# General Physics I

#### chapter 2

Sharif University of Technology Mehr 1401 (2022-2023)

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#### Chapter II

#### Motion in One Dimension



#### **Kinematics**

- Kinematics
  - In kinematics, you are interested in the description of motion
  - Not concerned with the cause of the motion

#### Position and Displacement

 Position is defined in terms of a frame of reference

Frame A:  $x_i > 0$  and  $x_f > 0$ 





#### Frame B: x' i<0 but x' f>0

 One dimensional, so generally the x- or y-axis

# Position and Displacement

- Position is defined in terms of a frame of reference
  - One dimensional, so generally the x- or y-axis
- Displacement measures the change in position
  - Represented as Δx (if horizontal) or Δy (if vertical)
  - Vector quantity
    - + or is generally sufficient to indicate direction for onedimensional motion



Units			
SI	Meters (m)		
CGS	Centimeters (cm)		

#### Displacement (example) Displacement measures the change in position

 $\square$  represented as  $\Delta x$  or  $\Delta y$ 



## **Distance or Displacement?**

 Distance may be, but is not necessarily, the magnitude of the displacement



# Position-time graphs



Note: position-time graph is not necessarily a straight line, even though the motion is along x-direction

# ConcepTest 1

An object (say, car) goes from one point in space to another. After it arrives to its destination, its displacement is

- either greater than or equal to
- always greater than
- always equal to
- either smaller or equal to
- either smaller or larger

than the distance it traveled.

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# ConcepTest 1 (answer)

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- always greater than
- always equal to
- either smaller or equal to
- either smaller or larger

than the distance it traveled.



Note: displacement is a vector from the final to initial points, distance is total path traversed

# Average Velocity

- It takes time for an object to undergo a displacement
- The average velocity is rate at which the displacement occurs

$$\vec{v}_{average} = \frac{\Delta \vec{x}}{\Delta t} = \frac{\vec{x}_f - \vec{x}_i}{\Delta t}$$

 It is a vector, direction will be the same as the direction of the displacement (Δt is always positive)

+ or - is sufficient for one-dimensional motion

# More About Average Velocity

• Units of velocity:

Units			
SI	Meters per second (m/s)		
CGS	Centimeters per second (cm/s)		

 Note: other units may be given in a problem, but generally will need to be converted to these

#### Example: Suppose that in both cases truck covers the distance in 10 seconds:



+70*m*  $\Delta t$ 10s= +7 m/s

 $\vec{v}_{2 average} = \frac{\Delta \vec{x}_2}{\Delta t} = \frac{-60m}{10s}$ = -6m/s

14

## Speed

- Speed is a scalar quantity
  - same units as velocity
  - speed = total distance / total time
- May be, but is not necessarily, the magnitude of the velocity

#### Graphical Interpretation of Average Velocity

 Average velocity can be determined from a position-time graph

$\rightarrow$ 12	$\Delta \vec{x}$	+ 40 <i>m</i>	
<i>V</i> average	$-\Delta t$	3.0 <i>s</i>	
	$=$ $\pm 13 m/s$		



16

Average velocity equals the slope of the line joining the initial and final positions

#### Instantaneous Velocity

 Instantaneous velocity is defined as the limit of the average velocity as the time interval becomes infinitesimally short, or as the time interval approaches zero

$$\vec{v}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t} = \lim_{\Delta t \to 0} \frac{\vec{x}_f - \vec{x}_i}{\Delta t}$$

 The instantaneous velocity indicates what is happening at every point of time

# **Uniform Velocity**

- Uniform velocity is constant velocity
- The instantaneous velocities are always the same
  - All the instantaneous velocities will also equal the average velocity

#### Graphical Interpretation of Instantaneous Velocity

 Instantaneous velocity is the slope of the tangent to the curve at the time of interest



#### Average vs Instantaneous Velocity



Average velocity

Instantaneous velocity

ConcepTest 2

The graph shows position as a function of time for two trains running on parallel tracks. Which of the following is true:

- 1. at time  $t_B$  both trains have the same velocity
- 2. both trains speed up all the time
- 3. both trains have the same velocity at some time before  $t_{B}$
- 4. train A is longer than train B
- 5. all of the above statements are true



ConcepTest 2

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# ConcepTest 2 (answer)

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- 4. train A is longer than train B
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position



Note: the slope of curve B is parallel to line A at some point t< t<sub>B</sub>

## **Average Acceleration**

- Changing velocity (non-uniform) means an acceleration is present
- Average acceleration is the rate of change of the velocity

$$\vec{a}_{average} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$



Average acceleration is a vector quantity

## **Average Acceleration**

- When the sign of the velocity and the acceleration are the same (either positive or negative), then the speed is increasing
- When the sign of the velocity and the acceleration are opposite, the speed is decreasing

Units		
SI	Meters per second squared (m/s <sup>2</sup> )	
CGS	Centimeters per second squared (cm/s <sup>2</sup> )	

## Instantaneous and Uniform Acceleration

 Instantaneous acceleration is the limit of the average acceleration as the time interval goes to zero

$$\vec{a}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \lim_{\Delta t \to 0} \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

- When the instantaneous accelerations are always the same, the acceleration will be uniform
  - The instantaneous accelerations will all be equal to the average acceleration

## Graphical Interpretation of Acceleration

 Average acceleration is the slope of the line connecting the initial and final velocities on a velocity-time graph

 Instantaneous acceleration is the slope of the tangent to the curve of the velocity-time graph



# Example 1: Motion Diagrams



- Uniform velocity (shown by red arrows maintaining the same size)
- Acceleration equals zero

## Example 2:



- Velocity and acceleration are in the same direction
- Acceleration is uniform (blue arrows maintain the same length)
- Velocity is increasing (red arrows are getting longer)

### Example 3:



- Acceleration and velocity are in opposite directions
- Acceleration is uniform (blue arrows maintain the same length)
- Velocity is decreasing (red arrows are getting shorter)



31<sup>31</sup>



32<sup>32</sup>

One-dimensional Motion With Constant Acceleration

• If acceleration is uniform (i a = a ):



$$a = \frac{v_f - v_o}{t_f - t_0} = \frac{v_f - v_o}{t}$$

$$v_f = v_o + at$$

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Shows velocity as a function of acceleration and time

#### One-dimensional Motion With Constant Acceleration

Used in situations with uniform acceleration

$$\Delta x = v_o t + \frac{1}{2} a t^2$$

$$v_f^2 = v_o^2 + 2a\Delta x$$
Velocity changes
uniformly!!!

#### Notes on the equations

$$\Delta x = v_{average} \ t = \left(\frac{v_o + v_f}{2}\right) t$$

Gives displacement as a function of velocity and time

$$\Delta x = v_o t + \frac{1}{2}at^2$$

$$v_f = v_o + at$$

Gives displacement as a function of time, velocity and acceleration

$$v_f^2 = v_o^2 + 2a\Delta x$$

Gives velocity as a function of acceleration and displacement

35

# Summary of kinematic equations

# **TABLE 2.3**Equations for Motion in a Straight LineUnder Constant Acceleration

Equation	Information Given by Equation	
$v = v_0 + at$	Velocity as a function of time	
$\Delta x = \frac{1}{2}(v_0 + v)t$	Displacement as a function of velocity and time	
$\Delta x = v_0 t + \frac{1}{2}at^2$	Displacement as a function of time	
$v^2 = v_0^2 + 2a\Delta x$	Velocity as a function of displacement	

*Note:* Motion is along the x axis. At t = 0, the velocity of the particle is  $v_0$ . © 2003 Thomson - Brooks/Cole

#### Free Fall

Near the surface of the Earth, all objects experience approximately the same acceleration due to gravity.



This is one of the most common examples of motion with constant acceleration.



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**Figure 2-12** A feather and an apple free fall in vacuum at the same magnitude of acceleration g. The acceleration increases the distance between successive images. In the absence of air, the feather and apple fall together.



In the absence of air resistance, all objects fall with the same acceleration, although this may be hard to tell by testing in an environment where there is air resistance.

38

## https://www.youtube.com/ watch?v=E43-CfukEgs





#### The acceleration due to gravity at the Earth's surface is approximately 9.80 m/s<sup>2</sup>.



*Galileo Galilei* Italian physicist and astronomer (1564–1642)

40<sup>40</sup>

#### Free Fall

- All objects moving under the influence of only gravity are said to be in free fall
- All objects falling near the earth's surface fall with a constant acceleration
- This acceleration is called the acceleration due to gravity, and indicated by g

#### Acceleration due to Gravity

- Symbolized by g
- $g = 9.8 \text{ m/s}^2$  (can use  $g = 10 \text{ m/s}^2$  for estimates)
- g is always directed downward
  - toward the center of the earth!!

## Free Fall -- an Object Dropped

y

- Initial velocity is zero
- Frame: let up be positive
- Use the kinematic equations
  - Generally use y instead
  - of x since vertical

$$\Delta y = \frac{1}{2}at^2$$
$$a = -9.8 \, m/s^2$$



# Free Fall -- an Object Thrown Downward

#### • a = g

 With upward being positive, acceleration will be negative, g = -9.8 m/s<sup>2</sup>

#### • Initial velocity $\neq 0$

• With upward being positive, initial velocity will be negative



# Free Fall -- object thrown upward



# Thrown upward

- The motion may be symmetrical
  - then t<sub>up</sub> = t<sub>down</sub>
  - then  $v_f = -v_o$
- The motion may not be symmetrical
  - Break the motion into various parts
    - generally up and down

# Non-symmetrical Free Fall

- Need to divide the motion into segments
- Possibilities include
  - Upward and downward portions
  - The symmetrical portion back to the release point and then the non-symmetrical portion



#### **Combination Motions**



48

#### Fun QuickLab: Reaction time



#### example

39. Why is the following situation impossible? Emily challenges her friend David to catch a \$1 bill as follows. She holds the bill vertically as shown in Figure P2.39, with the center of the bill between but not touching David's index finger and thumb. Without warning, Emily releases the bill. David catches the bill



Figure P2.39

without moving his hand downward. David's reaction time is equal to the average human reaction time.

#### Another Look at Constant Acceleration\*

The first two equations in Table 2-1 are the basic equations from which the others are derived. Those two can be obtained by integration of the acceleration with the condition that a is constant. To find Eq. 2-11, we rewrite the definition of acceleration (Eq. 2-8) as

dv = a dt.

We next write the *indefinite integral* (or *antiderivative*) of both sides:

$$\int dv = \int a \, dt.$$

Since acceleration *a* is a constant, it can be taken outside the integration. We obtain

$$\int dv = a \int dt$$

or

$$v = at + C. \tag{2-25}$$

To evaluate the constant of integration C, we let t = 0, at which time  $v = v_0$ . Substituting these values into Eq. 2-25 (which must hold for all values of t, including t = 0) yields

$$v_0 = (a)(0) + C = C.$$

To derive Eq. 2-15, we rewrite the definition of velocity (Eq. 2-4) as

dx = v dt

and then take the indefinite integral of both sides to obtain

$$\int dx = \int v \, dt.$$

Next, we substitute for v with Eq. 2-11:

$$\int dx = \int (v_0 + at) \, dt.$$

Since  $v_0$  is a constant, as is the acceleration *a*, this can be rewritten as

$$\int dx = v_0 \int dt + a \int t \, dt$$

Integration now yields

$$x = v_0 t + \frac{1}{2} a t^2 + C', \qquad (2-26)$$

where C' is another constant of integration. At time t = 0, we have  $x = x_0$ . Substituting these values in Eq. 2-26 yields  $x_0 = C'$ . Replacing C' with  $x_0$  in Eq. 2-26 gives us Eq. 2-15.

#### **Graphical Integration in Motion Analysis**

**Integrating Acceleration.** When we have a graph of an object's acceleration *a* versus time *t*, we can integrate on the graph to find the velocity at any given time. Because *a* is defined as a = dv/dt, the Fundamental Theorem of Calculus tells us that

$$v_1 - v_0 = \int_{t_0}^{t_1} a \, dt. \tag{2-27}$$

The right side of the equation is a definite integral (it gives a numerical result rather than a function),  $v_0$  is the velocity at time  $t_0$ , and  $v_1$  is the velocity at later time  $t_1$ . The definite integral can be evaluated from an a(t) graph, such as in Fig. 2-14*a*. In particular,

$$\int_{t_0}^{t_1} a \, dt = \begin{pmatrix} \text{area between acceleration curve} \\ \text{and time axis, from } t_0 \text{ to } t_1 \end{pmatrix}.$$
 (2-28)

If a unit of acceleration is 1 m/s<sup>2</sup> and a unit of time is 1 s, then the corresponding unit of area on the graph is

$$(1 \text{ m/s}^2)(1 \text{ s}) = 1 \text{ m/s},$$

which is (properly) a unit of velocity. When the acceleration curve is above the time axis, the area is positive; when the curve is below the time axis, the area is negative.

**Integrating Velocity.** Similarly, because velocity v is defined in terms of the position x as v = dx/dt, then

$$x_1 - x_0 = \int_{t_0}^{t_1} v \, dt, \qquad (2-29)$$

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where  $x_0$  is the position at time  $t_0$  and  $x_1$  is the position at time  $t_1$ . The definite integral on the right side of Eq. 2-29 can be evaluated from a v(t) graph, like that shown in Fig. 2-14b. In particular,

$$\int_{t_0}^{t_1} v \, dt = \begin{pmatrix} \text{area between velocity curve} \\ \text{and time axis, from } t_0 \text{ to } t_1 \end{pmatrix}.$$
 (2-30)

If the unit of velocity is 1 m/s and the unit of time is 1 s, then the corresponding unit of area on the graph is

$$(1 \text{ m/s})(1 \text{ s}) = 1 \text{ m},$$

which is (properly) a unit of position and displacement. Whether this area is positive or negative is determined as described for the a(t) curve of Fig. 2-14a.



**Figure 2-14** The area between a plotted curve and the horizontal time axis, from time  $t_0$  to time  $t_1$ , is indicated for (*a*) a graph of acceleration *a* versus *t* and (*b*) a graph of velocity *v* versus *t*.

#### **Examples:**



Figure 2-4 Calculation of the average velocity between t = 1 s and t = 4 s as the slope of the line that connects the points on the x(t) curve representing those times. The swirling icon indicates that a figure is available in WileyPLUS as an animation with voiceover.



To find average velocity, first draw a straight line, start to end, and then find the slope of the line.

-3 x(t)-4 -5Start of interval

4

3

2

1

0

-1

-2





#### Sample Problem 2.03 Acceleration and dv/dt

A particle's position on the x axis of Fig. 2-1 is given by

 $x = 4 - 27t + t^3$ ,

with x in meters and t in seconds.

(a) Because position x depends on time t, the particle must be moving. Find the particle's velocity function v(t) and acceleration function a(t).

(1) To get the velocity function v(t), we differentiate the position function x(t) with respect to time. (2) To get the acceleration function a(t), we differentiate the velocity function v(t) with respect to time.

Calculations: Differentiating the position function, we find

$$v = -27 + 3t^2, \qquad (Answer)$$

with v in meters per second. Differentiating the velocity function then gives us

a = +6t, (Answer)

with a in meters per second squared.

(b) Is there ever a time when v = 0?

**Calculation:** Setting v(t) = 0 yields

 $0 = -27 + 3t^2,$ 

which has the solution

 $t = \pm 3$  s. (Answer)

Thus, the velocity is zero both 3 s before and 3 s after the clock reads 0.

(c) Describe the particle's motion for  $t \ge 0$ .

#### Table 2-1 Equations for Motion with Constant Acceleration<sup>a</sup>

Equation Numbe <del>r</del>	Equation	Missing Quantity
2-11	$v = v_0 + at$	$x - x_0$
2-15	$x - x_0 = v_0 t + \frac{1}{2}at^2$	v
2-16	$v^2 = v_0^2 + 2a(x - x_0)$	t
2-17	$x - x_0 = \frac{1}{2}(v_0 + v)t$	а
2-18	$x - x_0 = vt - \frac{1}{2}at^2$	$v_0$

"Make sure that the acceleration is indeed constant before using the equations in this table.

#### Checkpoint 4

4

The following equations give the position x(t) of a particle in four situations: (1) x = 3t - 4; (2)  $x = -5t^3 + 4t^2 + 6$ ; (3)  $x = 2/t^2 - 4/t$ ; (4)  $x = 5t^2 - 3$ . To which of these situations do the equations of Table 2-1 apply?

https://www.youtube.com/ watch?v=4JI9m--j2F4



A popular web video shows a jet airplane, a car, and a motorcycle racing from rest along a runway (Fig. 2-10). Initially the motorcycle takes the lead, but then the jet takes the lead, and finally the car blows past the motorcycle. Here let's focus on the car and motorcycle and assign some reasonable values to the motion. The motorcycle first takes the lead because its (constant) acceleration  $a_m = 8.40 \text{ m/s}^2$  is greater than the car's (constant) acceleration  $a_c = 5.60 \text{ m/s}^2$ , but it soon loses to the car because it reaches its greatest speed  $v_m = 58.8 \text{ m/s}$  before the car reaches its greatest speed  $v_c = 106 \text{ m/s}$ . How long does the car take to reach the motorcycle?



Figure 2-10 A jet airplane, a car, and a motorcycle just after accelerating from rest.



Figure 2-11 Graph of position versus time for car and motorcycle.

#### Sample Problem 2.05 Time for full up-down flight, baseball toss

In Fig. 2-13, a pitcher tosses a baseball up along a y axis, with an initial speed of 12 m/s.

(a) How long does the ball take to reach its maximum height?

6



#### Table 2-1 Equations for Motion with Constant Acceleration<sup>a</sup>

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2-11	$v = v_0 + at$	$x - x_0$
2-15	$x - x_0 = v_0 t + \frac{1}{2}at^2$	v
2-16	$v^2 = v_0^2 + 2a(x - x_0)$	t
2-17	$x - x_0 = \frac{1}{2}(v_0 + v)t$	a
2-18	$x - x_0 = vt - \frac{1}{2}at^2$	$\nu_0$

"Make sure that the acceleration is indeed constant before using the equations in this table.

8 The following equations give the velocity v(t) of a particle in four situations: (a) v = 3; (b)  $v = 4t^2 + 2t - 6$ ; (c) v = 3t - 4; (d)  $v = 5t^2 - 3$ . To which of these situations do the equations of Table 2-1 apply?

• 7

••8 Panic escape. Figure 2-24 shows a general situation in which a stream of people attempt to escape through an exit door that turns out to be locked. The people move toward the door at speed  $v_s = 3.50$  m/s, are each d = 0.25 m in depth, and are sepa-

rated by L = 1.75 m. The arrangement in Fig. 2-24 occurs at time t = 0. (a) At what average rate does the layer of people at the door increase? (b) At what time does the layer's depth reach 5.0 m? (The answers reveal how quickly such a situation becomes dangerous.)



Figure 2-24 Problem 8.

**•••12** Traffic shock wave. An abrupt slowdown in concentrated traffic can travel as a pulse, termed a shock wave, along the line of cars, either downstream (in the traffic direction) or upstream, or it can be stationary. Figure 2-25 shows a uniformly spaced line of cars moving at speed v = 25.0 m/s toward a uniformly spaced line of slow cars moving at speed  $v_s = 5.00$  m/s. Assume that each faster car adds length L = 12.0 m (car length plus buffer zone) to the line of slow cars when it joins the line, and assume it slows abruptly at the last instant. (a) For what separation distance d between the faster cars does the shock wave remain stationary? If the separation is twice that amount, what are the (b) speed and (c) direction (upstream or downstream) of the shock wave?



#### Module 2-4 Constant Acceleration

•23 SSM An electron with an initial velocity  $v_0 = 1.50 \times 10^5$  m/s enters a region of length L = 1.00cm where it is electrically accelerated (Fig. 2-26). It emerges with  $v = 5.70 \times 10^6$  m/s. What is its acceleration, assumed constant?



Figure 2-26 Problem 23.

74 A pilot flies horizontally at 1300 km/h, at height h = 35 m above initially level ground. However, at time t = 0, the pilot begins to fly over ground sloping upward at angle  $\theta = 4.3^{\circ}$  (Fig. 2-41). If the pilot does not change the airplane's heading, at what time t does the plane strike the ground?

11





**69.** Two thin rods are fastened to the inside of a circular ring as shown in Figure P2.69. One rod of length D is vertical, and the other of length L makes an angle  $\theta$  with the horizontal. The two rods and the ring lie in a vertical plane. Two small beads are free to slide without friction along the rods. (a) If the two beads are released from rest simultaneously from the positions shown, use your intuition





66. A man drops a rock into a well. (a) The man hears the sound of the splash 2.40 s after he releases the rock from rest. The speed of sound in air (at the ambient temperature) is 336 m/s. How far below the top of the well is the surface of the water? (b) What If? If the travel time for the sound is ignored, what percentage error is introduced when the depth of the well is calculated?