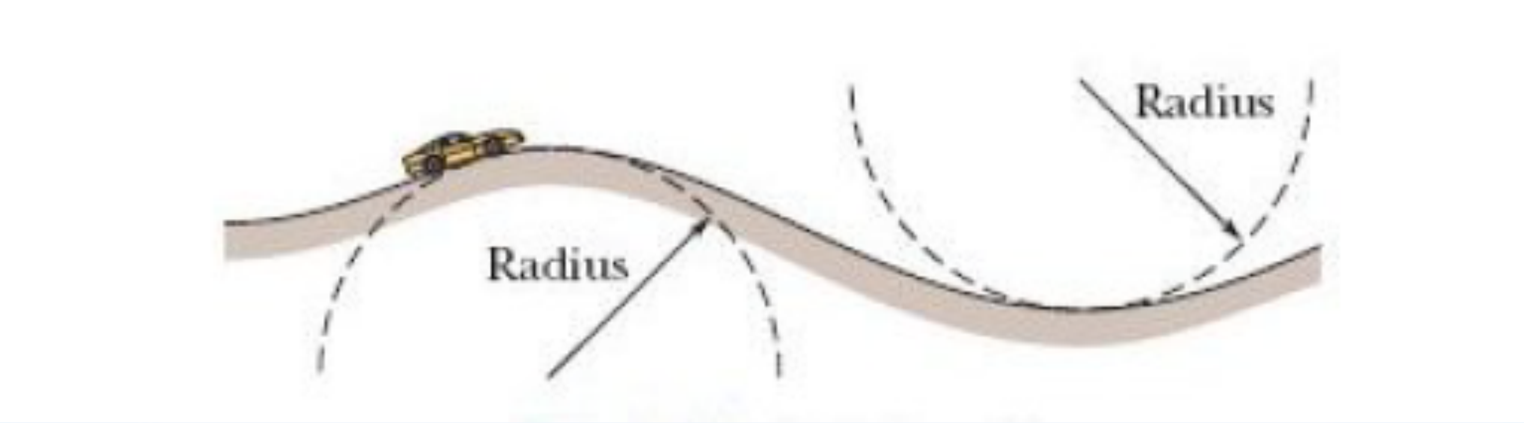
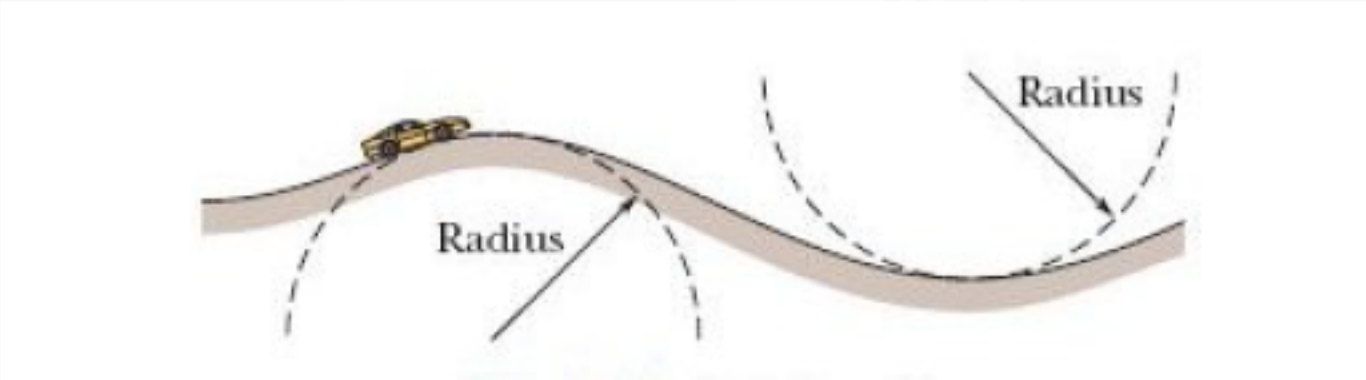


Figure 6-43 Problem 57.

radius of curvature:





If the curve is given in Cartesian coordinates as *y(x)*, then the radius of curvature is (assuming the curve is differentiable up to order 2):

$$R = \left| \frac{(1 + y'^2)^{3/2}}{y''} \right|, \qquad \text{where} \quad y' = \frac{dy}{dx}, \quad y'' = \frac{d^2y}{dx^2},$$

and | *z* | denotes the absolute value of *z*.

If the curve is given parametrically by functions *x(t)* and *y(t)*, then the radius of curvature is

$$R = \left| \frac{ds}{d\varphi} \right| = \left| \frac{(\dot{x}^2 + \dot{y}^2)^{3/2}}{\dot{x}\ddot{y} - \dot{y}\ddot{x}} \right|, \qquad \text{where} \quad \dot{x} = \frac{dx}{dt}, \quad \ddot{x} = \frac{d^2x}{dt^2}, \quad \dot{y} = \frac{dy}{dt}, \quad \ddot{y} = \frac{d^2y}{dt^2}.$$

Example

82 In Fig. 6-57, a stuntman drives a car (without negative lift) over the top of a hill, the cross section of which can be approximated by a circle of radius $R = 250$ m. What is the greatest speed at which he can drive without the car leaving the road at the top of the hill?

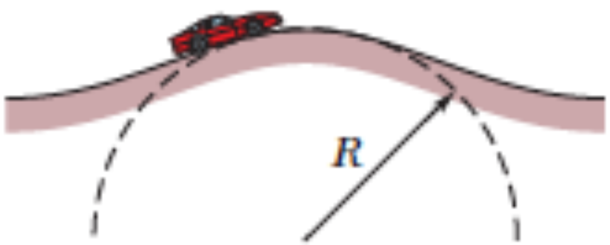


Figure 6-57 Problem 82.