BLIND SOURCE SEPARATION OF DISCRETE FINITE ALPHABET SOURCES USING A SINGLE MIXTURE

Mohammad Rostami[1], *Massoud Babaie-Zadeh*[2*], *Samareh Samadi*[3,4], *and Christian Jutten*[4]

[1]Department of Electrical and Computer Engineering, University of Waterloo, Waterloo, Canada
[2]Department of Electrical Engineering, Sharif University of Technology, Tehran, Iran
[3]School of Electrical and Computer Engineering, University of Tehran, Tehran, Iran
[4]GIPSA-LAB and Institut Universitaire de France, Grenoble, France

ABSTRACT

This paper deals with blind separation of finite alphabet sources where we have n sources and only one observation. The method is applied directly in time (spatial) domain and no transformation is needed. It follows a two stage procedure. In the first stage the mixing coefficients are estimated, and in the second stage the sources are separated using the estimated mixing coefficients. We also study restrictions of this method and conditions for which its performance is acceptable. Simulation results are presented to show the ability of this method to source separation in images and pulse amplitude modulation (PAM) signals.

1. INTRODUCTION

The main purpose of blind source separation (BSS) is separating some source signals from a number of their mixtures. BSS has found applications in different fields such as feature extraction, telecommunications, medical imaging and audio separation [1]. A simple case is where source signals are mixed linearly. By assuming source independance it has been shown that linear mixtures are separable, provided that the number of observed signals is equal to or larger than the source number [1]. When the number of sources is less than the number of the observed signals, then the problem is called underdetermined BSS (UBSS). Most algorithms have been developed for regular BSS, where the number of sources and observations are equal, while UBSS methods are limited. The main reason is that in the general form, the problem is not solvable. The solution is however possible if the sources have special characteristics. As an example, in [2] the sources are separated using sparsity of the sources. Source sparsity can be enhanced by using transforms like Fourier, wavelets or discrete cosine transform. Since these transforms preserve linearity, the linear relationship between sources and observed signals is preserved in the transformed space. This property has led to some UBSS methods [2, 3, 4]. Some methods are based on Maximum Likelihood estimation of the mixing coefficients and then in the second step by using estimated mixing coefficients, sources are separated [5, 6]. A third approach for UBSS is mainly based on the geometric properties of the signals [7, 8].

In this paper, we extend the idea presented in [8], which has been proposed for the simple case where there are two sources with finite alphabets, and only one mixture. The algorithm of [8] separates the sources in two stages. In the first stage mixing coefficients are estimated and in the second stage the sources are estimated. In this paper, we consider the extended case in which we have n finite alphabet sources and only one observation signal. We extend the first stage of the algorithm of [8] so we can use it for this general case. In the second step, using the resulting mixing coefficients, we separate the sources similar to the approach of [8].

This paper is organized as follows. In section 2, we describe the problem and basic assumptions. Section 3 is devoted to our separation method. In section 4, we discuss the limitations of the proposed method. Finally, simulation results are presented in section 5.

2. PROBLEM DESCRIPTION

Consider n real source signals $s_1(\cdot), s_2(\cdot), ..., s_n(\cdot)$ that only take the values from the finite set $V = \{v_0, v_1, ..., v_{m-1}\}$ and suppose that the values are equally spaced, that is, $v_{i+1} - v_i = D$ for $i = 0, \dots, m-2$. Some practical signals such as telecommunications signals produced using Pulse Amplitude Modulation (PAM), in which $V = \{-(m-1)D/2, -(m-3)D/2, ..., (m-3)D/2, (m-1)D/2\}$, or digital images, for which $V = \{0, 1, ..., 255\}$, satisfy this condition. Without loss of generality assume $D = 1$.

Our problem is estimation of source signals $s_1(\cdot), ..., s_n(\cdot)$, using the observation signal:

$$x(t) = a_1 s_1(t) + a_2 s_2(t) + ... + a_n s_n(t), t = 1, ..., T, \quad (1)$$

where n is assumed to be known, $a_1, a_2, ..., a_n$ are real valued

*This work has been partially funded by Iran Telecom Research Center (ITRC).

unknown mixing coefficients, and T is the number of available samples.

Since the input source signals have a finite number of values, the observation signal has also a finite number of values belonging to the multiset[1] $S = \{c_1, c_2, ..., c_K\}$, where each member of S comes from a linear combination:

$$c_k = a_1 v_1(k) + a_2 v_2(k) + ... + a_n v_n(k), \tag{2}$$

where $v_1(k), v_2(k), ..., v_n(k) \in V$. In this paper, as in [8], we assume that all members of the multiset S are distinct. It means that:

$$\begin{aligned} &c_k = c_l \Rightarrow \\ &v_1(k) = v_1(l), v_2(k) = v_2(l), \dots, v_n(k) = v_n(l). \end{aligned} \tag{3}$$

Under this assumption S has $K = m^n$ distinct members and thus mathematically would be a set with cardinality $K = m^n$.

3. SEPARATION METHOD

In this section we present our idea for separating the n sources. This idea is a method for estimating the mixing coefficients a_i's. After estimating a_i's, separating the sources would be easy because of the assumption (3).

3.1. Estimating mixing coefficients

Assume all the observation signal values are put in the column vector $\mathbf{x}$ with length T. Subtracting any two entries of $\mathbf{x}$ we obtain the difference:

$$\begin{aligned} &\delta_{i,j} = x_i - x_j = c_k - c_l = a_1(v_1(k) - v_1(l)) + \\ &a_2(v_2(k) - v_2(l)) + \dots + a_n(v_n(k) - v_n(l)), \end{aligned} \tag{4}$$

where $1 \leq i, j \leq T$, x_i and x_j are the i^{th} and j^{th} entry of $\mathbf{x}$ respectively and c_k and c_l are corresponding values of x_i and x_j, respectively.

We call every pair in the form $v(k) - v(l)$ a difference pair. The value of every difference pair in (4) belongs to the set:

$$\begin{aligned} &\Delta = \{v_i - v_j | \forall v_i, v_j \in V\} = \\ &\{-(m-1), -(m-2), ..., 0, ..., m-2, m-1\}. \end{aligned} \tag{5}$$

We assume the input sequence $[s_1(\cdot), ..., s_n(\cdot)]$ is rich enough in the sense that it contains all possible pairs of input values in V^n.

We put all the differences into the multiset:

$$X_\Delta = \{x_i - x_j | \forall i, j : 1 < i, j < T\}. \tag{6}$$

If the observation vector $\mathbf{x}$ has T entries, then X_Δ has $\binom{T}{2} = \frac{T(T-1)}{2}$ members but since Δ has only $2m - 1$ members we conclude that X_Δ will have members with repeated values (assuming $T > m$). We now look at the distribution of the elements of X_Δ (assuming that the elements of Δ are equiprobable).

Consider the case $\delta_{i,j} = 0$. For having this case, according to (3), every difference pair in (4) must be equal to zero. Since for each pair this can happen in m different ways when $v(k) = v(l)$ for some $v(k) = 0, 1, \dots, m-1$, in general $\delta_{i,j}$ can be equal to zero in m^n different cases. Next consider the case $\delta_{i,j} = \pm a_q$ for some $q = 1, \dots, n$. This situation occurs in $(m-1)m^{n-1}$ different cases, since except the difference pair $v_q(k) - v_q(l)$ which must be equal to ± 1, other pairs must be equal to zero. The latter can happen in m^{n-1} cases and the former occurs in $m - 1$ cases. Based on the rule of products we have $(m-1)m^{n-1}$ different cases overall. In general, for greater $|\delta_{i,j}|$ the cases that it happens decrease so that for the case $|\delta_{i,j}| = (m-1)(|a_1| + |a_2| + \dots + |a_n|)$ we have only one possible case, while the most probable case is $\delta_{i,j} = 0$. We propose to use this property to estimate mixing coefficients. These results suggest that if sources take all the values relatively uniformly, most likely (6) will have a nonuniform distribution with $\delta_{i,j} = 0$ as the most frequent value. Using this property we propose the following algorithm for estimation of mixing coefficients:

1. Supposes we have the observation vector $\mathbf{x}$ which is a $T \times 1$ vector, then calculate $\mathbf{M}$ as follows:

$$\begin{aligned} &\mathbf{Y} = \mathbf{x} \cdot \mathbf{1}^T \\ &\mathbf{M} = \mathbf{Y} - \mathbf{Y}^T, \end{aligned} \tag{7}$$

where $\mathbf{1}$ is T dimensional all ones column vector and $(\cdot)^T$ stands for matrix transposition. So, $\mathbf{M}$ contains all members of the multiset (6).

2. Convert all nonzero entries of $\mathbf{M}$ into the vector $\mathbf{k}$ and compute the histogram of this vector.

3. Set $\pm\hat{a}_1, \pm\hat{a}_2, \dots, \pm\hat{a}_n$ equal to the n most frequent values in the histogram of $\mathbf{k}$.

3.2. Separation

Since our sources have finite alphabet, the observation signal has m^n alphabets. We assumed these values to be completely distinct. As an important result, each value of the observation signal corresponds to unique values of source signals. Therefore the values of sources $s_1(\cdot), s_2(\cdot), \dots, s_n(\cdot)$ at point t can be estimated by finding the source values $v_1, v_2, \dots, v_n$ by minimizing the reconstruction error, that is:

$$[\hat{s}_1(t), \dots, \hat{s}_n(t)] = \underset{v_1,\dots,v_n}{\operatorname{argmin}} [x(t) - (\hat{a}_1 v_1 + \dots + \hat{a}_n v_n)]^2. \tag{8}$$

This minimization is done using a full search over all possible values for $v_1, ..., v_n$.

An ambiguity that remains is the order of the sources which is resulted from ambiguity in the order of the mixing coefficients. This is a common ambiguity in BSS problems. However, we don't have scaling ambiguity, as in usual BSS.

[1] By a multiset, we mean a set that can have repeated members.

4. RESTRICTIONS OF THE ALGORITHM

In this section, we describe the conditions for which this algorithm performs well. Consider mixing coefficients estimation first. Our method is based on the probability of values in the difference set. Now suppose that we have four 10-level signals with $V = \{0, 1, \ldots, 9\}$. If we form the difference set we have the following probability of happening (assuming that all levels occur with the same probability in the sources):

$$P_0 = \frac{10^4}{(10^2)^4} = 0.0001, P_{\pm a_i} = \frac{(10-1)10^3}{(10^2)^4} = 0.00009. \quad (9)$$

The difference between these probabilities is very small, so the cardinality of the difference set must be large enough so we can detect this difference between probabilities.

Another limitation that we encounter is the value of the mixing coefficients. Mixing coefficients must be greater than D, the quantification level. If the mixing coefficients values are less than D, we cannot identify them using this method, since our method is based on differences. This is because when we subtract observation signal values, the minimum possible value would be zero and the next number would be D, so we cannot detect mixing coefficients if this difference is less than D.

Next consider the reconstruction step. This stage is based on a search among linear combination of the mixing coefficients. So it is an important issue that there should be no repeated values in S. If S has repeated values during search we encounter more than one possible combination of source signals, so the sources cannot be recovered.

5. SIMULATION RESULTS

5.1. PAM coded signals

In this experiment we used four randomly generated PAM signals, taking values from the set $V = \{-\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}\}$ ($m = 4$) each containing 10000 samples. The mixing model is described below:

$$x(t) = a_1 s_1(t) + a_2 s_2(t) + a_3 s_3(t) + a_4 s_4(t), t = 1, ..., 10000. \quad (10)$$

Results of separation are given for three different set of mixing coefficients in Table 1. As the performance measure, we used $\text{SNR} = 10 \log(\|\mathbf{x}\|_2^2 / \|\mathbf{x} - \hat{\mathbf{x}}\|_2^2)$, where $\| \cdot \|_2$ stands for the Euclidean norm of a vector.

It can be seen that the output SNR in the third case is not acceptable, though mixing coefficients have been estimated well. This problem has occurred due to repeated values in S. For example $[s_1, s_2, s_3, s_4] = [\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}]$ and $[s_1, s_2, s_3, s_4] = [-\frac{3}{2}, \frac{3}{2}, \frac{1}{2}, \frac{1}{2}]$ both result in $x = 5$, thus if $x = 5$ we cannot use (8) for reconstruction.

Table 1. Results of separation on PAM coded signals

$a_1/\hat{a}_1$	$a_2/\hat{a}_2$	$a_3/\hat{a}_3$	$a_4/\hat{a}_4$	$\text{SNR}_1, \text{SNR}_2, \text{SNR}_3, \text{SNR}_4$
$1.22/1.18$	$1.92/1.92$	$2.32/2.36$	$0.82/0.86$	$32.21, 36.53, 37.21, 34.11$
$0.37, 0.36$	$0.63/0.61$	$1/0.99$	$2.07/2.09$	$27.32, 34.32, 30.21, 21.21$
$1/0.99$	$2/2.04$	$3/2.98$	$4/3.96$	$-15.53, -12.22, -16.87, -8.43$

5.2. Images

We do this experiment in two stages to show that quantization noise can decrease the quality of separation.

First we used two images of size 240×140 (see Fig. 1). In order to meet the conditions of the method, the original image gray levels were reduced from 256 down to 6. Next, for executing our algorithm for each image, we stacked rows next to each other to form a 1-dimensional signal. The observed image $x(t)$ was generated by the following linear mixture x quantized to levels 0, ..., 255, (see Fig. 1.b)

$$x(t) = 24.1 s_1(t) + 16.9 s_2(t) + \nu(t), \quad (11)$$

where $\nu(\cdot)$ is quantization noise (resulting from rounding $x(t)$ to an integer between 0 to 255).

We applied our method to obtain the following estimated coefficients: $\hat{a}_1 = 24, \hat{a}_2 = 17$.

The reconstructed images are shown in Fig. 1. The reconstruction has been done perfectly ($s_i(t) = \hat{s_i}(t)$ for all t). So the performance of the algorithm is perfect in this example.

Next we used three images of size 170×150 (Fig. 2). The original image gray levels were reduced from 256 down to 4. The observed image $x(t)$ was generated by the following linear mixture quantized to levels 0, ..., 255 (see Fig. 2)

$$x(t) = 35.6 s_1(t) + 19.5 s_2(t) + 29.7 s_3(t) + \nu(t). \quad (12)$$

After executing the first step we obtain the following estimated coefficients: $\hat{a}_1 = 35.5, \hat{a}_2 = 19.5, \hat{a}_3 = 30$.

The reconstructed images are shown in Fig. 3. It can be seen that in some parts of images the separation is not perfect. This situation has happened due to more gray level values in the observation signal. The observation signal has $4^3 = 64$ levels (in the range 0 to 255). So the levels are close to each other and quantization noise has resulted in the change of levels in the observation signal.

6. CONCLUSIONS

In this paper, we extended a previously introduced method for blind separation of two sources and one sensor to the general case when we have multiple sources. This method applies to finite alphabet sources. We showed necessity of some conditions for the method to work properly. We studied experimentally the performance of the algorithm on separating PAM signals and image signals. An important assumption is the knowledge of the number of the sources. One good motivation for future work is solving its weaknesses and extending it to noisy cases.

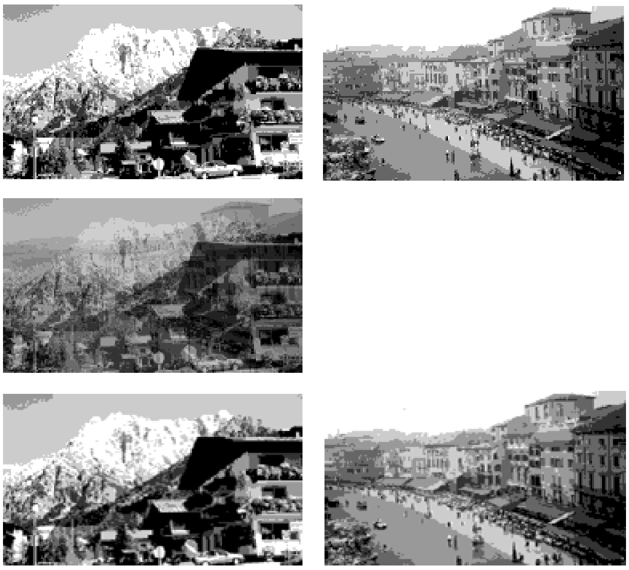

Fig. 1. Top: Original images; Middle: The observation image; Bottom: Separated images using the algorithm.

Fig. 2. The original images and observation image.

Fig. 3. Separated images using the algorithm form the observation image depicted in Fig. 2.

7. ACKNOWLEDGMENT

The authors would like acknowledge Prof. H. Soltanian-Zadeh for his helpful comments.

8. REFERENCES

[1] P. Comon and C. Jutten, *Handbook of Blind Source Separation*, Academic Press, 2010.

[2] P. Bofill and M. Zibulevsky, "Underdetermined blind source separation using sparse representations," *Signal Processing*, vol. 81, pp. 2353–2362, 2001.

[3] Y. Li, S. Amari, A. Cichocki, D.W.C. Ho, and S. Xie, "Underdetermined blind source separation based on sparse representation," *IEEE Transactions On Signal Processing*, vol. 54, no. 2, pp. 423–437, February 2006.

[4] A. Aissa-El-Bey, L.-T. Nguyen, K. Abed-Meraim, A. Belouchrani, and Y. Grenier, "Underdetermined blind separation of nondisjoint sources in the time frequency domain," *IEEE Transactions On Signal Processing*, vol. 55, no. 3, pp. 897–907, March 2007.

[5] M.S. Lewicki and T.J. Sejnowski, "Learning overcomplete representations," *Neural Computation*, vol. 12, no. 2, pp. 337–365, February 2000.

[6] T.W. Lee, M.S. Lewicki, M. Girolami, and T.J. Sejnowski, "Blind source separation of more sources than mixtures using overcomplete representations," *IEEE Signal Processing Letters*, vol. 4, no. 4, pp. 87–90, April 1999.

[7] Y. Li, A. Cichocki, and L. Zhang, "Blind separation and extraction of binary sources," *IEICE Transactions on Fundamentals*, vol. E86-A, no. 3, pp. 580–589, March 2003.

[8] K. I. Diamantaras, "Blind separation of two multi-level sources from a single linear mixture," in *18th IEEE Int. Workshop on Machine Learning for Signal Processing (MLSP-2008)*, Cancun, Mexico, October 2008, pp. 93–98.